

## Structural Behaviour of Origami Inspired Folded Shell Surface

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**Abstract:** Paper folding art provides rich idea for application in shell and spatial structures. The combination of shell surface and folded straight lines is an excellent example of source of idea for shell and spatial structure. Such structural form combines the advantage of superior load carrying capacity of shell structure and aesthetical value of created by the folds. The structural behaviour of shell surface with curved folds line (SSCFL) originated previous researcher is presented in this study. Two types of curve with four parameters affecting the surfaces have been investigated. The procedures for plan configuration and surface measurement have been proposed. The best model in term of aesthetic value from the folded surface models has been chosen for analysis under self-weight condition. Highest deflection has been found to occur at the centre of the surface with the lowest deflection observed in the case of fixed support. The in-plane tensile stresses are found to have exceeded the limit. However, the compressive and out-of-plane shear stresses have been found to be within the limit of the material used. High stresses have been found to occur around the curved folds and at the convergent points of the curved folds of the models. The upward and downward curved folds of the surface experience tension and compression, respectively. The ground and top supports experienced the highest value of shear stress.

**Key words:** Shell surface • Folded plate • Origami • Curved fold line

### INTRODUCTION

Shell structures has contributed huge impact to the engineering field and became landmark in many countries. The capability to carry load by combining stretching and bending effects make the shell structure among the best selection for roof systems [1]. The variation of thickness from thin to thick shell governed the effect of bending and the aesthetical value of the shell. On the other side, folded plate structure also became one of the best choices for the roofing systems. It is well known that the load carrying capacity, stiffness and the strength of a flat and thin structure increased if the surface is folded. The span-to-depth ratio will determine the effectiveness of the folded plate structure towards the load carrying capacity. That is the reasons many types of folded plate have been studied, analysed and built [2]. Folded plates can also be found in concrete stairs, folded lampshades and lanterns, corrugated sheet pile and others. The richness of the

curvature of shell surface and geometries of folded plates were commonly found in the ancient Japanese and Chinese art of paper folding or *origami*. The combination of the curve and folding could provide an amazing aesthetical structural form or art especially when the vertices of the folding paper join at several folds [3]. Many types of surface structures have been inspired from the ability and flexibility of origami to create various forms and shapes.

Intensive studies on shell surface and folded plate have been carried out throughout the years. Nevertheless the work on origami, the folding method and the creation of complex form have been enhanced and improved from time to time. However, the studies on the combination of those three fields for structural use purpose are still lacking. Recently, Buri *et al.* [4] have proposed new methods to generate rapidly complex folded plate structures that can be built with cross laminated timber panels. A. Razzack [5] conducted a pioneer study on folded shell surface found in nature.

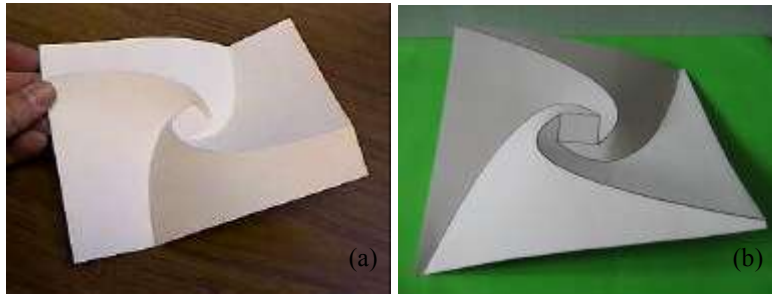


Fig. 1: (a) A square shape of SSCFL produce by Huffman (b) A square shape of SSCFL used in this study

Huffman [6] has created a beautiful shell surface with curve fold lines (SSCFL) by using paper. Inspired by his work, MD Resadi [7] has studied the constructability by constructing a small-scale ellipse shapes physical model. The data from Harun [8] has been used in that study. By using the same type of SSCFL as the previous authors, Rohim *et al.* [9] has proposed the rapid generation of SSCFL and Ng [10] has studied the effect of folds on structural behaviour of origami by changing the curvature of the model.

Rohim *et al.* [11] also carried out the systematic generation of square shape type of SSCFL. However, the generation of data for elevation has not been discussed. The objective of this paper is to enhance the procedures proposed by studying the structural behaviour of square shape SSCFL by using the procedures proposed by Rohim *et al.* [9, 11] earlier. The elevation data of the model must be made available in order to create 3D model of SSCFL.

### Shell Surface with Curve Fold Lines (SSCFL)

**Creation of 2D Data:** The idea of SSCFL created by the folding paper art of Huffman [6] is shown in Figure 1a. Figure 1b shows the model that has been folded and used in this study.

In order to produce the folded shape as presented by Huffman [6], the parameters affecting the surface form are first identified. By studying the model created by Huffman [6], the shape of folded curves is assumed to be either from parabolic or elliptical curve. Starting from the above assumption, the generation of plan configuration of the models has been conducted. Three parameters have been identified for each type of curve. Each model has a small square located at the centre of the model with a dimension of  $b$  by connecting points  $A1$ ,  $A2$ ,  $A3$  and  $A4$ . The outer parameter for both models has been taken as  $B$ .

For the parabolic curve, the identified parameters are  $b$ ,  $d$  and  $h$  as shown in Figure 2.  $P1$ ,  $P2$  and  $P3$  are the three points that are required to form a parabolic curve

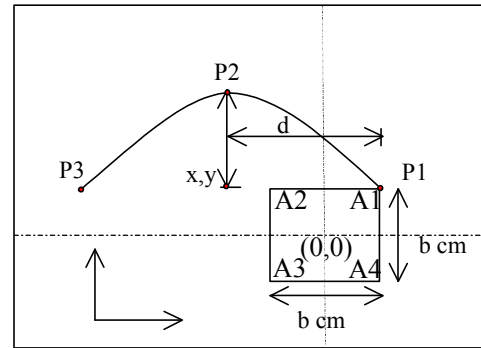


Fig. 2: Construction of centre square and first parabolic curve

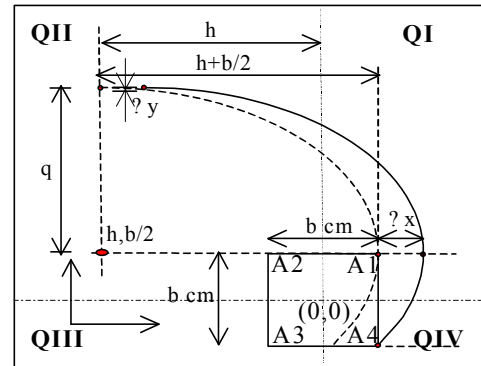


Fig. 3: Construction of centre square and first ellipse curve

from the parabolic general equation  $y = ax^2 + bx + c$ . The curve must start in quadrant one (QI) with point  $P1$  and end up at the edge of quadrant three (QIII) after passing through point  $P3$ . Parameter  $d$  is to control the width of the parabolic curve opening while  $h$  is to control the peak of the parabolic curve.

For the elliptical curve, the parameters identified are  $b$ ,  $h$  and  $q$  as shown in Figure 3.  $h$  is the distance from the centre point (0,0) and  $q$  is the distance from the edge of small square. Both  $h+b/2$  and  $q$  are the values for major and the minor axis to form the elliptical curve by using equation of ellipse as shown in Equation (1).





Fig. 5: Variation of folded model



Fig. 6: SSCFL-E8-the selected model

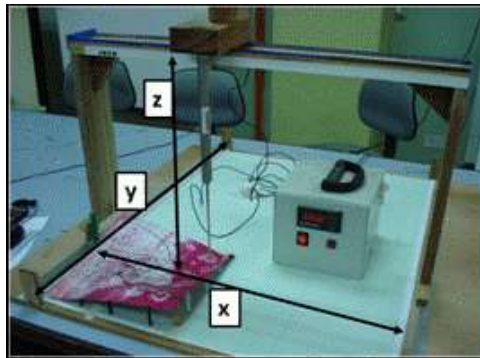


Fig. 7: Fabricated measuring device

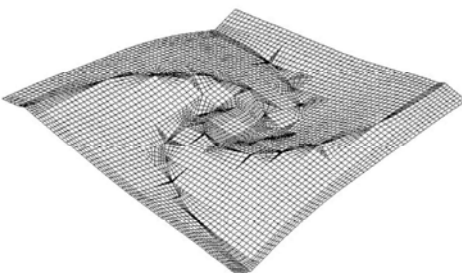


Fig. 8: Generation of spatial model

model. Figure 5 shows the folded model with parabolic and elliptical curves. Model number 8 with elliptical curve (SSCFL-E8) with ( $B = 30$  unit length,

$b = 4$  unit length,  $h = 19$  unit length,  $q = 10$  unit length) has been chosen as the best model as shown in Figure 6.

The model then marked with a grid of 2 units in  $x$  and  $y$  direction on its surface. Special fabricated device has been used to record the surface data of  $x$ ,  $y$  and  $z$  coordinate of the SSCFL-E8 as shown in Figure 7. The obtained data has been transferred into the finite element software, ADINA[12] to create a spatial model for the purpose of analysis as in Figure 8.

The type of element used for the finite element analysis is shell element. Three different thicknesses have been used to determine the effect of thickness on the behaviour of shell structures; they are 200 mm, 350 mm and 500 mm. Material used for the model is concrete of grade C30. Analysis has been carried out under self-weight condition only. The material properties such as Young's Modulus, density and Poisson ratio used are  $E = 24.0 \times 10^9$  N/m<sup>2</sup>,  $\rho = 2.40 \times 10^3$  Kg/m<sup>3</sup> and  $\nu = 0.2$ , respectively. The support fixity used were pinned and fixed support for each case of surface thickness.

## RESULTS AND DISCUSSION

From the analysis results, it can be seen that surface thickness and fixity of support affect the deflection of the model as summarized in Table 2. The deflection increases with decreasing thickness of the model. Models with pinned supports produce a higher deflection compared to that of fixed supports.

For all the models, the highest deflection has been found to occur at the centre of all the models. The maximum deflections of all the models are within the limit stated in BS8110 [13]. The presence of curved folds on the model has been found to have the effect of stiffening and changing the load distribution of the model depending on the shape of the curve folds.

Table 3 and 4 show summary of the maximum value of stress in the local  $rr$ -direction,  $\sigma_{rr}$ , local  $ss$ -direction,  $\sigma_{ss}$ , shear stress in the  $rt$ -direction,  $\sigma_{rt}$  and  $st$ -direction,  $\sigma_{st}$ , respectively.

For all models, the section of the model with curved folds show greater in-plane and out-of-plane stresses compared to sections on the model without curved folds. The section on the model with upward curved folds experiences tension, while the section with downward curved folds experiences compression

The convergence of the curves at the centre square of the model analysed also results in the compression of this region. The compressive stress has been found to be

Table 2: Maximum deflection for each of the models

Model	Thickness (mm)	Support Fixity	Maximum Deflection (mm)
1	500	Fixed	0.740
2	500	Pinned	0.838
3	350	Fixed	0.876
4	350	Pinned	0.989
5	200	Fixed	1.668
6	200	Pinned	1.925

Table 3: Maximum positive and negative shear stress for  $\sigma_{xy}$  and  $\sigma_{yz}$ 

Model	Support Fixity	Thickness (mm)	$\sigma_{xy}$ (kN/m <sup>2</sup> )		$\sigma_{yz}$ (kN/m <sup>2</sup> )	
			Positive	Negative	Positive	Negative
1	Fixed	500	5960	1766	10205	1951
2	Pinned	500	2939	4723	7405	4296
3	Fixed	350	6954	2711	11591	2107
4	Pinned	350	3597	5075	8633	4438
5	Fixed	200	8959	14786	14786	2359
6	Pinned	200	4854	5694	11462	4697

Table 4: Maximum tensile and compressive stress for  $\sigma_{xx}$  and  $\sigma_{yy}$ 

Model	Support Fixity	Thickness (mm)	$\sigma_{xx}$ (kN/m <sup>2</sup> )		$\sigma_{yy}$ (kN/m <sup>2</sup> )	
			Positive	Negative	Positive	Negative
1	Fixed	500	1319	1127	1062	913
2	Pinned	500	1686	1252	1784	1893
3	Fixed	350	1433	1245	1056	845
4	Pinned	350	1619	1161	2075	1838
5	Fixed	200	1754	1509	1045	798
6	Pinned	200	1606	1222	3260	2889

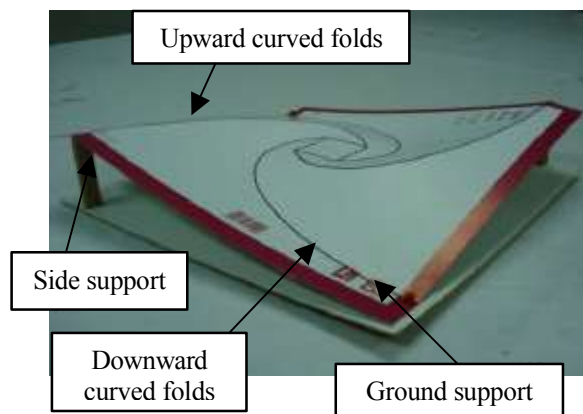


Fig. 9: Curved folds and supports of the model.

within the limit of the material used. As for tensile stress, the stress limit has been found to have exceeded the limit for Model 2.

From the stress contours obtained, out-of-plane shear stress has been found to be concentrated near the support of the models. Maximum and minimum shear stress occur at two of the different supports as the model

consists of two side supports and two ground supports as shown in Figure 9. The ground supports experience negative shear stress while the side supports experience positive shear stress. The shear stress of the model is within the shear stress limits stated in BS8110 [13].

## CONCLUSION

This study has been carried out to investigate the effects of the configuration of shell surface with curve fold lines (SSCFL) on its load carrying characteristics under self-weight condition. The effects of the thickness of the shell surface and support fixity have also been investigated. The model with elliptical curves and parameters  $B=30$  cm,  $b=4$  cm,  $d=19$  cm,  $h=10$  cm has been selected as the model with the best aesthetical value.

Based on the analysis results, the highest deflection has been found to occur at the centre of the models. It is found that a thicker surface experiences a lower deflection because higher depth reduces the surface's deflection. However, a thinner surface shows higher values of in-plane stresses.

Models with fixed support show lower value of deflection compared to those with pinned support. However models with fixed support exhibits higher value of in-plane tensile stresses but lower in-plane compressive stresses compared to pinned support which exhibits lower value of in-plane tensile stresses and but higher in-plane compressive stresses.

The stresses are more concentrated on the curved portions of the model. The upward and downward curved folds experience tensile and compressive stress respectively. The ground and side supports experience the highest value of shear stress.

It can be concluded that the model is able to carry its self-weight with minimal deflection. The introduction of tension reinforcement into the design of the shell surface will be able to resist the tensile stress of the surface. The measuring device fabricated has been found to be able to yield sufficiently accurate surface data for the modelling process of SSCFL.

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