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The Analysis of the Dynamics of the Voltage Regulator with an Electronic Relay Element with Hysteresis

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Abstract: Presented a schematic diagram of developed voltage regulator with electronic switching element with a hysteresis and also its replacement scheme. Obtained a mathematical model of equivalent circuit of voltage regulator in the as a system of differential equations the 4th order. Based on the use of numerical methods software developed and studied dynamic regimes encountered in the resulting mathematical model. Studied implemented physical model of the voltage regulator. Shown the results of the research of dynamic modes of mathematical and physical models in the time domain and in the phase space: the periodic oscillations, quasi-periodic oscillation and chaotic dynamics of the movement or the so-called "strange attractor."

Key words: The voltage regulator · Relay element · Hysteresis · Mathematical model · Physical model · periodic oscillations · Quasi-periodic oscillation · Chaotic dynamics · "Strange attractor" · Chaotic oscillations

INTRODUCTION

Relay systems are an important class of nonlinear automatic systems that are widely used in various industries [1-4]. The main mode of stable operation of relay systems - the mode of oscillations with constant amplitude and constant frequency that is, supported by not periodic external action, but determined by the properties of the system [5]. At the same time, experimental and computer modeling detected more complex modes, including a quasi-periodic and random variations. Furthermore, for multidimensional relay systems typical situation where a wide range of parameters coexist multiple attractors different dynamic characteristics. Under these conditions, the influence of external noise even arbitrarily small, may lead to a sudden transition from one dynamic state to another. As a result, perhaps not only a significant increases dynamic errors and deterioration of quality indicators, but also the sudden failures of process equipment [5, 6].

Main: To obtain a nonlinear dynamical system with chaotic dynamics (order of the system $n \geq 3$), developed a physical model of the voltage regulator with the control element in the form of an electronic relay with hysteresis circuit diagram is shown in Fig. 1. Relay systems of this type are widely used to control the pulse-modulation energy converters process automation systems and energy-saving technologies [1].

Realized voltage regulator consists of the following main parts: the input RLC filter, the semiconductor voltage converter, output RLC filter, electronic relay with hysteresis, which also acts as a comparator element. Relay hysteresis implemented on the basis of two comparators and DA1.1 DA1.2, defining upper and lower thresholds of the relay switch element (hysteresis) and the RS-flip-flop used for stabilization the current value of the relay output.

For certain values of the parameters (U_S - the supply voltage, U_r - reference voltage, [chi] - the width of the hysteresis band, Rl - the resistance of the load resistor, R1, C1, L1 and R2, C2, L2 - parameters of input and output RLC-filters, respectively) system shows chaotic motion, which is typical nonlinear dynamic systems described by differential equations of the third and the higher orders.

Fig. 2 shows the equivalent circuit for the voltage regulator designed based on which was obtained a simplified mathematical model of the system [7,8].

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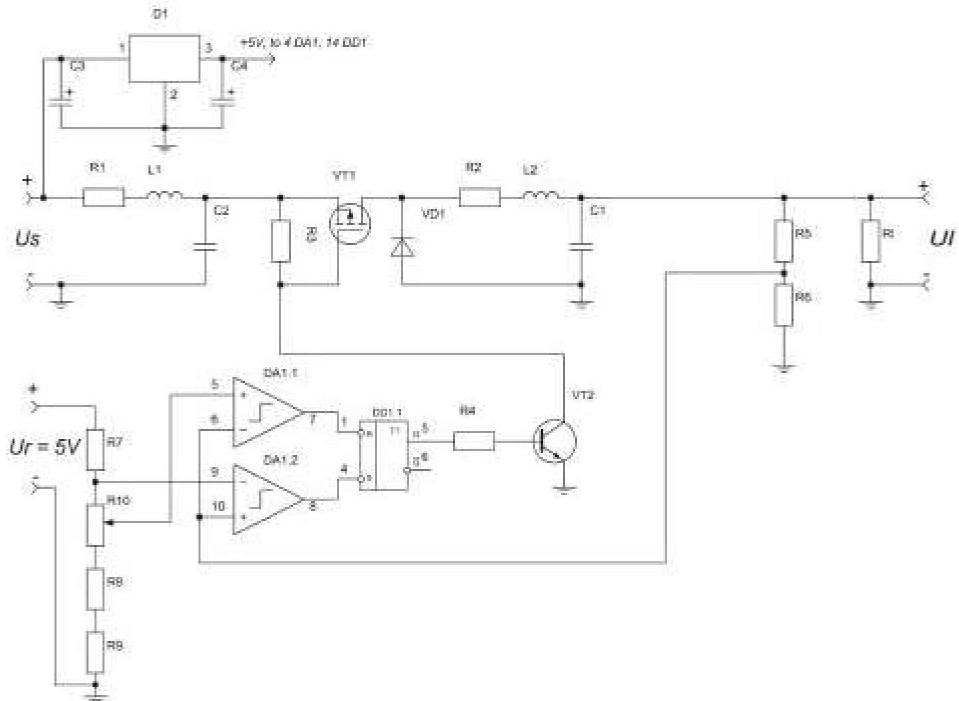


Fig. 1: Schematic diagram of the voltage regulator with electronic relay element

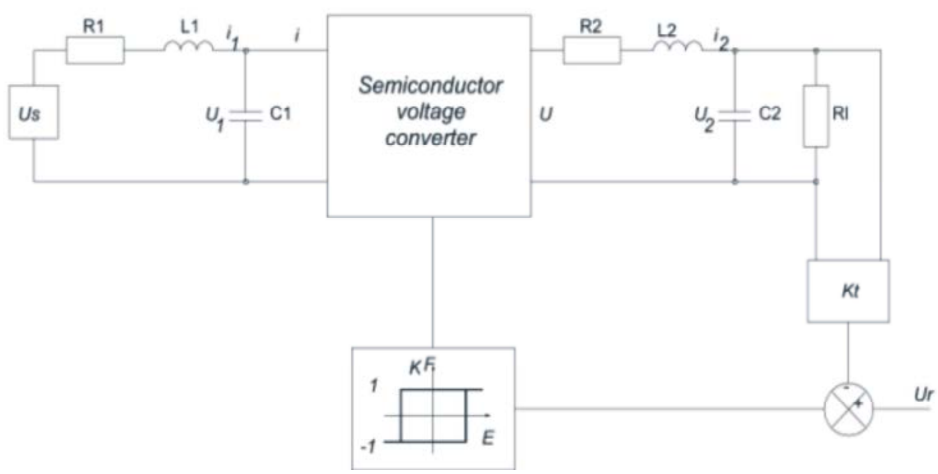


Fig. 2: The equivalent circuit for the voltage regulator

Based on the known postulate Papaleksi-Mandelstam for capacitance and inductance and the 1st and 2nd laws of Kirchhoff:

$$C\frac{dU_C}{dt}=i_C; L\frac{di_L}{dt}=U_L; C\frac{dU_C}{dt}=i_C; L\frac{di_L}{dt}=U_L; \qquad (1)$$

describe the equivalent circuit of the stabilizer (Fig. 2) in the form of the following differential equations [6, 8]:

$$\begin{gathered} C_1\frac{dU_1}{dt}+i=i_1;\ C_2\frac{dU_2}{dt}+\frac{U_2}{R_l}=i_2; \\ L_1\frac{di_1}{dt}+R_1i_1+U_1=U_S; L_2\frac{di_2}{dt}+R_2i_2+U_2=U; \end{gathered} \qquad (2)$$

where

$$U=\begin{cases} U_1,\ K^F=1; \\ 0,\ K^F=-1; \end{cases} \quad i=\begin{cases} i_2,\ \ K^F=1; \\ 0,\ K^F=-1; \end{cases} \qquad (3)$$

$$U = \frac{1}{2}(1+K^F(\varepsilon))U_1; i = \frac{1}{2}(1+K^F(\varepsilon))i_2. \quad (4)$$

Here [epsilon] - signal applied to the input of the electronic relay element, K^F - the output relay signal.

Then the system of differential equations describing the simplified nonlinear system under study, will look like this:

$$\begin{cases} \frac{di_1}{dt} = -\frac{R_1}{L_1}i_1 - \frac{1}{L_1}U_1 + \frac{U_S}{L_1}; \\ \frac{dU_1}{dt} = \frac{1}{C_1}i_1 - \frac{1+K^F(\varepsilon)}{2C_1}i_2; \\ \frac{di_2}{dt} = \frac{1+K^F(\varepsilon)}{2L_2}U_1 - \frac{R_2}{L_2}i_2 - \frac{1}{L_2}U_2; \\ \frac{dU_2}{dt} = \frac{1}{C_2}i_2 - \frac{1}{C_2R_l}U_2; \end{cases} \quad (5)$$

Introduce the following notations:

$$x_1 = R_1i_1,\ x_2 = U_1, x_3 = R_1i_2, x_4 = U_2.$$

Then the system of differential equations can be written as:

$$\frac{\mathbf{dX}}{\mathbf{dt}} = \mathbf{G(X)},\ \mathbf{X} = (x_1, x_2, x_3, x_4)^T;\ \mathbf{G} = (g_1, g_2, g_3, g_4)^T, \quad (6)$$

where

$$g_1 = \eta(-x_1 - x_2 + \Omega);\ g_2 = \gamma(x_1 - \frac{1}{2}(1+K_k^F(\varepsilon))x_3);$$

$$g_3 = \frac{\mu(1+K_k^F(\varepsilon))}{2}x_2 - \nu x_3 - \mu x_4;\ g_4 = \lambda\left(\frac{x_3}{\alpha} - \frac{x_4}{\beta}\right);$$

$$K_k^F = K_{k-1}^F sign(\chi + (-1)^{N_{k-1}}\varepsilon(X))),\ K_0^F = -1;$$

$$N_k = N_{k-1} + \frac{1 - sign(K_k^F K_{k-1}^F)}{2}, N_0 = 0, k = 1,2...;$$

$$\varepsilon(X) = U_r - K_t x_4;$$

$$\eta = \frac{R_1}{L_1}, \Omega = U_S, \gamma = \frac{1}{C_1R_1}, \mu = \frac{R_1}{L_2},$$

$$\nu = \frac{R_2}{L_2}, \lambda = \frac{1}{C_2}, \beta = R_l, \alpha = R_1,$$

where K_t - transfer efficiency of voltage the load resistor R_l.

In matrix form, the system will be as follows:

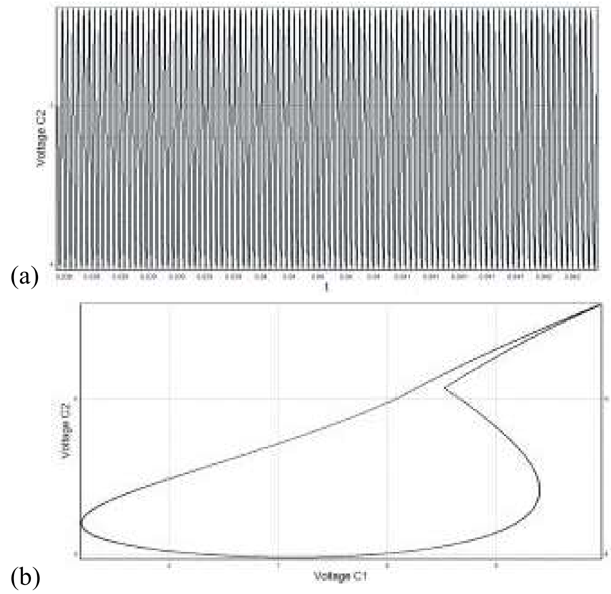


Fig. 3: The results of the study at $U_s = 8V$: *a* - graph of $U_{C_2}(t)$; *b* - phase portrait of the representative point $U_{C_2}(U_{C_1})$

$$\frac{\mathbf{dX}}{\mathbf{dt}} = \mathbf{A_k X} + \mathbf{B};$$

$$\mathbf{A_k} = \begin{pmatrix} -\eta & -\eta & 0 & 0 \\ \gamma & 0 & -\frac{\gamma}{2}(1+K_k^F(\varepsilon)) & 0 \\ 0 & \frac{\mu}{2}(1+K_k^F(\varepsilon)) & -\nu & -\mu \\ 0 & 0 & \frac{\lambda}{\alpha} & -\frac{\lambda}{\beta} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \eta\Omega \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad (7)$$

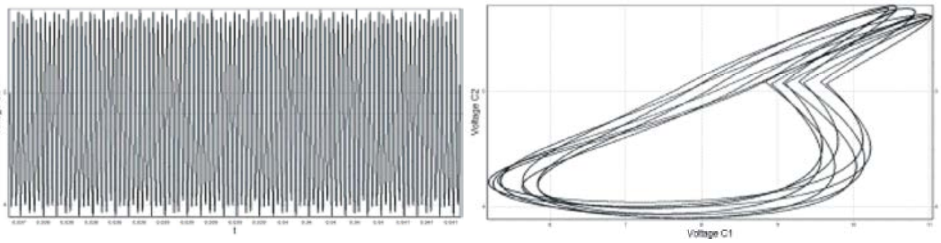
$$K_k^F = K_{k-1}^F sign(\chi + (-1)^{N_{k-1}}\varepsilon(X))),\ K_0^F = -1;$$

$$N_k = N_{k-1} + \frac{1 - sign(K_k^F K_{k-1}^F)}{2}, N_0 = 0, k = 1,2...;$$

$$\mathbf{e(X)} = \mathbf{U_r} - \mathbf{U_t X}; \mathbf{U_t} = (0,0,0,K_t).$$

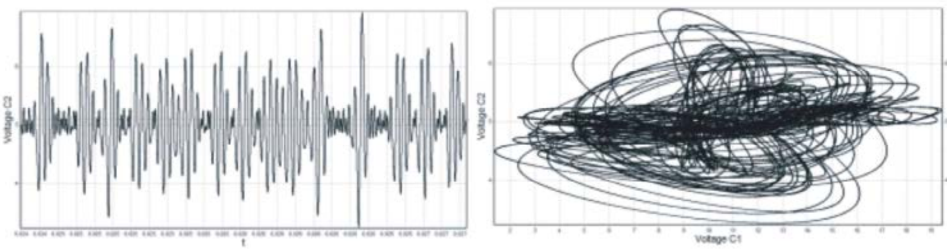
To solve the system of differential equations with discontinuous right-hand side of (6) was created the software, based on numerical integration of Runge-Kutta 4th order.

In research have chosen the following parameters of the system: R_1 = 0,47 Ohm, L_1 = 0,709.10^{-3} H, C_1 = 0,478.10^{-6} F, R_2 = 2,2 Ohm, L_2 = 3,440.10^{-3} H, L_2 = 0,97.10^{-6} F, R_l = 100 Ohm, U_r = 5V, [chi] = 0,05 V, U_s = 7...15V - variable parameter.



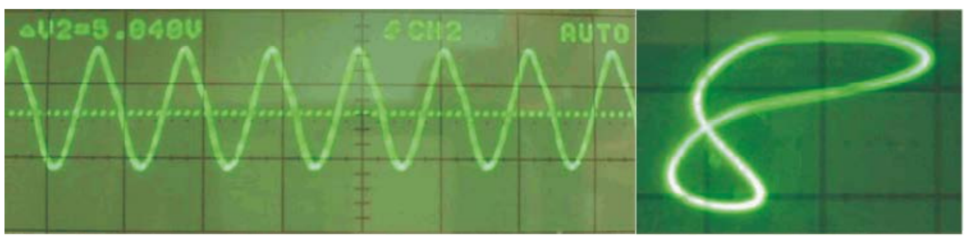
(a) (b)

Fig. 4: The results of the study at U_s = 8,5 V: a - graph of $U_{C_2}(t)$; b - phase portrait of the representative point $U_{C2}(U_{C1})$



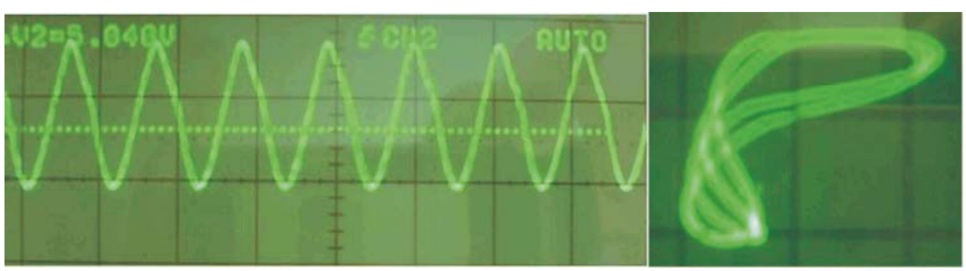
(a) (b)

Fig. 5: The results of the study at U_s = 10,5V: a - graph of $U_{C_2}(t)$; b - phase portrait of the representative point $U_{C2}(U_{C1})$



(a) (b)

Fig. 6: The results of research on the physical model of the system with U_s = 8V: a - oscillogram of $U_{C_2}(t)$; b - oscillogram of the representative point in the plane $U_{C2}(U_{C1})$

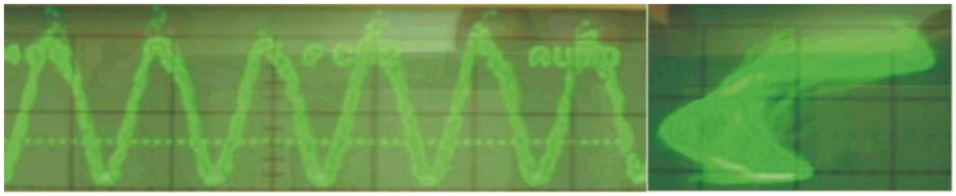


(a) (b)

Fig. 7: The results of research on the physical model of the system with U_s = 8,5V: a - oscillogram of $U_{C_2}(t)$; b - oscillogram of the representative point in the plane $U_{C2}(U_{C1})$

As can be seen from Fig. 3 - Fig. 5, in the studied system, depending on the varied parameter - U_s appear three types of motion: periodic oscillations (Fig. 3), quasi-periodic oscillations (Fig. 4) and the random motion (Fig. 5) or the movement of the so-called "strange attractor".

In addition to research received a simplified mathematical model was also performed an analysis of the dynamics of the implemented physical model, resulting in a waveform (Fig. 6 - Fig. 8) showing the three types of moving at the values varied parameter U_S, used in the software research of mathematical model system.



(a) (b)

Fig. 8: The results of research on the physical model of the system with U_s = 10,5V: a - oscillogram of $U_{C_2}(t)$; b - oscillogram of the representative point in the plane $U_{C2}(U_{C1})$

CONCLUSION

Thus, designed and implemented a nonlinear dynamic system - a voltage regulator with electronic switching element with hysteresis. Obtain a simplified mathematical model of the system. In theory, based on the obtained mathematical model and experimentally, using a physical model, investigated the dynamics of the system and obtained illustrate the periodic, quasi-periodic and chaotic regimes of motion.

Consequence: As a result of theoretical and experimental research of the dynamic properties of the voltage regulator with electronic switching element with hysteresis, identified three types of motion: periodic oscillations, quasi-periodic oscillations and the motion by the "strange attractor." Thus, the results show that the commonly used system [7 - 11], described by differential equations of the third and the older orders, demonstrate the presence of complex dynamic modes of operation (random oscillations) and require a qualitative theoretical analysis in the development or use of devices, damping the chaotic dynamics.

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REFERENCES

1. Zhusubaliyev, Z.T. and V.S. Titov, 2007. Bifurkatsii v shirotno-impulsnykh sistemakh avtomaticheskogo upravleniya: uchebnoye posobiye. Kursk. gos. tekhn. un-t. Kursk, pp: 100.
2. Porkhalo, V.A., V.G. Rubanov and V.G. Shaptala 2013. Avtomatizatsiya pechi obzhiga klinkera na osnove kaskadnoy i mnogosvyaznoy sistem upravleniya //Vestnik BGTU im. V.G. Shukhova. 2: 69-72.
3. Rubanov, V.G., 2011. Sistemnyy analiz i proyektirovaniye upravlyayemykh mobilnykh logisticheskikh sredstv, obladayushchikh svoystvom zhivuchesti. Nauchnyye vedomosti BelGU. Sektsiya Istoriya. Politologiya. Sotsiologiya. Informatika. 1(96): 17/1.
4. Rubanov, V.G., 2006. Teoriya avtomaticheskogo upravleniya (nelineynyye, optimal'nyye i tsifrovyye sistemy): ucheb. Posobiye - Part.II. - Belgorod: Izd-vo BGTU, pp: 256.
5. Zhusubaliyev, Z.T., V.G. Polishchuk and V.S. Titov, 2000. Kolebaniya, bifurkatsii i khaos v tekhnicheskikh sistemakh / Kursk. gos. tekhn. un-t. Kursk. pp: 166.
6. Zhusubaliyev, Z.T., V.G. Polishchuk and V.S. Titov, 2008. Khaoticheskiye kolebaniya v tekhnicheskikh sistemakh: monografiya. Kursk. gos. tekhn. un-t. Kursk, pp: 201.
7. Nabokov, R.A. and V.G. Rubanov, 2011. Razrabotka i matematicheskoye opisaniye releynoy sistemy s gisterezisom, obladayushchey khaoticheskoy dinamikoy. Matematicheskiye metody v tekhnike i tekhnologiyakh - MMTT-24: sb. trudov XXIV Mezhdunar. nauchn. konf.: v 10 t. T.6. Sektsiya 6,7 / pod obshch. red. V.S. Balakireva. - Kiyev: Natsion. tekhn. un-t Ukrainy «KPI», pp: 102-104.
8. Zhusubaliyev, Z.T. and E. Mosekilde, 2003. Bifurcations and Chaos in Piecewise-smooth Dynamical System. World Scientific, pp: 113-133.
9. Zhusubaliyev, Z.T. and E. Mosekilde, 2008. Chaos in Pulse-Width Modulated Control Systems. Handbook of Chaos Control, pp: 771-791.
10. Zhusubaliyev, Z.T. and E. Mosekilde, 2008. Formation and destruction of multilayered tori in coupled map systems. Chaos, 18: 037124.
11. Zhusubaliyev, Z.T. and E. Mosekilde, 2006. Birth of bilayered torus and torus breakdown in a piecewise-smooth dynamical system. Physics Letters A, pp: 167-174.