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Using the Reduced Differential Transform Method to Solve Nonlinear PDEs Arises in Biology and Physics

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Abstract: In this paper, we present an algorithm called the Reduced Differential Transform Method (RDTM) to obtain approximate solutions for the Fitzhugh-Nagumo (FN) equation, Ito equation and find an exact solution to nonlinear PDE. The numerical results show that this method is a powerful tool for solving nonlinear PDEs and the results show that the method reduces the numerical calculations. Also, the approximate solutions we present in this paper reveals that the proposed method is very effective, simple and can be applied to other nonlinear partial differential equations (NLPDEs) models in the area of Biology, population genetics, Physics and Engineering.

AMS Classification: 35J05 · 35J10 · 35K05 · 35L05
Key words: Reduced Differential Transform Method (RDTM) · Differential Transform Method (DTM) · Fitzhugh-Nagumo (FN) equation · Ito equation

INTRODUCTION

Nonlinear partial differential equations are widely used to describe many important phenomena and dynamic processes in physics, mechanics, chemistry, biology, etc. The study of nonlinear partial differential equations plays an important role in physical sciences and engineering fields. The investigation of exact solutions of nonlinear PDEs plays an important role in the study of nonlinear physical phenomena. Many methods, exact, approximate and purely numerical are available in literature for the solution of nonlinear partial differential equations.

The RDTM was first introduced by a Turkish Mathematician, Y. Keskin in his Ph.D. [1-3]. This method based on the use of the traditional DTM techniques. Usually, a few numbers of iteration needed of the series solution for numerical purposes to get high accuracy.

The RDTM has been used by many authors to obtain analytical and approximate solutions to nonlinear wave equations. Keskin and Oturanc, [1-3] used the RDTM to solve linear and nonlinear wave equations and they showed the effectiveness and the accuracy of the proposed method. Moreover, Keskin and Oturanc showed that the number of iterations it takes to get an approximate solutions is less than the one used by the DTM and other well-known methods in the field. Also, S. M. Sayed and G. M. Gharib, [6] used the Sine-Cosine Method to solve the FN equation. In addition, M. Rawashdeh, [12] used the RDTM to find exact and approximate solution for Gardner equation, Variant Nonlinear Water Wave equation (VNWW) and the Fifth-Order Korteweg-de Vries (FKdV) equation. Finally, Ibis and Bayram [11] used the RDTM to find approximate solutions for the (KdVB) equation, Drinefel'd-Sokolov-Wilson equations, coupled Burgers equations and modified Boussinesq equation. The standard form of the Fitzhugh-Nagumo equation [9] is given by $u_t - u_{xx} + u(1-u)(a-u) = 0$, where is a constant. Note that when a = -1 , the FN equation becomes the Newell-Whitehead (NW) equation which is an important nonlinear reaction-diffusion equation and usually is used to model the transmission of nerve impulse, also used in circuit theory, biology and the area of population genetics as mathematical models. In this paper, we were being able to find approximate and exact solutions for the following NLPDEs:

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First, the Fitzhugh-Nagumo (FN) equation:

$$u_t = u_{xx} - u(1-u)(a-u) \tag{1.1}$$

subject to the initial conditions

$$u(x,0) = \frac{1}{2}\left(1\text{-tanh}\left(\frac{\sqrt{2}x}{4}\right)\right) \tag{1.2}$$

Second, the Ito equation:

$$u_t + u_{xxxxx} + 3uu_{xx} + 6u_x u_{xx} + 2u^2 u_x = 0 \tag{1.3}$$

subject to the initial condition

$$u(x,0) = -10\mu\left(2 - 3\tanh^2\left(\sqrt{-\mu}x\right)\right) \tag{1.4}$$

Third, consider the nonlinear PDE:

$$u_t - uu_{xx} - (u_x)^2 - u = 0 \tag{1.5}$$

subject to the initial condition

$$u(x,0) = \sqrt{x} \tag{1.6}$$

The aim of our study is to be able to use the RDTM as an alternative method to the existing methods in solving different types of nonlinear partial differential equations (NLPDEs). Many authors used different methods to solve the NLPDEs mentioned above, to name few: The DTM, ADM, VIM, Tanh-Coth method and Sine-Cosine method.

The rest of this paper is organized as follows: In Section 2, the RDTM is introduced. Section 3 is devoted to apply the method to the PDEs mentioned above and present tables to show the effectiveness of the RDTM for some values of and . Section 4 is for discussion and conclusion of this paper.

The Reduced Differential Transform Method (RDTM): In this section, we will give the methodology of the RDTM. So let's start with a function of two variables $u(x,t)$ which is analytic and k-times continuously differentiable with respect to time t and space x in the domain of our interest. Assume we can represent this function as a product of two single-variable functions $u(x,t)= f(x).g(t)$. From the definitions of the DTM, the function can be represented as

$$u(x,t) = \left(\sum_{i=0}^{\infty} F(i)x^i\right)\left(\sum_{j=0}^{\infty} G(j)t^j\right) = \sum_{k=0}^{\infty} U_k(x)t^k \tag{2.1}$$

where $U_k(x)$ is the transformed function of which can be defined as:

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0} t^k \tag{2.2}$$

From equations (2.1) and (2.2) we can deduce

$$u(x,t) = \sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0} t^k = \sum_{k=0}^{\infty}\frac{1}{k!}U_k(x)t^k \tag{2.3}$$

Some basic operations of the reduced differential transformation obtained from equations (2.1) and (2.2) are given in the table below:

Now, we illustrate the RDTM by using the Newell-Whitehead (NW) equation in standard form

$$L(u(x,t)) + R(u(x,t)) + N(u(x,t)) + F(u(x,t)) = 0 \tag{2.4}$$

with initial conditions

$$u(x,0) = f(x), u_t(x,0) = g(x) \tag{2.5}$$

where, $L = \frac{\partial}{\partial t}, R = \frac{\partial^2}{\partial x^2}, F = u$ and $N = u^3$ are the linear operators that have partial derivatives.

Using the RDTM formulas in Table 1, we can find the following recursive relation:

$$(k+1)U_k(x) = R(U_k(x)) - N(U_k(x)) + U_k(x) \tag{2.6}$$

where, $R(U_k(x)), U_k(x)$ and $N(u(x,t))$ are the transformations of $R(u(x,t))$, $F(u(x,t))$ and $N(u(x,t))$ respectively.

Now from equation (2.5), we can write the initial condition as:

$$U_0(x) = f(x), U_1(x) = g(x) \tag{2.7}$$

Table 1: Basic operations of the RDTM [1, 2, 3]

Functional Form	Transformed form
$u(x,t)$	$\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}$
$\alpha u(x,t)\pm\beta v(x,t)$	$\alpha U_k(x)\pm\beta V_k(x)$, α and β are contant.
$u(x,t).v(x,t)$	$\sum_{i=0}^{k}U_i(x)\,V_{k-i}(x)$
$u(x,t).v(x,t).w(x,t)$	$\sum_{i=0}^{k}\sum_{j=0}^{i}U_j(x)\,V_{i-j}(x)W_{k-i}(x)$
$\frac{\partial^n}{\partial t^n}u(x,t)$	$\frac{(k+n)!}{K!}U_{k+n}(x)$
$\frac{\partial^n}{\partial x^n}u(x,t)$	$\frac{\partial^n}{\partial x^n}U_k(x)$
$x^m t^n u(x,t)$	$x^m U_{k-n}(x)$
$x^m t^n$	$x^m\delta(k-n)$, where $\delta(k-n)=\begin{cases}1, & k=n\\ 0, & k\neq n\end{cases}$
$\frac{\partial^{n+m}}{\partial x^n\partial t^m}u(x,t)$	$\frac{\partial^n}{\partial x^n}\left[\frac{(k+m)!}{k!}U_{k+m}(x)\right]$

To find all other iterations, we first substitute equation (2.7) into equation (2.6) and then we find the values of $U_k(x)$. Finally, we apply the inverse transformation to all the values $\{U_k(x)\}_{k=0}^{n}$ to obtain the approximate solution:

$$\hat{u}(x,t)=\sum_{k=0}^{n}U_k(x)t^k \tag{2.8}$$

where n is the number of iterations we need to find the intended approximate solution.

Hence, the exact solution of our problem is given by $u(x,t)=\lim_{n\to\infty}\hat{u}(x,t)$.

Applications: In this section, we apply the RDTM to three numerical examples and then compare our approximate solutions to the exact solutions.

Examples: In this section, we present three examples to show the efficiency of the RDTM.

Example 3.1.1: Consider the Fitzhugh-Nagumo (FN) equation: $u_t=u_{xx}-u(1-u)(a-u)$, where a is a constant. It is worth mentioning that this equation has an exact solution given by $u(x,t)=\frac{1}{2}\left(1+tanh\left(\left(\frac{1}{4}-\frac{a}{2}\right)t-\frac{x}{2\sqrt{2}}\right)\right)$.

Case 1: (a=-1) This is called the Newell-Whitehead (NW) equation which is given by

$$u_t-u_{xx}-u(1-u)(1+u)=0 \tag{3.1}$$

subject to the initial conditions

$$u(x,0)=\frac{1}{2}\left(1-tanh\left(\frac{\sqrt{2}}{4}x\right)\right),u_t(x,0)=\frac{3}{8}sech^2\left(\frac{x}{2\sqrt{2}}\right) \tag{3.2}$$

where the exact solution is

$$u(x,t)=\frac{1}{2}\left(1+tanh\left(\frac{3t}{4}-\frac{x}{2\sqrt{2}}\right)\right) \tag{3.3}$$

Applying the RDTM to (3.1) and (3.2), we obtain the recursive relation

$$U_{k+1}(x)=\left(\frac{1}{k+1}\right)\left(\begin{array}{l}\frac{\partial^2}{\partial x^2}\left(U_k(x)\right)+U_k(x)-\\ \sum_{i=0}^{k}\sum_{j=0}^{i}U_j(x)\,U_{i-j}(x)U_{k-i}(x)\end{array}\right) \tag{3.4}$$

where the $U_k(x)$, is the transform function of the dimensional spectrum. Note that

$$U_0(x)=\frac{1}{2}\left(1-tanh\left(\frac{\sqrt{2}}{4}x\right)\right)\text{ and }U_1(x)=\frac{3}{8}sech^2\left(\frac{x}{2\sqrt{2}}\right) \tag{3.5}$$

Now, substitute Eq. (3.5) into Eq. (3.4) to obtain the following:

$$U_2(x)=\frac{9}{4}csch^3\left(\frac{x}{\sqrt{2}}\right)sinh^4\left(\frac{x}{2\sqrt{2}}\right),U_3(x)=\frac{\left(1440sech^2\left(\frac{x}{2\sqrt{2}}\right)-2160sech^4\left(\frac{x}{2\sqrt{2}}\right)\right)}{10240} \tag{3.6}$$

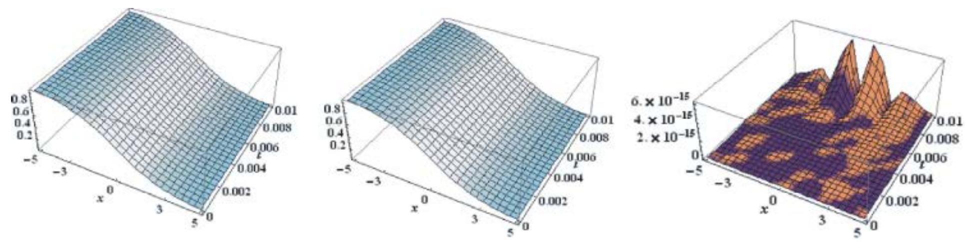

Fig. 1: The approximate, exact solutions and absolute error respectively for example 3.1.1 ( ) when -5< x <5 and 0< t <0.01.

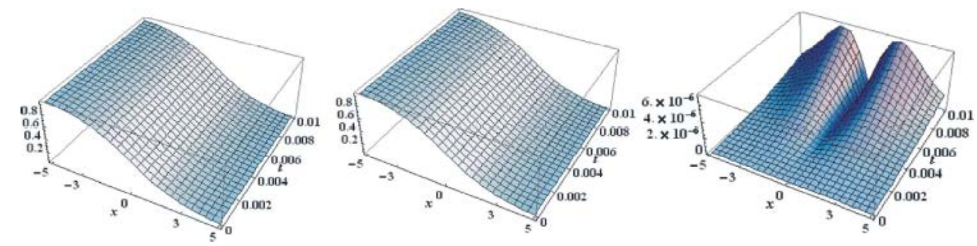

Fig. 2: The approximate, exact solutions and absolute error respectively for example 3.1.1 ( ) when -5< x <5 and 0< t <0.01.

We continue in this manner and after the fifth iteration, the differential inverse transform of $\{U_k(x)\}_{k=0}^{\infty}$ will provide us with the following approximate solution:

$$\hat{u}(x,t)=\sum_{k=0}^{\infty}U_k(x)t^k=U_0(x)+U_1(x)t+U_2(x)t^2+....$$

$$=\frac{1}{2}\left(1-tanh\left(\frac{\sqrt{2}}{4}x\right)\right)+\frac{3}{8}\sec h^2\left(\frac{x}{2\sqrt{2}}\right)$$

$$t+\frac{9}{4}csch^3\left(\frac{x}{\sqrt{2}}\right)sinh^4\left(\frac{x}{2\sqrt{2}}\right)t^2+...$$

Case 2: ($a = 1$) Now the Fitzhugh-Nagumo (FN) equation becomes

$$u_t-u_{xx}+u(1-u)(1-u)=0 \tag{3.7}$$

subject to the conditions

$$u(x,0)=\frac{1}{2}\left(1-tanh\left(\frac{\sqrt{2}}{4}x\right)\right),u_t(x,0)=-\frac{3}{8}sech^2\left(\frac{x}{2\sqrt{2}}\right) \tag{3.8}$$

where the exact solution is

$$u(x,t)=\frac{1}{2}\left(1-tanh\left(\frac{3t}{4}+\frac{x}{2\sqrt{2}}\right)\right) \tag{3.9}$$

Applying the RDTM to (3.7) and (3.8), we obtain the recursive relation

$$U_{k+1}(x)=\left(\frac{1}{k+1}\right)\left(\begin{array}{l}\frac{\partial^2}{\partial x^2}\left(U_k(x)\right)-U_k(x)+\\ 2\sum\limits_{i=0}^{k}U_i(x)U_{k-i}(x)-\\ \sum\limits_{i=0}^{k}\sum\limits_{j=0}^{i}U_{i-j}(x)U_j(x)U_{k-i}(x)\end{array}\right) \tag{3.10}$$

where $U_k(x)$ the, is the transform function of the dimensional spectrum. Note that

$$U_0(x)=\frac{1}{2}\left(1-tanh\left(\frac{\sqrt{2}}{4}x\right)\right)\text{ and }U_1(x)=-\frac{3}{8}sech^2\left(\frac{x}{2\sqrt{2}}\right) \tag{3.11}$$

Now, substitute Eq. (3.11) into Eq. (3.10) to obtain the following:

$$U_2(x)=\frac{3}{32}sech^2\left(\frac{x}{2\sqrt{2}}\right)\tanh\left(\frac{x}{2\sqrt{2}}\right),$$

$$U_3(x)=\frac{1}{128}sech^2\left(\frac{x}{2\sqrt{2}}\right)\left(sech^2\left(\frac{x}{2\sqrt{2}}\right)\left(5+6\tanh\left(\frac{x}{2\sqrt{2}}\right)\right)-2\right) \tag{3.12}$$

We continue in this manner and after the fifth iteration, the differential inverse transform of $\{U_k(x)\}_{k=0}^{\infty}$ will provide us with the following approximate solution:

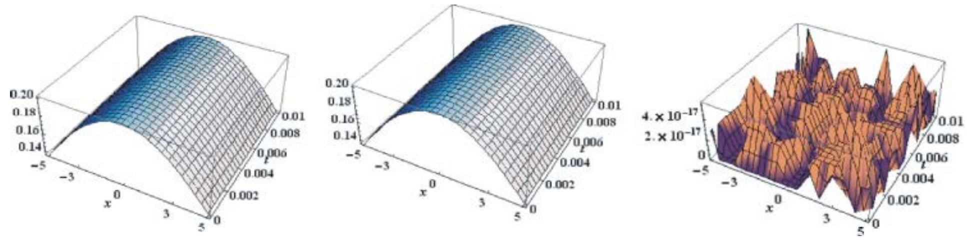

Fig. 3: The approximate, exact solutions and absolute error respectively for example 3.1.2 ( ) when -5< x <5 and 0< t <0.01.

$$\hat{u}(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k = U_0(x) + U_1(x)t + U_2(x)t^2 + ...$$
$$= \frac{1}{2}\left(1 - tanh\left(\frac{\sqrt{2}}{4}x\right)\right) - \frac{3}{8}\sec h^2\left(\frac{x}{2\sqrt{2}}\right)t$$
$$+ \frac{3}{32}sech^2\left(\frac{x}{2\sqrt{2}}\right)\tanh\left(\frac{x}{2\sqrt{2}}\right)t^2 + ...$$

Example 3.1.2:
We consider the Ito equation

$$u_t + u_{xxxxx} + 3uu_{xx} + 6u_x u_{xx} + 2u^2 u_x = 0 \quad (3.13)$$

subject to the initial condition

$$u(x,0) = -10\mu\left(2 - 3\tanh^2\left(\sqrt{-\mu}x\right)\right) \quad (3.14)$$

where the exact solution

$$u(x,t) = -10\mu\left(2 - 3\tanh^2\left(\sqrt{-\mu}\left(x - 96\mu^2 t\right)\right)\right) \quad (3.15)$$

Now, we apply the RDTM to Eq. (3.13) and Eq. (3.14) we get

$$(x) = \left(\frac{-1}{(k+1)}\right)\begin{pmatrix} \frac{\partial^5}{\partial x^5}\left(U_k(x)\right) + 3\sum_{i=0}^{k} U_k(x)\frac{\partial^2}{\partial x^2}U_{k-i}(x) \\ +6\sum_{i=0}^{k}\frac{\partial}{\partial x}U_k(x)\frac{\partial^2}{\partial x^2}U_{k-i}(x) + \\ \sum_{i=0}^{k}\sum_{j=0}^{i} U_{i-j}(x)U_j(x)U_{k-i}(x) \end{pmatrix} \quad (3.16)$$

where the $U_k(x)$, is the transform function of the *t*-dimensional spectrum. Note that when μ = -0.01, then

$$U_0(x) = 0.1\left(2 - 3\tanh^2(0.1x)\right) \quad (3.17)$$

Now, substitute Eq. (3.17) into Eq. (3.16) to obtain the following:

$$U_1(x) = sech^2(0.1x)\tanh(0.1x)(0.0048 - 0.001344 sech^4(0.1x)$$
$$-0.01296\tanh^2(0.1x) + 0.008736\tanh^4(0.1x) + sech^2(0.1x)$$
$$(-0.00288 + 0.007392\tanh^2(0.1x))$$

$$U_2(x) = 0.000244 sech^{12}(0.1x)(-0.000495 - 0.00055$$
$$\cosh(0.2x) - 0.0002\cosh(0.4x) - 0.00001327$$
$$\cosh(0.6x) + 0.000017\cosh(0.8x) + (4.42368)10^{-6}$$
$$\cosh(x) + (7.10366)10^{-18}\sinh(0.2x) + (1.9645)10^{-20}$$
$$\sinh(0.4x) + \sinh(0.6x) - (4.911)10^{-22}\sinh(x))$$

So after the third iteration, the differential inverse transform of $\{U_k(x)\}_{k=0}^{3}$ will give the following approximate solution:

$$\hat{u}(x,t) = \sum_{k=0}^{3} U_k(x)t^k = U_0(x) + U_1(x)t + U_2(x)t^2 + U_3(x)t^3.$$

Note this will converge to the exact solution.

Example 3.1.3:

We consider the nonlinear PDE

$$u_t - uu_{xx} - (u_x)^2 - u = 0 \quad (3.18)$$

subject to the condition

$$u(x,0) = \sqrt{x} \quad (3.19)$$

where the exact solution

$$u(x,t) = \sqrt{x}\,e^t \quad (3.20)$$

Applying the RDTM to (3.13) and (3.14), we obtain the recursive relation

$$\begin{cases} U_{k+1}(x) = \dfrac{1}{k+1}\left(\begin{array}{l} \displaystyle\sum_{i=0}^{k} U_i(x) \frac{\partial^2}{\partial x^2} U_{k-i}(x) + \\ \displaystyle\sum_{i=0}^{k} \frac{\partial}{\partial x} U_i(x) \frac{\partial}{\partial x} U_{k-i}(x) + U_k(x) \end{array} \right) \\ U_0(x) = \sqrt{x} \end{cases} \tag{3.21}$$

Now for $k \geq 1$ we obtain

$$U_1(x) = \sqrt{x}, U_2(x) = \frac{\sqrt{x}}{2}, U_3(x) = \frac{\sqrt{x}}{6}, U_4(x) = \frac{\sqrt{x}}{24}$$

Thus,

$$u(x,t) = \sqrt{x} + t\sqrt{x} + \frac{t^2\sqrt{x}}{2} + \frac{t^3\sqrt{x}}{6} + \frac{t^4\sqrt{x}}{24} + \frac{t^5\sqrt{x}}{120} + \frac{t^6\sqrt{x}}{720} + \ldots$$

$$= \sqrt{x}\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24} + \frac{t^5}{120} + \frac{t^6}{720} + \ldots\right) = \sqrt{x}\, e^t.$$

This is an exact solution of Eq. (3.13).

Tables of Numerical Calculations: We employed the RDTM successfully to three applications. The RDTM showed it is accurate and efficient method. Tables {2, 3,4} show the exact solution, the approximate solution and the absolute error obtained by the RDTM for different values of x and t.

Table 2: Comparison of the absolute error of the solution for the Newell-Whitehead equation, by RDTM for different values of *x* and *t*

x	t	Exact Solution	RDTM Solution	Abs error (RDTM) (n=5)
−5	0.002	0.9717645127934744	0.9717645127934744	$1.11022302E^{-16}$
	0.004	0.9718467111304566	0.9718467111304567	$1.11022302E^{-16}$
	0.006	0.971928677086468	0.971928677086468	$1.11022302E^{-16}$
	0.01	0.9720919143212237	0.9720919143212234	$2.22044605E^{-16}$
−3	0.002	0.893244612236795	0.8932446122367951	$1.11022302E^{-16}$
	0.004	0.5543178019390701	0.5543178019390702	$1.11022302E^{-16}$
	0.006	0.8938154157815541	0.893815415781554	$1.11022302E^{-16}$
	0.01	0.8943835281984087	0.894383528198407	$1.66533454E^{-15}$
3	0.002	0.10732889125532619	0.1073288912553262	$1.38777878E^{-17}$
	0.004	0.10761665823390826	0.10761665823390831	$4.16333634E^{-17}$
	0.006	0.10790510350024035	0.10790510350024045	$8.32667268E^{-17}$
	0.01	0.1084840332887782	0.10848403328877992	$1.72084569E^{-15}$
5	0.002	0.4132482317409408	0.41324823174094083	$5.55111512E^{-17}$
	0.004	0.028483482692001405	0.028483482692001398	$6.93889390E^{-18}$
	0.006	0.028566616748610607	0.028566616748610604	$3.46944695E^{-18}$
	0.01	0.028733591944711923	0.02873359194471206	$1.38777878E^{-16}$

Table 3: Comparison of the absolute error of the solution for the Fitzhugh-Nagumo equation , by RDTM for different values of *x* and *t*

x	t	Exact Solution	RDTM Solution	Abs error (RDTM) (n=5)
−5	0.002	0.971599416502523	0.971599494464562	$7.79620389E^{-8}$
	0.004	0.9715165173079986	0.971516829513273	$3.12205274E^{-7}$
	0.006	0.9714333832513894	0.9714340865177219	$7.03266333E^{-7}$
	0.01	0.9712664080552881	0.9712683660500422	$1.95799475E^{-6}$
−3	0.002	0.8926711087446738	0.892671334245639	$2.25500965E^{-7}$
	0.004	0.5513513090196914	0.5513516193356554	$3.10315964E^{-7}$
	0.006	0.8920948964997597	0.892096928493779	$2.03199402E^{-6}$
	0.01	0.8915159667112218	0.8915216180335443	$5.65132232E^{-6}$
3	0.002	0.10675538776320498	0.10675516262488272	$2.25138322E^{-7}$
	0.004	0.10646964904829925	0.10646874939284344	$8.99655456E^{-7}$
	0.006	0.10618458421844595	0.1061825620140173	$2.02220443E^{-6}$
	0.01	0.10561647180159123	0.10561086579296386	$5.60600863E^{-6}$
5	0.002	0.41179414818488336	0.4117940207223016	$1.27462582E^{-7}$
	0.004	0.02815328886954338	0.02815297815869499	$3.10710048E^{-7}$
	0.006	0.028071322913531982	0.02807062469073936	$6.98222793E^{-7}$
	0.01	0.027908085678776395	0.02790615103239626	$1.93464638E^{-6}$

Table 4: Comparison of the absolute error of the solution for the Ito equation (μ = -0.01), by RDTM for different values of x and t

x	t	Exact Solution	RDTM Solution	Abs error (RDTM) (n=3)
−5	0.002	0.13593390121696444	0.13593390121696441	$2.77555756E^{-17}$
	0.004	0.13593348254352552	0.13593348254352552	0
	0.006	0.13593306386946158	0.13593306386946158	0
	0.01	0.13593222651945855	0.13593222651945855	0
−3	0.002	0.17454078143447252	0.17454078143447255	$2.77555756E^{-17}$
	0.004	0.19973009287589155	0.19973009287589155	0
	0.006	0.17454016720291363	0.17454016720291363	0
	0.01	0.17453955296531953	0.17453955296531956	$2.77555756E^{-17}$
3	0.002	0.1745413956599962	0.1745413956599962	0
	0.004	0.17454170277049483	0.17454170277049483	$2.77555756E^{-17}$
	0.006	0.1745420098794846	0.1745420098794846	$2.77555756E^{-17}$
	0.01	0.17454262409293766	0.17454262409293766	0
5	0.002	0.19925130563878277	0.19925130563878277	0
	0.004	0.1359351572335306	0.1359351572335306	$2.77555756E^{-17}$
	0.006	0.13593557590446917	0.13593557590446917	$2.77555756E^{-17}$
	0.01	0.135936413244471	0.13593641324447098	0

CONCLUSION

In this paper, we applied the Reduced Differential Transform Method (RDTM) to all three physical models, namely, the Fitzhugh-Nagumo equation, Ito equation and one NLPDEs equation. We successfully found approximate solutions for the Fitzhugh-Nagumo equation and Ito equation and found an exact solution to another NLPDE. We only used 3-iteration in the case of Ito equation to get a very good error.

Also, we were being able to find exact solutions to examples (3.1.3). The results we obtained in example (3.1.1) and (3.1.2) were in excellent agreement with the exact solutions. The RDTM introduces a significant improvement in the fields over existing techniques because it takes less calculations and the number of iteration is less compared by other methods. My goal in the future is to apply this method to other nonlinear PDEs which arise in other areas of science such as Biology, Medicine and Engineering. Computations of this paper have been carried out using the computer package of Mathematica 7.

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