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Chinese Bond Market: A Need for Sound Estimation of Term Structure Interest Rates

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Abstract: Prior to Asian economic crisis in 1997, bond markets in lots of Asian countries were small and relatively undeveloped. The Asian financial crisis demonstrated the need for a broader range of funding sources, which led the governments of Asian countries to embark on a major program to develop a local bond market. China avoided this crisis due to the special regulation and the strict rules for assets movement in and out of the country. Nowadays the Chinese local bond market is the most rapidly growing in Asia. In this article we discuss than the zero-coupon yield curve estimation method used by the Chinese government, apply it to bond trading data, compare it with the most popular methods and with a novel non-parametric zero-coupon yield curve estimation method developed by the authors.

Key words: China · Bond market · Zero-coupon yield curve · Splines

INTRODUCTION

In finance, the yield curve is a curve showing several yields or interest rates across different contract lengths for a similar debt contract. The curve shows the relation between the (level of) interest rate and the time to maturity, known as the "term", of the debt for a given borrower in a given currency. More formal mathematical descriptions of this relation are often called the term structure of interest rates.

Yield curves are used by fixed income analysts, who analyze bonds and related securities, to understand the state of financial market and to seek trading opportunities. Economists use the curves to understand economic conditions and forward rates to forecast behavior of interest rates in the future.

There are several widely used techniques to strip zero-coupon yields from prices of coupon-bearing bonds. They might be divided into three large groups.

Naïve Methods: These methods yield a more or less acceptable answer, but the reasoning behind the procedures employed is very weak or even absent. These include bootstrapping [1], using a similar bond to determine the yield (without any yield curve at all), using yield-to-maturity curve instead of zero-coupon curve and kernel estimators [2].

Parametric Methods: These methods start from a supposition of a specific parametric form of the zero-coupon yield curve (or of the spot forward rate curve). The equation employed might originate from economic reasoning or be just a useful expression. Although parametric approach had been used long before, it seems that as a consistent paradigm it was first described in [3]. Widely used Nelson-Siegel [4] and Svensson [5] methods might serve as examples. The number of parameters does not necessarily have to be low. For example, [6] and [7] propose parametric models with arbitrary number of parameters. Strictly speaking, some spline methods can also be attributed to this category, although a discriminating line may be drawn between the two (see below).

Spline Methods: Quadratic splines were used in yield curve construction in [8], cubic in [9; 10], B-splines were employed in [11] and exponential splines in [12]. Smoothness was addressed in [13] and [14]. We would also like to point out the work [15] as our construction builds heavily upon it.

The distinction between spline methods and parametric methods in our opinion lies in the reasoning behind the formulae. If the main supposition is the extreme property to be satisfied by the curve, then it is a spline method (one usually gets splines solving such problems).

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And if, on the contrary, spline form is postulated "just because" it is a clear example of a parametric method with the parametric form being the spline equation.

Parametric methods offer one major advantage – that is simplicity – while suffering from several drawbacks: possibility of economically absurd results (negative interest rates or increasing discount functions), inconsistent results for insufficient date, numerical instability. Spline methods (and non-parametric methods in general), on the other hand, suffer from high complexity of models and calculations, difficulty of estimating the model and doubtable economic interpretation of the model and its results.

The main problem with the zero-coupon yield curve estimation is with the data. If the data is good, i.e. consistent, not noisy and sufficient, then any reasonable method will succeed in constructing an acceptable zero-coupon yield curve. This is the case, for example, for US Treasuries. On the contrary, developing markets often exhibit poor data quality: scarce, incomplete and noisy/unreliable data. A yield curve construction method must be specially designed to cope with these problems in order to succeed. Chinese bond markets provide clear examples of such unfriendly environment for modeling.

The CCDC yield curve is considered to be the benchmark of the government term structure of interest rates. The method itself consists in several stages. At the first stage input data is filtered. Bond yields, either quoted or from deals, which lie far from the last day's and cannot be explained by the financial variation or the relational economic policy on that day then the zero-coupon yield curve are excluded. This adds some robustness to the method. At the second stage trading data is augmented with expert estimates and sometimes with historical data. Next, key yields are chosen for the interpolation. And finally, Hermit spline interpolation is used to form the entire yield curve.

The official CCDC yield curve is updated every day and it is available online at http:// www.chinabond.com.cn/ Site/cb/en. We use the interbank market bond transaction prices supplied by CCDC (available on their web site). Here we give an example of CCDC method to get the yield to maturity curve and spot rates yield curve [16] (Figure 1):

If we do only one or two small changes in choosing key yields for interpolation, it will result in a totally different yield curve (Figure 2).

So the yield curve is quite sensitive to the choice of the key yields and this choice has to be made by experts.

We have fitted the zero coupon yield curve using three most widespread methods: Nelson-Siegel [5], Svensson [6] and penalized cubic splines [13]. We also report the fitting results with sinusoidal-exponential splines [14,17].

Zero-Coupon Yield Curve Construction: A plausible zero-coupon yield curve has to possess several properties in order to be acceptable. It has to:

- Exclude arbitrage opportunities. That means the spot forward rates should be positive and the discount function decreasing.
- Approximate the real data with a sufficient degree of precision. There is no sense in approximating quotes up to the fifth digit and different instruments might possibly require different degrees of approximation precision. We argue that the bid-ask spread is a good measure of the necessary precision.

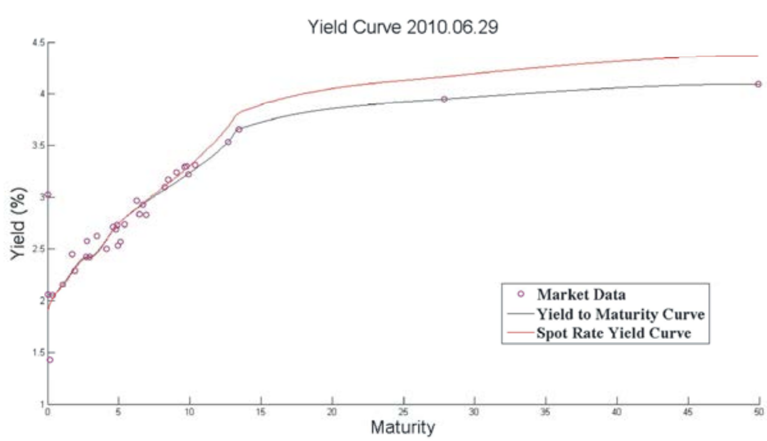

Fig. 1: YTM curve and Spot rate curve on 29th June, 2010.

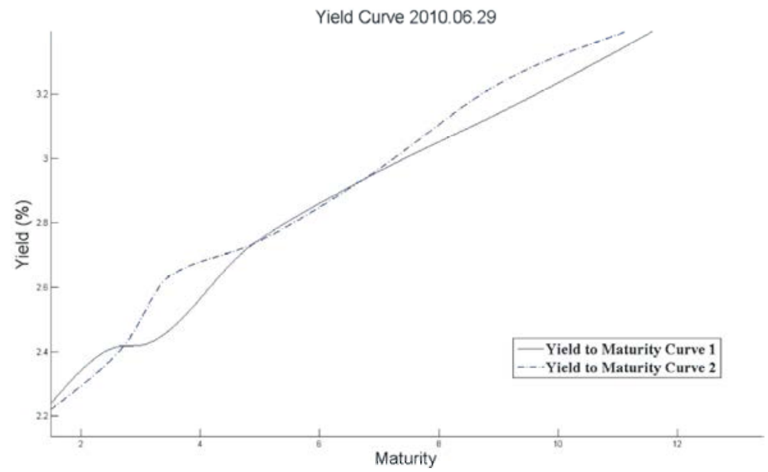

Fig. 2:

- Be sufficiently smooth. It is easy to construct an interest rate term structure to fit the given prices exactly (i.e. via bootstrapping). However, the resulting curve is usually awfully shaped and bears no economic sense. Investors' expectations are usually continuous in time, which means that the spot forward rates should be continuous. Moreover, additional degrees of smoothness usually take place (i.e. piecewise differentiable forward rates).

Note that it is essential that the data approximated be from the price domain and not from the yield domain. Yield to maturity for an instrument is calculated with the assumption of flat interest rate term structure (and this flat level, certainly, varies from instrument to instrument since it is the yield to maturity itself). So yield to maturity values for different instruments are calculated within different assumptions (and therefore within different models). And uniting them within the same model and fitting with a single non-flat yield curve is methodologically incorrect since these quantities were not meant to be replicated.

A correct yield-based approach would consist in a two-step objective function: first, obtain theoretical bond prices and then recalculate theoretical yields to maturity using theoretical prices; then theoretical yields should be compared to the actually observed yields (that is, calculated from the actually observed prices). This procedure is rather complicated and a price-based approach seems more appropriate while being equally sound economically.

We now describe our construction to deal with all these items. First, we set up some useful notation. We denote the spot zero-coupon yield for the term t as $r(t)$, the instantaneous forward rate for the time t as $f(t)$ and the discount factor for the time t as $d(t)$. We also use continuous compounding, which implies that $r(t) = \frac{1}{t}\int_0^t f(\tau)d\tau$ and $d(t) = \exp(-r(t)t) = exp\left(-\int_0^t f(\tau)d\tau\right)$.

We also suppose that there exist N bonds on the market, each promising payments in times $t_1,...,t_n$ from the present moment. $F_{i,k}$ is the cash flow amount for the k-th bond at time t_i. In order to make cash flow times t_i universal for all bonds, we introduce zero cash flows where necessary. The bid and ask quotes for the k-th bond are b_k and a_k respectively. We also suppose that all bonds possess the same credit quality the bear approximately equal liquidity risk. This means that a bond may be viewed and priced as a portfolio of bullet payments: $P_k \approx \sum_{i=1}^{n} F_{i,k} d(t_i)$.

- To ensure positive spot forward rates we let $f(t) = g^2(t)$, where $f(t)$ is the spot forward rate for time t. All subsequent calculations will be done in terms of $g(.)$ instead of spot forward rates $f(.)$.
- To approximate the data with a reasonable degree of precision we formulate our objective as minimizing the weighted residual functional: $J_2(g) = \sum_{k=1}^{N}\left[\frac{1}{a_k - b_k}\left(q_k(f) - P_k\right)\right]^2$, where a_k, b_k are the ask and bid quotes for the k-th bond respectively, $q_k = \sum_{i=1}^{n} F_{i,k} \exp\left[-\int_0^{t_i} g^2(\tau)d\tau\right]$ is the model price of the k-th bond, calculated as the sum of the future cash flows discounted via given interest rates term structure and P_k is the quoted bond price.

Since we have only bid and ask quotes, the effective price is best represented by the mid-price $P_k = \frac{1}{2}(a_k + b_k)$.

- To ensure that the spot forward rates be smooth we add a second objective functional measuring the spot forward rates non-smoothness: $J_1(g) = \int_0^T g'(\tau)^2 d\tau$. The overall objective functional to be minimized is $J(g) = \alpha J_1(g) + J_2(g)$, where α is the regularization parameter governing the desired interplay between precision and smoothness. One may even assign economic meaning to this parameter and propose meaningful procedures of choosing its numerical value, but this will be the subject of a subsequent research.

We end up with the following problem: given bid and ask quotes for N bonds a_k, b_k and given promised cash flow times t_i and amounts $F_{i,k}$ for each bond, find a function $g(.)$ to minimize the following functional.

$$J(g) = \alpha \int_0^T g'(\tau)^2 d\tau + \sum_{k=1}^{N} \frac{1}{a_k - b_k} \left[\left(\sum_{i=1}^{n} F_{i,k} \exp\left(-\int_0^{t_i} g^2(\tau) d\tau \right) \right) - P_k \right]^2 \to \min_{g(\cdot)}$$

As it is shown in (Lapshin, 2009 [Error! Bookmark not defined.]), the solution to this problem is an exponential-sinusoidal spline with the coefficients to be determined from the given data. The resulting $g(.)$ is piecewise continuously differentiable, so the zero-coupon yield curve $r(t) = \frac{1}{t}\int_0^t g^2(\tau) d\tau$ is piecewise twice continuously differentiable.

With an appropriate choice for the regularization parameter α to govern the necessary precision, the resulting spline delivers an economically sound solution to the zero-coupon yield curve fitting problem: the spot forward rates are positive, the bond prices are fitted with a reasonable precision and the curve is the smoothest one among all curves yielding the same precision. The choice of the regularization parameter is a separate task. In a seminal work on ill-posed problems (of which the present one is a clear example), Tikhonov proposed the following principle: the regularization parameter should be chosen so that the residual be close to the data measurement error [18]. Other techniques widely used to choose the smoothing parameter for splines are the General Cross-Validation and Maximum Likelihood [19]. In what follows we regard the smoothing parameter as exogenous (set by the analyst).

Figure 3 shows the CCDC yield curves and zero-coupon yield curves fitted exponential-sinusoidal splines. Dots mark the yield to maturity plotted versus maturity. It is economically sounder to plot yield to maturity versus duration, but we adhere to the practice employed by CCDC for the ease of comparison.

Our curve and the CCDC curve are built from approximately the same data. As a consequence, we end up with several zero-coupon yield curves each constructed from its own dataset and one CCDC curve, constructed from all available data. In what follows we also show that different datasets imply different curves and claim that using the same curve regardless of the purpose is not advisable.

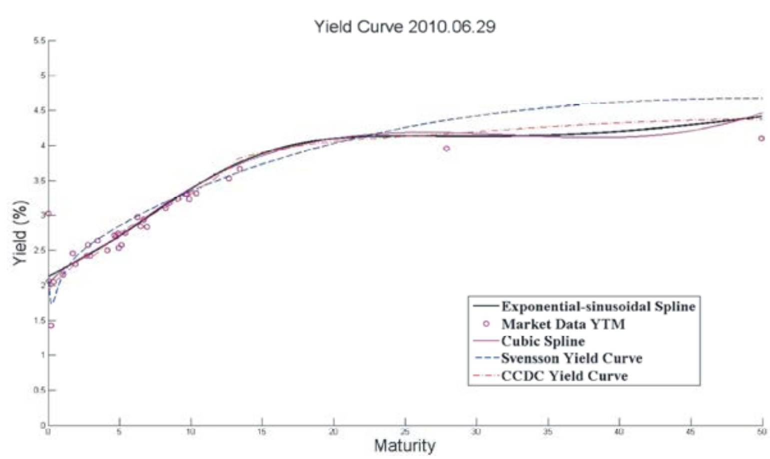

Fig. 3: Constructing zero-coupon yield curve via different methods

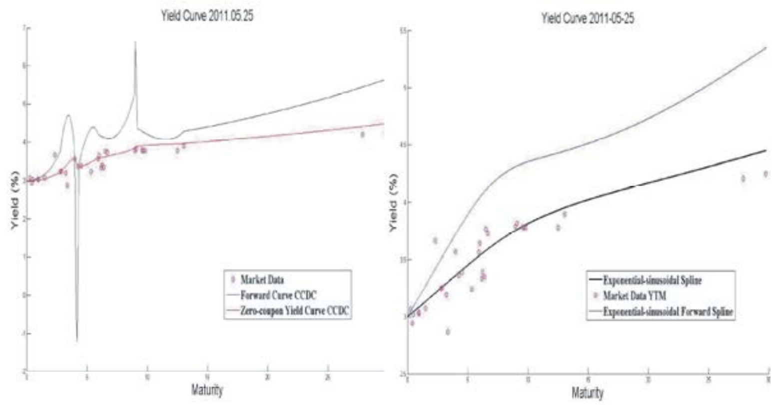

Fig. 4: CCDC and Exponential-sinusoidal forward yield curve on 25th May, 2011

An advantage of the Exponential-sinusoidal spline is the forward yield is always positive, since the yield curve is based on the pricing formula with continuous compounding:

$$\mathrm{d(x)} = \exp[-\int_0^t g^2(x)dx]$$

$$\mathrm{P_k} = \sum_{i=1}^{n} F_{i,k} * d(t_i)$$

Where $d(t)$ is a discount factor for the time of term to maturity: t. $F_{i,k}$ is the cash flow amount for the k-th bond at time t_i, To ensure positive spot forward rates we let $f(t) = g^2(t)$, where $f(t)$ is the spot forward rate for time t. And all subsequent calculations will be done in terms of $g(t)$ instead of spot forward rates $f(t)$.

On 25th May, 2011, due to the CCDC method, the forward rate falls below 0 around the time to maturity of 4.8 years (Figure 4), but the Exponential-sinusoidal forward yield curve is smooth and always above 0

The average fitting errors from 2009 to 2011 for different methods are reported in the following table:

	SE Spline	Cubic Spline	Svensson	CCDC
MAE	0.4147608	0.4244038	0.5852014	0.4203772
RMS	0.3981206	0.4339859	0.6081669	0.5254672

From this table we note that the Exponential-sinusoidal spline and CCDC model has a good precision while we restrain ourselves with only a subset of available information focusing (we use either only quotes or only transaction prices from only one of the markets – namely the interbank market) while CCDC uses all available data and for each date explicitly decides which pieces of the data to use in the actual construction.

CONCLUSIONS

A sound method for construction of zero-coupon yield curve on China's bond market should satisfy several requirements.

- The spot forward rates should be positive (that is the discount function should be decreasing).
- Approximate the real data with a sufficient degree of precision. We argue that the bid-ask spread is a sound economic basis, taking into account market liquidity to measure price residual; moreover no arbitrage principle can be applied to measure precision of zero coupon yield curve construction for different maturities.
- Reasonable smoothness of forward rates and yield curve should be achieved. It is easy to construct an interest rate term structure to fit the given prices exactly (for example via bootstrapping). However, the resulting curve is usually of unacceptable nonrealistic shape, having oscillating forward rates with no economic sense. It is natural to assume that investors' expectations are continuous in term, so that the spot forward rates should be sufficiently smooth.

Based on these three considerations and the comparison among the exponential sinusoidal spline, CCDC method, cubic spline and Svensson model, we conclude that the sinusoidal-exponential spline method is economically sensible as it always have positive forward rate and provides a flexible tool to get reasonable interplay between sufficient degree of precision and smoothness, taking into account market liquidity, which is suitable solution for the term structure evaluation for the Chinese bond market.

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