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Improved Estimators of Finite Population Variance in Stratified Random Sampling

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Abstract: This article proposes a class of exponential type estimators for estimating population variance of a study variable using information of auxiliary variable under stratified random sampling. The bias and mean square error of the estimators belonging to proposed class are obtained and the optimum parameters of class are given. It has been shown that the proposed class of estimators is more efficient than other estimators in the literature under stratified random sampling. Efficiency comparison is carried out using a data set.

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Key words: Auxiliary variable . ratio estimator . exponential estimator . stratified random sampling . efficiency

INTRODUCTION

The use of supplementary information provided by auxiliary variables in survey sampling was discussed by many survey statisticians. The problem of constructing efficient estimators for the population variance has been discussed by various authors such as Das and Tripathi [4], Isaki [8], Singh *et al*. [20], Prasad and Singh [15, 16], Garcia and Cebrain [5], Agrawal and Sthapit [1], Arcos *et al*. [2], Kadilar and Cingi [9, 10], Gupta and Shabbir [7], Shabbir and Gupta [18] and Singh and Solanki [19], Koyuncu [14]. In sampling literature, Bahl and Tuteja [3], Shabbir and Gupta [17], Grover [6] and Koyuncu [13] have studied exponential type estimators to get more efficient estimates. The aim of this paper is to make an improvement over variance estimators in the sampling literature and to see the performance of proposed class of estimators in stratified random sampling. Moving along this direction, we have proposed a class of exponential type estimators under stratified random sampling.

STRATIFIED RANDOM SAMPLING

Assume that the population of size N is divided into L strata with N_h elements in the hth stratum. Let n_h be the size of the sample drawn from hth stratum of size N_h by using simple random sampling without replacement. The total sample size

$$\sum_{h=1}^{L} n_h = n$$

and the population size

$$\sum_{h=1}^{L} N_h = N$$

Let y and x be the study and the auxiliary variables, respectively, assuming values y_{hi} and x_{hi} for the ith unit in hth stratum. Moreover, let

$$\bar{y}_h = \sum_{i=1}^{n_h} \frac{y_{hi}}{n_h}, \ \bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$$

and

$$\bar{Y}_h = \sum_{i=1}^{N_h} \frac{y_{hi}}{N_h}, \ \bar{Y} = \bar{Y}_{st} = \sum_{h=1}^{L} W_h \bar{Y}_h$$

be the sample and population means of y, respectively, where

$$W_h = \frac{N_h}{N}$$

is the stratum weight. Similar expressions for x can also be defined. When the finite population correction

$$\frac{N_h - n_h}{N_h}$$

is ignored, the classical variance of $\bar{y}_{st}$ is given by

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$$\mathrm{Var}\left(\bar{y}_{st}\right)=\sum_{h=1}^{L}W_h^2\frac{S_{y(h)}^2}{n_h}=S_{y(st)}^2$$

where

$$S_{y(h)}^2=\sum_{i=1}^{N_h}\frac{\left(y_{hi}-\bar{Y}_h\right)^2}{N_h}$$

is the population variance of y in the hth stratum. To obtain the bias and mean square error let us define

$$\delta_{0h}=\frac{s_{y(h)}^2-S_{y(h)}^2}{S_{y(h)}^2},\ \delta_{1h}=\frac{s_{x(h)}^2-S_{x(h)}^2}{S_{x(h)}^2}$$

Using these notations

$$E\left(\delta_{0h}\right)=E\left(\delta_{1h}\right)=0,\ E\left(\delta_{0h}^2\right)=\frac{\left(\beta_{2y(h)}-1\right)}{n_h}$$

$$E\left(\delta_{1h}^2\right)=\frac{\left(\beta_{2x(h)}-1\right)}{n_h},\ E\left(\delta_{0h}\delta_{1h}\right)=\frac{\left(\theta_{22(h)}-1\right)}{n_h}$$

where

$$\beta_{2y(h)}=\frac{\mu_{40(h)}}{\mu_{20(h)}^2},\ \beta_{2x(h)}=\frac{\mu_{04(h)}}{\mu_{02(h)}^2}$$

are the coefficients of kurtosis of y and x, respectively, in the hth stratum and

$$\theta_{22(h)}=\frac{\mu_{22(h)}}{\mu_{02(h)}\mu_{20(h)}}$$

where

$$\mu_{ab(h)}=\sum_{i=1}^{N_h}\frac{\left(y_{hi}-\bar{Y}_h\right)^a\left(x_{hi}-\bar{X}_h\right)^b}{N_h}$$

We consider the estimators of $S_{y(st)}^2$, as considered by Shabbir and Gupta [18]

(i) Conventional unbiased estimator

$$\hat{S}_{y(st)}^2=\sum_{h=1}^{L}W_h^2\frac{s_{y(h)}^2}{n_h}$$

where

$$s_{y(h)}^2=\sum_{i=1}^{n_h}\frac{\left(y_{hi}-\bar{y}_h\right)^2}{\left(n_h-1\right)}$$

is the sample variance of y in the hth stratum.

$$\mathrm{Var}\left(\hat{S}_{y(st)}^2\right)\cong\sum_{h=1}^{L}\frac{\left(W_hS_{y(h)}\right)^4}{n_h^3}\left(\beta_{2y(h)}-1\right)\qquad(2.1)$$

(ii) Customary stratified ratio estimator

$$\hat{S}_{R(st)}^2=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s_{y(h)}^2\frac{S_{x(h)}^2}{s_{x(h)}^2}\right)$$

where

$$s_{x(h)}^2=\sum_{i=1}^{n_h}\frac{\left(x_{hi}-\bar{x}_h\right)^2}{\left(n_h-1\right)}$$

is the sample variance of x in the hth stratum. The expression of bias and MSE, up to the first order of approximation, are as follows:

$$\mathrm{Bias}\left(\hat{S}_{R(st)}^2\right)\cong\sum_{h=1}^{L}\left(\frac{W_hS_{y(h)}}{n_h}\right)^2\left[\left(\beta_{2x(h)}-1\right)-2\left(\theta_{22(h)}-1\right)\right]\qquad(2.2)$$

$$\mathrm{MSE}\left(\hat{S}_{R(st)}^2\right)\cong\sum_{h=1}^{L}\frac{\left(W_hS_{y(h)}\right)^4}{n_h^3}\left[\left(\beta_{2y(h)}-1\right)+\left(\beta_{2x(h)}-1\right)-2\left(\theta_{22(h)}-1\right)\right]\qquad(2.3)$$

An unbiased stratified estimator

$$\hat{S}_{PS(st)}^2=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s_{y(h)}^2-\frac{s_{x(h)}^2}{S_{x(h)}^2}+1\right)$$

defined by Prasad ve Singh [16].

$$\mathrm{MSE}\left(\hat{S}_{PS(st)}^2\right)\cong\sum_{h=1}^{L}\frac{\left(W_hS_{y(h)}\right)^4}{n_h^3}\left[\left(\beta_{2y(h)}-1\right)+\frac{\left(\beta_{2x(h)}-1\right)}{S_{y(h)}^4}-2\frac{\left(\theta_{22(h)}-1\right)}{S_{y(h)}^2}\right]\qquad(2.4)$$

(iv) Usual stratified regression estimator

$$\hat{S}^2_{Reg(st)} = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{ s^2_{y(h)} + b_h \left(S^2_{x(h)} - s^2_{x(h)} \right) \right\}$$

where, b_h is the sample regression coefficient of y_h on x_h with corresponding population regression coefficient given by

$$\beta_h = \frac{S^2_{y(h)} \left(\theta_{22(h)} - 1 \right)}{S^2_{x(h)} \left(\beta_{2x(h)} - 1 \right)}$$

in the stratum h.

$$Var\left(\hat{S}^2_{Reg(st)}\right) = \sum_{h=1}^{L} \frac{\left(W_h S_{y(h)}\right)^4}{n_h^3} \left(\beta_{2y(h)} - 1\right)\left(1 - \rho_h^2\right) \tag{2.5}$$

where

$$\rho_h = \frac{\left(\theta_{22(h)} - 1\right)}{\sqrt{\left(\beta_{2y(h)} - 1\right)}\sqrt{\left(\beta_{2x(h)} - 1\right)}}$$

The ratio-type estimators

$$\hat{S}^2_{Pi(st)} = \sum_{h=1}^{L} \left(\frac{W_h^2}{n_h} \right) \left[\alpha_{1h} s^2_{y(h)} - \alpha_{2h} \left(\frac{s^2_{x(h)}}{S^2_{x(h)}} - 1 \right) \right] \left(\frac{\eta_h S^2_{x(h)} + \varpi_h}{\eta_h s^2_{x(h)} + \varpi_h} \right)$$

(i = 1,2,3,4,5) where α_{1h} and α_{2h} are suitably chosen constants and η_h and ϖ_h are known population parameters of auxiliary variable which may be $\beta_{2x(h)}$, $C_{x(h)}$ or $\theta_{22(h)}$, suggested by Shabbir and Gupta [18]. The bias is given by

$$Bias\left(\hat{S}^2_{Pi(st)}\right) = \sum_{h=1}^{L} \left(\frac{W_h^2}{n_h} \right) Bias\left(\hat{S}^2_{Pi(h)}\right) \tag{2.6}$$

$$Bias\left(\hat{S}^2_{Pi(h)}\right) \cong \left(\alpha_{1h} - 1\right) S^2_{y(h)} + \alpha_{1h} S^2_{y(h)} \frac{1}{n_h} \left(\tau_{ih}^2 \left(\beta_{2x(h)} - 1\right) - \tau_{ih} \left(\theta_{22(h)} - 1\right) \right) + \alpha_{2h} \tau_{ih} \frac{1}{n_h} \left(\beta_{2x(h)} - 1\right) \tag{2.7}$$

where

$$\alpha_{1h(opt)} = \frac{1 - \frac{1}{n_h} \tau_{ih}^2 \left(\beta_{2x(h)} - 1\right)}{1 + \frac{1}{n_h} \left[\left(\beta_{2y(h)} - 1\right)\left(1 - \rho_h^2\right) - \tau_{ih}^2 \left(\beta_{2x(h)} - 1\right) \right]}, \quad \alpha_{2h(opt)} = S^2_{y(h)} \left\{ \tau_{ih} + \alpha_{1h(opt)} \left(\frac{\left(\theta_{22(h)} - 1\right)}{\left(\beta_{2x(h)} - 1\right)} - 2\tau_{ih} \right) \right\}$$

The expressions of mean square errors (MSE), up to first order of approximation, is as follows:

$$MSE\left(\hat{S}^2_{Pi(st)}\right) = \sum_{h=1}^{L} \left(\frac{W_h^4}{n_h^2} \right) \frac{S^4_{y(h)} \left(1 - \frac{1}{n_h} \tau_{ih}^2 \left(\beta_{2x(h)} - 1\right) \right) \left(\frac{1}{n_h} \left(\beta_{2y(h)} - 1\right)\left(1 - \rho_h^2\right) \right)}{\left(1 - \frac{1}{n_h} \tau_{ih}^2 \left(\beta_{2x(h)} - 1\right) \right) + \left(\frac{1}{n_h} \left(\beta_{2y(h)} - 1\right)\left(1 - \rho_h^2\right) \right)} \tag{2.8}$$

where

$$\tau_{ih} = \frac{\eta_h S^2_{x(h)}}{\eta_h S^2_{x(h)} + \varpi_h}, \; (i = 1, \ldots, 5). \quad \tau_{1h} = \frac{S^2_{x(h)}}{S^2_{x(h)} + \beta_{2x(h)}}, \quad \tau_{2h} = \frac{S^2_{x(h)}}{S^2_{x(h)} + C_{x(h)}}$$

$$\tau_{3h} = \frac{C_{x(h)} S^2_{x(h)}}{C_{x(h)} S^2_{x(h)} + \beta_{2x(h)}}, \quad \tau_{4h} = \frac{\beta_{2x(h)} S^2_{x(h)}}{\beta_{2x(h)} S^2_{x(h)} + C_{x(h)}} \quad \tau_{5h} = \frac{S^2_{x(h)}}{S^2_{x(h)} + \theta_{22(h)}}$$

We consider stratified version of Grover [6] estimator as given by

$$\hat{S}^2_{P(st)} = \sum_{h=1}^{L}\left(\frac{W_h^2}{n_h}\right)\hat{S}^2_{P(h)} \tag{2.9}$$

where

$$\hat{S}^2_{P(h)} = \left[k_{1h}s^2_{y(h)} + k_{2h}\left(S^2_{x(h)} - s^2_{x(h)}\right)\right]\exp\left(\frac{S^2_{x(h)} - s^2_{x(h)}}{S^2_{x(h)} + s^2_{x(h)}}\right)$$

k_{1h} and k_{2h} are any constants.

$$\mathrm{Bias}\left(\hat{S}^2_{P(st)}\right) = \sum_{h=1}^{L}\left(\frac{W_h^2}{n_h}\right)\mathrm{Bias}\left(\hat{S}^2_{P(h)}\right) \tag{2.10}$$

$$\mathrm{Bias}\left(\hat{S}^2_{P(h)}\right) = \left(k_{1h}-1\right)S^2_{y(h)} - \frac{1}{2}k_{1h}S^2_{y(h)}\frac{\left(\theta_{22(h)}-1\right)}{n_h} + \frac{1}{2}\left(S^2_{x(h)}k_{2h} + \frac{3}{4}k_{1h}S^2_{y(h)}\right)\frac{\left(\beta_{2x(h)}-1\right)}{n_h} \tag{2.11}$$

Up to the first order of approximation, the mean square error of $\hat{S}^2_{P(st)}$ is as follow:

$$\mathrm{MSE}\left(\hat{S}^2_{P(st)}\right) = \sum_{h=1}^{L}\left(\frac{W_h^4}{n_h^2}\right)\mathrm{MSE}\left(\hat{S}^2_{P(h)}\right)$$

$$\begin{aligned}\mathrm{MSE}\left(\hat{S}^2_{P(h)}\right) = S^4_{y(h)}&\left\{\left(k_{1h}-1\right)^2 + k_{1h}^2\left(\frac{\left(\beta_{2y(h)}-1\right)}{n_h} + \frac{\left(\beta_{2x(h)}-1\right)}{n_h} - 2\frac{\left(\theta_{22(h)}-1\right)}{n_h}\right) + k_{1h}\left(\frac{\left(\theta_{22(h)}-1\right)}{n_h} - \frac{3}{4}\frac{\left(\beta_{2x(h)}-1\right)}{n_h}\right)\right\}\\ &+ S^4_{x(h)}k_{2h}^2\frac{\left(\beta_{2x(h)}-1\right)}{n_h} + 2S^2_{y(h)}S^2_{x(h)}\left(k_{1h}k_{2h}\left(\frac{\left(\beta_{2x(h)}-1\right)}{n_h} - \frac{\left(\theta_{22(h)}-1\right)}{n_h}\right) - \frac{k_{2h}}{2}\frac{\left(\beta_{2x(h)}-1\right)}{n_h}\right)\end{aligned} \tag{2.12}$$

The optimum value of k_{1h} and k_{2h} are:

$$k_{1h} = \frac{\left(\beta_{2x(h)}-1\right)\left\{2 - \frac{1}{4n_h}\left(\beta_{2x(h)}-1\right)\right\}}{2\left[\left(\beta_{2x(h)}-1\right)\left[1+\frac{A}{n_h}\right] - \frac{B^2}{n_h}\right]},\quad k_{2h} = \left[\frac{S^2_{y(h)}}{2S^2_{x(h)}} + \frac{-S_y^2\left(2 - \frac{1}{4n_h}\left(\beta_{2x(h)}-1\right)\right)B}{2S^2_{x(h)}\left[\left(\beta_{2x(h)}-1\right)\left(1+\frac{A}{n_h}\right) - \frac{B^2}{n_h}\right]}\right]$$

where

$$A = \left(\beta_{2y(h)}-1\right) + \left(\beta_{2x(h)}-1\right) - 2\left(\theta_{22(h)}-1\right),\quad B = \left(\beta_{2x(h)}-1\right) - \left(\theta_{22(h)}-1\right)$$

Substituting the optimum values of k_{1h} and k_{2h}, we get the minimum MSE as given by

$$\mathrm{MSE}\left(\hat{S}^2_{P(h)}\right) = \frac{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)}{\left[1+\frac{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)}{S^4_{y(h)}}\right]} - \frac{\frac{\left(\beta_{2x(h)}-1\right)}{n_h}\left\{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right) + \frac{S^4_{y(h)}\left(\beta_{2x(h)}-1\right)}{16n_h}\right\}}{4\left[1+\frac{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)}{S^4_{y(h)}}\right]} \tag{2.13}$$

where

$$\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right) = \frac{S^4_{y(h)}}{n_h}\left(\beta_{2y(h)}-1\right)\left(1-\rho_h^2\right)$$

We propose following estimators for population variance of stratified sample mean, which is given by

$$\hat{S}^2_{Ni(st)} = \sum_{h=1}^{L}\left(\frac{W_h^2}{n_h}\right)\hat{S}^2_{N(h)} \quad (i=1,2,3,4,5) \tag{2.14}$$

where

$$\hat{S}^2_{N(h)} = \left[w_{1h}s^2_{y(h)} + w_{2h}\left(\frac{s^2_{x(h)}}{S^2_{x(h)}}\right)^{\gamma_h}\right]\exp\left[\frac{\eta_h\left(S^2_{x(h)} - s^2_{x(h)}\right)}{\eta_h\left(S^2_{x(h)} + s^2_{x(h)}\right) + 2\varpi_h}\right]$$

w_{1h} and w_{2h} are suitably chosen constants and η_h and λ_h are known population parameters of auxiliary variable which may be $\beta_{2x(h)}$, $C_{x(h)}$ or $\theta_{22(h)}$, as defined by Shabbir and Gupta [18].
The bias of estimator is given by

$$\text{Bias}\left(\hat{S}^2_{Ni(st)}\right) = \sum_{h=1}^{L}\left(\frac{W_h^2}{n_h}\right)\text{Bias}\left(\hat{S}^2_{Ni(h)}\right) \tag{2.15}$$

$$\text{Bias}\left(\hat{S}^2_{N(h)}\right) = \left(w_{1h}-1\right)S^2_{y(h)} + w_{2h} - \frac{\tau_{ih}}{2}w_{1h}S^2_{y(h)}\frac{\left(\theta_{22(h)}-1\right)}{n_h} + \left(w_{2h}\left(\frac{\gamma(\gamma-1)}{2} - \frac{\gamma}{2}\tau_{ih} + \frac{3}{8}\tau^2_{ih}\right) + \frac{3}{8}\tau^2_{ih}w_{1h}S^2_{y(h)}\right)\frac{\left(\beta_{2x(h)}-1\right)}{n_h} \tag{2.16}$$

Squaring both side of (3.16) and neglecting terms of δ's having power greater than two and taking expectations of both sides we get the MSE of $\hat{S}^2_{N(h)}$ to the first order of approximation as:

$$\text{MSE}\left(\hat{S}^2_{Ni(st)}\right) = \sum_{h=1}^{L}\left(\frac{W_h^4}{n_h^2}\right)\text{MSE}\left(\hat{S}^2_{N(h)}\right)$$

$$\text{MSE}\left(\hat{S}^2_{N(h)}\right) = \left[S^4_{y(h)} + w^2_{1h}S^4_{y(h)}A + w^2_{2h}B + w_{1h}S^4_{y(h)}D + w_{2h}S^2_{y(h)}G + w_{1h}w_{2h}S^2_{y(h)}F\right] \tag{2.17}$$

where

$$A = \left(1 + \frac{\left(\beta_{2y(h)}-1\right)}{n_h} + \tau^2_{ih}\frac{\left(\beta_{2x(h)}-1\right)}{n_h} - 2\tau_{ih}\frac{\left(\theta_{22(h)}-1\right)}{n_h}\right),\quad B = \left(1 + \left\{\gamma_h^2 + \tau^2_{ih} + \gamma_h\left(\gamma_h - 1\right) - 2\gamma_h\tau_{ih}\right\}\frac{\left(\beta_{2x(h)}-1\right)}{n_h}\right)$$

$$D = \left(-2 + \tau_{ih}\frac{\left(\theta_{22(h)}-1\right)}{n_h} - \frac{3}{4}\tau^2_{ih}\frac{\left(\beta_{2x(h)}-1\right)}{n_h}\right),\quad G = \left(-2 + \left\{\tau_{ih}\gamma_h - \gamma_h\left(\gamma_h - 1\right) - \frac{3}{4}\tau^2_{ih}\right\}\frac{\left(\beta_{2x(h)}-1\right)}{n_h}\right)$$

$$F = \left(2 + \left\{\gamma_h\left(\gamma_h - 1\right) + 2\tau^2_{ih} - 2\gamma_h\tau_{ih}\right\}\frac{\left(\beta_{2x(h)}-1\right)}{n_h} + 2\left\{\gamma_h - \tau_{ih}\right\}\frac{\left(\theta_{22(h)}-1\right)}{n_h}\right)$$

The MSE is minimized for

$$w_{1h} = \frac{GF - 2DB}{4AB - F^2}$$

$$w_{2h} = \frac{S^2_{y(h)}\left(DF - 2AG\right)}{4BA - F^2} \tag{2.18}$$

Substituting the optimum values of w_{1h} and w_{2h}, we get the minimum MSE as given by

$$\text{MSE}_{\min}\left(\hat{S}^2_{Ni(h)}\right) = S^4_{y(h)}\left[1 - \frac{BD^2 - DGF + AG^2}{\left(4AB - F^2\right)}\right] \tag{2.19}$$

COMPARISON OF ESTIMATORS

In sampling literature many ratio estimator suggested by various authors isn't efficient than regression estimator, as an exception Grover [6], Koyuncu and Kadilar [12]. These studies are important because they found that their estimators are efficient than regression estimator. So here we want to emphasize that stratified version of Grover [6] is always efficient than stratified regression estimator and proposed estimator is efficient than stratified regression estimator and Grover [6] when the conditions are satisfied.

We first compare the stratified regression estimator with the stratified version of Grover [6] estimator

$$\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right) - \mathrm{MSE}\left(\hat{S}^2_{G(h)}\right) > 0$$

$$\frac{\dfrac{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)}{S^4_{y(h)}}}{\left[1+\dfrac{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)}{S^4_{y(h)}}\right]} + \frac{\dfrac{\left(\beta_{2x(h)}-1\right)}{n_h}\left\{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)+\dfrac{S^4_{y(h)}\left(\beta_{2x(h)}-1\right)}{16n_h}\right\}}{4\left[1+\dfrac{\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right)}{S^4_{y(h)}}\right]} > 0 \qquad (3.1)$$

The condition (3.1) is always satisfied, so we can say that stratified version of Grover [6] estimator is always more efficient than stratified regression estimator.

Secondly we compare the proposed class of estimators with stratified regression estimator

$$\mathrm{Var}\left(\hat{S}^2_{Reg(h)}\right) - \mathrm{MSE}\left(\hat{S}^2_{N(h)}\right) > 0$$

$$\frac{S^4_{y(h)}}{n_h}\left(\beta_{2y(h)}-1\right)\left(1-\rho^2_h\right) - \left\{S^4_{y(h)} + w^2_{1h}S^4_{y(h)}A + w^2_{2h}B \right. \qquad (3.2)$$
$$\left. + w_{1h}S^4_{y(h)}D + w_{2h}S^2_{y(h)}G + w_{1h}w_{2h}S^2_{y(h)}F\right\} > 0$$

When the condition (3.2) is satisfied, the proposed estimator $\hat{S}^2_{Ni(st)}$ is more efficient than stratified regression estimator.

EMPRICAL STUDY

To see the performance of proposed estimators of population variance in stratified random sampling scheme. Summary of the data for each strata is as follows:

(Singh and Mangat [21], p.212, y: leaf area for the newly developed strain of wheat, x: weight of leaves).

$N = 39$, $n = 14$, $N_1 = 12$, $N_2 = 13$
$N_3 = 14$, $n_1 = 4$, $n_2 = 5$, $n_3 = 5$
$C_{x1} = 0.1118928$, $C_{x2} = 0.0733753$, $C_{x3} = 0.1193784$
$S_{y1} = 6.0664603$, $S_{y2} = 5.2915229$, $S_{y3} = 6.4961301$
$S_{x1} = 11.5715804$, $S_{x2} = 8.139014$, $S_{x3} = 12.4494617$
$\beta_2(y_1) = 1.9394547$, $\beta_2(y_2) = 2.9819269$

Table 1: Some members of $\hat{S}^2_{Ni}$, $\hat{S}^2_{Ni(st)}$, $\hat{S}^2_{Nid}$

Estimator	γ_h	η_h	ϖ_h
$\hat{S}^2_{N1(st)}$	0	1	$\beta_{2x(h)}$
$\hat{S}^2_{N2(st)}$	0	1	$C_{x(h)}$
$\hat{S}^2_{N3(st)}$	0	$C_{x(h)}$	$\beta_{2x(h)}$
$\hat{S}^2_{N4(st)}$	0	$\beta_{2x(h)}$	$C_{x(h)}$
$\hat{S}^2_{N5(st)}$	0	1	$\theta_{22(h)}$
$\hat{S}^2_{N6(st)}$	1	1	$\beta_{2x(h)}$
$\hat{S}^2_{N7(st)}$	1	1	$C_{x(h)}$
$\hat{S}^2_{N8(st)}$	1	$C_{x(h)}$	$\beta_{2x(h)}$
$\hat{S}^2_{N9(st)}$	1	$\beta_{2x(h)}$	$C_{x(h)}$
$\hat{S}^2_{N10(st)}$	1	1	$\theta_{22(h)}$

$\beta_2(y_3) = 2.3448986$, $\beta_2(x_1) = 2.2748233$
$\beta_2(x_2) = 3.436904$, $\beta_2(x_3) = 2.8955496$
$\lambda_{22(1)} = 1.9123464$, $\lambda_{22(2)} = 2.970998$, $\lambda_{22(3)} = 2.5134376$

We have generated ten estimators from suggested class by substituting suitable variable γ_h, η_h, ϖ_h in stratified random as shown in Table 1. Using the bias and MSE of each estimator we have calculated bias and MSE. The results are reported in Table 2. From Table 2 we observe that members of suggested estimator are considerable more efficient than all estimators in survey sampling literature. By the theorotical way we can say that stratified version of Grover [6] estimator is always

Table 2: MSE and bias of estimators in stratified random sampling

Estimator	MSE	Bias	Estimator	MSE	Bias
$\hat{S}^2_{y(st)}$	0.649830316	Unbiased	$\hat{S}^2_{N1(st)}$	0.023928836	-0.02829
$\hat{S}^2_{R(st)}$	0.161345817	-0.55313	$\hat{S}^2_{N2(st)}$	0.024395811	-0.02888
$\hat{S}^2_{PS(st)}$	0.613226536	Unbiased	$\hat{S}^2_{N3(st)}$	0.019058494**	-0.0218
$\hat{S}^2_{Reg(st)}$	0.116676239	Unbiased	$\hat{S}^2_{N4(st)}$	0.02440685	-0.0289
$\hat{S}^2_{P(st)}$	0.094564711	-0.11327	$\hat{S}^2_{N5(st)}$	0.024003481	-0.02839
$\hat{S}^2_{P1(st)}$	0.106589594	-0.12739	$\hat{S}^2_{N6(st)}$	0.156913494	-0.18241
$\hat{S}^2_{P2(st)}$	0.106211405	-0.12683	$\hat{S}^2_{N7(st)}$	0.149461117	-0.17278
$\hat{S}^2_{P3(st)}$	0.108249991	-0.12977	$\hat{S}^2_{N8(st)}$	0.214113793	-0.25758
$\hat{S}^2_{P4(st)}$	0.106203465	-0.12682	$\hat{S}^2_{N9(st)}$	0.149292451	-0.17257
$\hat{S}^2_{P5(st)}$	0.106540528	-0.12739	$\hat{S}^2_{N10(st)}$	0.155862965	-0.18106

more efficient than stratified regression estimator. When we compare suggested estimators with Shabbir and Gupta [17] estimator, we can say that suggested family of estimator is highly efficient than Shabbir and Gupta [17] estimator.

CONCLUSION

In this paper, we have suggested a class of exponential type of estimators for population variance. To see the performance of suggested estimators we use stratified random sampling scheme. We can generate many estimators from suggested class by substituting suitable variable. We observe that members of suggested estimator are considerable more efficient than all estimators in survey sampling literature.

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