World Applied Sciences Journal 24 (3): 364-369, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.24.03.932

Heat and Mass Transfer over an Isothermal Inclined Plate at Constant Concentration Gradient and with Heat Source

Hitesh Kumar
Department of Information Technology,
IBRI College of Technology, Ibri, P.O. Box: 466, Oman

Submitted: Jul 3, 2013; **Accepted:** Aug 10, 2013; **Published:** Aug 25, 2013

Abstract: In this paper a numerical solution of steady laminar flow of viscous electrically conducting incompressible fluid, over a semi-infinite inclined plate, which is at prescribed mass flux with heat generation is presented. The numerical solutions using an implicit finite difference scheme known as Keller-box method for velocity, concentration and temperature are found. The effects of various parameters like Schmidt number, magnetic field, heat source, Prandtl number and angle of inclination on the velocity, temperature and concentration, skin friction coefficient and Nusselt number are presented in graphical or tabular form.

AMS Classification: 76D10, 76R50, 80A20
Key words: Heat transfer · Mass transfer · Heat generation · Concentration · Inclined wall

INTRODUCTION

It is well known that natural convection heat transfer occurs as a result of temperature differences in an enclosure or near a heated or cooled flat plate. Natural convection along an inclined plate has received less attention than the cases of vertical and horizontal plates. However, this configuration is frequently encountered in engineering devices and in the natural environment. A number of researchers have considered an inclined, semi-infinite flat plate in their research because of its engineering applications. The study of natural convection flow of inclined plate is presented by the authors [Ganesan and Palani [1, 2] and Sparrow and Husar [3]]. Said *et al*. [4] studied the problem of turbulent natural convection between inclined isothermal plates. Chen [5] presented the analysis to study natural convection flow over a permeable inclined surface with variable wall temperature and concentration. Hossain *et al*. [6] studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Anghel *et al*. [7] presented a numerical solution of free convection flow past an inclined surface. Bhuvaneswari *et al*. [8] studied exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium. Sivasankaran *et al*. [9] presented a Lie group analysis of natural convection heat and mass transfer in an inclined surface. Shit and Haldar [10] studied MHD flow, heat and mass transfer over an inclined permeable stretching sheet with thermal radiation and Hall Effect.

Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, etc. The study of the heat generation or absorption in moving fluids is important with dissociating fluids. Specifically, the effects of heat generation may alter the temperature distribution, consequently affecting the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. In fact, the literature is replete with examples of heat transfer in the laminar flow of viscous fluids. For instance, Vajravelu and Hadjinicolaou [11] studied heat transfer characteristics in the laminar boundary-layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Kumar [12] investigated heat transfer over a stretching porous sheet subjected to power law heat flux in the presence of a heat source.

Corresponding Author: Hitesh Kumar, Department of Information Technology,
IBRI College of Technology, Ibri, P.O. Box: 466, Oman.

In the present work, heat and mass transfer over an isothermal inclined plate which is at a constant concentration gradient in the presence of heat generation is studied. The boundary layer equations are solved numerically using an implicit finite-difference scheme which is the Keller-box method [13, 14]. The effects of Schmidt number, Prandtl Number, heat generation parameter, magnetic field and angle of inclination is studied on velocity, concentration, temperature field, skin friction coefficient and Nusselt number are presented.

MATERIALS AND METHODS

A steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined wall with an acute angle ϕ from the vertical and at a constant concentration gradient is considered here. The wall is at a constant temperature T_w which is higher than the ambient temperature T_∞ and at prescribed mass flux. The flow is assumed to be in the x - direction, which is taken along the semi-infinite inclined plate and y - axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant. Then, under the usual Boussinesq's and boundary layer approximations, the governing equations of the mass, momentum, energy and concentration for the steady flow can be written as,

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} + g\,\beta_T\,(T - T_\infty)\cos\phi + g\,\beta_C\,(c - c_\infty)\cos\phi - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho\, c_p}\left(T - T_\infty\right) \tag{3}$$

$$v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} \tag{4}$$

with boundary conditions

$$u = 0,\ v = v_w,\ T = T_w,\ -D\frac{\partial c}{\partial y} = m_w,\ \text{at } y = 0$$

$$u \to 0,\ T \to T_\infty,\ c \to c_\infty,\ \text{as } y \to \infty \tag{5}$$

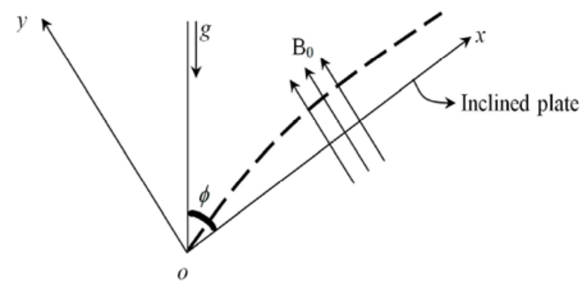

Fig. 1: Physical Model and Coordinate System

The equation of continuity (1) with boundary condition (5) changes to:

$$v = -\,v_w \tag{6}$$

On assuming the non-dimensional variables as follows:

$$Y = \frac{y v_w}{\vartheta},\ U = \frac{u}{u_w},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ C = \frac{c - c_\infty}{\left(m_w\vartheta/v_w D\right)},$$

$$Gr_T = \frac{g\beta_T(T_w - T_\infty)\vartheta}{u_w v_w^2},\ Gr_c = \frac{g\beta_c\, m_w\,\vartheta^2}{u_w v_w^3\, D},\ M^2 = \frac{B_0^2\vartheta\sigma}{v_w^2\,\rho},$$

$$\Pr = \frac{\rho\vartheta c_p}{k} = \frac{\vartheta}{\alpha},\ S = \frac{Q\,\vartheta}{\rho\, c_p\, v_w^2},\ Sc = \frac{\vartheta}{D}.$$

On using the above non-dimensional parameters, the equations (2) to (4) reduces to:

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} + Gr_T\,\theta\cos\phi + Gr_c\,C\cos\phi - M^2 U = 0 \tag{7}$$

$$\frac{d^2\theta}{dY^2} + \Pr\frac{d\theta}{dY} + \Pr S\,\theta = 0 \tag{8}$$

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} = 0 \tag{9}$$

With the corresponding boundary conditions,

$$U = 0,\ \ \theta = 1,\ \ \frac{\partial C}{\partial Y} = -1,\ \ at\ \ Y = 0$$

$$U \to 0,\ \ \theta \to 0,\ \ C \to 0,\ \ as\ \ Y \to \infty \tag{10}$$

Wall shear stress,

$$\tau_f = \mu\left(\frac{du}{dy}\right)_{y=0}$$

and the skin friction coefficient is defined as,

$$C_f = \frac{\tau_f}{\rho u_w v_w} = \left(\frac{dU}{dY}\right)_{Y=0} \tag{14}$$

The effect of convection on the wall heat transfer rate,

$$q_w = -k\left(\frac{dT}{dy}\right)_{y=0}$$

and the Nusselt number is defined as,

$$Nu = \frac{q_w \vartheta}{(T_w - T_\infty) k v_w} = -\left(\frac{d\theta}{dY}\right)_{Y=0} \tag{15}$$

RESULT AND DISCUSSION

Eqs. (7), (8), (9), (14) and (15) subject to the boundary conditions (10) are solved numerically using the Keller-box method as described by Cebeci and Bradshaw [13, 14]. A study of the velocity field, temperature field, mass transfer, Nusselt number and skin friction of the steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined wall with constant concentration gradient, has been carried out in the present research. Presented results are verified and found in good agreement as that of Kandasamy and Devi [15] and Ramadan and Chamkha [16].

The effects of Sc on concentration are presented in Fig. 2. Concentration decreases as Sc increases. The reductions in the concentration profiles are accompanied by simultaneous reductions in the concentration boundary layer thickness. Physically, the increase of Sc means decrease of molecular diffusivity (D). That results in decrease of concentration boundary layer. Hence, the concentration of the species is higher for small values of Sc and lower for larger values of Sc.

From Figure 3 it is noticed that, an increase in the magnetic field parameter M or angle of inclination (ϕ) leads to a decrease in the velocity. The application of a transverse magnetic field to an electrically conducting fluid rise to a resistive type force called Lorentz force. This force has the tendency to slow down the motion of the fluid. Velocity attains its high peak for vertical plate, the fluid has higher velocity when the surface is vertical

Table 1: Skin friction coefficient for different values of Pr, Sc, M and ϕ when S=0.1, Gr_T = 1.0 and Gr_c = 0.5

	C_f					
Parameters	-2.57625	-2.71239	-3.05917	-0.4505	-2.98445	-3.14781
Pr	1	5	1	1	1	1
Sc	0.22	0.22	0.6	0.6	0.6	0.6
M	3	3	3	1	3	3
ϕ	30°	30°	30°	30°	0°	45°

Table 2: Nusselt Number for different values of Pr and S

	N_u		
Parameters	0.887298	4.89792	0.723607
Pr	1	5	1
S	0.1	0.1	0.2

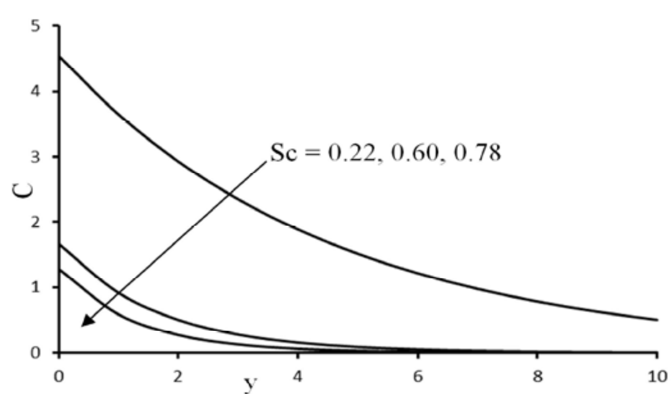

Fig. 2: Schmidt number effects on concentration when Pr = 0.71, S = 0.1, M = 3.0, Gr_T = 1.0, Gr_c = 0.5 and ϕ = 30°

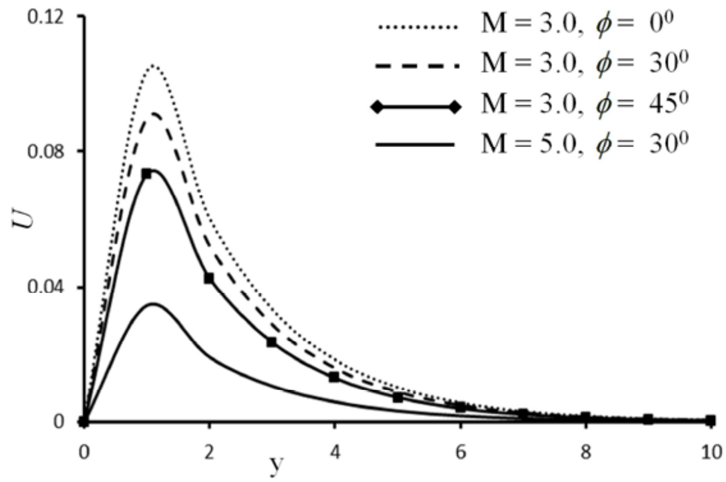

Fig. 3: The effects of magnetic field and angle of inclination on velocity when Sc = 0.6, Pr = 0.71, S = 0.1, Gr_T = 1.0 and Gr_c = 0.5

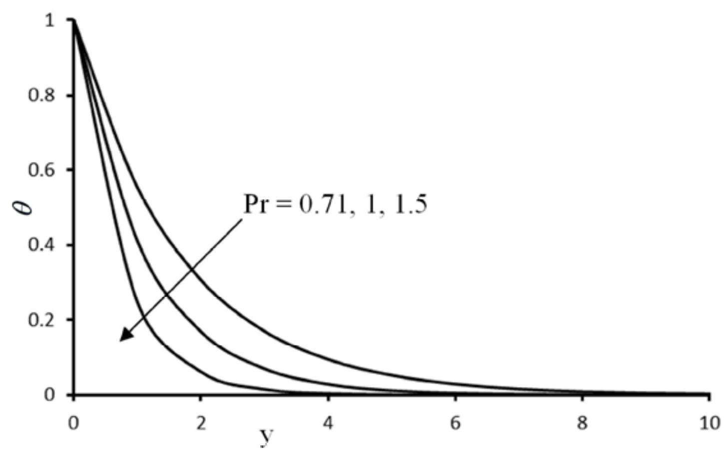

Fig. 4: The Prandtl Number effect on temperature when Sc = 0.6, S = 0.1, M = 3.0, Gr_T = 1.0, Gr_c = 0.5 and ϕ = 30°

than when inclined because of the fact that the buoyancy effect decreases due to gravity components (g cos ϕ) as the plate is inclined. It is also interesting to note that the velocity has very good and clear effects of the range 0° – 45°, (please refer [16]).

Figure 4 shows the effects of Pr on temperature. The increase of Pr results in the decrease of temperature distribution, it is because of the fact that fluid has a thinner thermal boundary-layer with higher values of Pr.

Table 1 and Table 2 represents the coefficients of skin friction and Nusselt number. It has been observed that C_f decreases as Pr or Sc or M or ϕ increases. Nusselt number increases with Pr whereas it decrease as S increases.

CONCLUSION

In the present research, the effects of Schmidt number, Prandtl number, heat generation parameter, magnetic field parameter, thermal Grashof number, solutal Grashof number and angle of inclination on velocity, concentration and temperature distributions are obtained which can be interpreted as follows:

The velocity decreases with an increase in magnetic parameter or angle of inclination, concentration decreases with the increase in Schmidt number. The Prandtl number reduces the thermal boundary layer. Shear stress decreases as Prandtl number or Schmidt number or magnetic parameter or angle of inclination increases and Nusselt number increases with Prandtl number and decrease as the heat source parameter increases.

Nomenclature:

y horizontal coordinate	(m)	c_p specific heat	(J Kg^{-1} K^{-1})
u axial velocity	(m/s)	Q heat generation coefficient	(W m^{-3} K^{-1})
v transverse velocity	(m/s)	ρ density	(kg/m^3)
T temperature of the fluid	(K)	D mass diffusion coefficient	(m^2 s^{-1})
T_∞ far field temperature	(K)	m_w wall mass flux	(mol/m^2s)
c species concentration	(mol/m^3)	u_w surface velocity	(m/s)
c_∞ far field concentration	(mol/m^3)	v_w suction velocity	(m/s)
c_w concentration on the surface	(mol/m^3)	Gr_T thermal Grashof number	
g acceleration due to gravity	(m/s^2)	Gr_c solutal Grashof number	
β_T coefficient of thermal expansion	(K^{-1})	M^2 magnetic field parameter,	
β_c coefficient of concentration expansion	(m^3/mol)	Pr Prandtl number,	
ϕ angle of inclination	(degree)	N radiation parameter,	
ϑ kinematic viscosity	(m^2/s)	S heat generation parameter	
σ electrical conductivity	(S/m)	Sc Schmidt number	
B_0 magnetic field coefficient	(T)	Y dimensionless horizontal coordinate	
α thermal diffusivity	(m^2/s)	U dimensionless axial velocity	
		θ dimensionless temperature	
		C dimensionless species concentration	

ACKNOWLEDGMENTS

The author is very much thankful to Prof. (Dr.) S.S. Tak, Jai Narain Vyas University, Jodhpur (India) for offering his valuable suggestions and assistance to improve the paper.

REFERENCES

1. Ganesan, P. and G. Palani, 2004. Finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux. Int. J. Heat Mass Transfer, 47: 4449-4457.
2. Ganesan, P. and G. Palani, 2003. Natural convection effects on impulsively started inclined plate with heat and mass transfer. Heat and Mass Transfer, 39: 277-283.
3. Sparrow, E.M. and R.B. Husar, 1969. Longitudinal vortices in natural convection flow on inclined plates. J. Fluid. Mech., 37: 251-255.
4. Said, S.A. M., M.A. Habib, H.M. Badr and S. Anwar, 2005. Turbulent natural convection between inclined isothermal plates. Computers & Fluid, 34: 1025-1039.
5. Chen, C.H., 2004. Heat and mass transfer in MHD flow with variable wall temperature and concentration. Acta Mechanica, 172: 219-235.
6. Hossain, M. A., I. Pop and M. Ahmad, 1996. MHD free convection flow from an isothermal plate. J. Theo. and Appl. Mech., 1: 194-207.
7. Anghel, M., M.A. Hossain and S. Zeb, 2001. Combined heat and mass transfer by free convection past an inclined flat plate. Int. J. Appl. Mech. and Engg., 2: 473-497.
8. Bhuvaneswari, M., S. Sivasankaran and Y.J. Kim, 2010. Exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium. World Applied Journal, 10: 774-778.
9. Sivasankaran, S., M. Bhuvaneswari, P. Kandaswamy and E.K. Ramasami, 2006. Lie Group Analysis of Natural Convection Heat and Mass Transfer in an Inclined Surface. Nonlinear Analysis: Modelling and Control, 11: 201-212.
10. Shit, G.C. and R. Haldar, 2011. Thermal radiation and Hall Effect on MHD flow, heat and mass transfer over an inclined permeable stretching sheet. Thermal Science, 15: S195-S204.
11. Vajravelu, K. and A. Hadjinicolaou, 1993. Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Int. Comm. Heat Mass Transfer, 20: 417-430.
12. Kumar, H., 2011. Heat transfer over a stretching porous sheet subjected to power law heat flux in presence of heat source. Thermal Sci., 15: S187-S194.
13. Cebeci, T. and P. Bradshaw, 1977. Momentum transfer in boundary layers. Hemisphere, New York, USA.

14. Cebeci, T. and P. Bradshaw, 1988. Physical and computational aspects of convective heat transfer. Springer-Verlag, New York, USA.
15. Kandasamy, R. and S.P.A. Devi, 2004. Effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection. Journal of Computational and Applied Mechanics, 5: 21-31.
16. Ramadan, H.M. and A.J. Chamkha, 1999. Two-phase free convection flow over an infinite permeable inclined plate with non-uniform particle-phase density. International Journal of Engineering Science, 37: 1351-1367.