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Exact Solutions for Nonlinear Evolution Equations with Jacobi Elliptic Function Rational Expansion Method

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Abstract: In this paper we implement the unified rational expansion methods, which leads to find exact rational formal polynomial solutions of nonlinear partial differential equations (NLPDEs), to the (1+1)-dimensional dispersive long wave and Clannish Random Walker's parabolic (CRWP) equations. By using this scheme, we get some solutions of the (1+1)-dimensional dispersive long wave and CRWP equations in terms of Jacobi elliptic functions.

Key words: Unified rational expansion methods . Jacobi elliptic function rational expansion method . (1+1)-dimensional dispersive long wave equation . CRWP equation

INTRODUCTION

The theory of nonlinear dispersive wave motion is an interesting area investigated in the numerous articles in which it appears in various subjects. We do not attempt to characterize the general form of nonlinear dispersive wave equations [1, 2]. These studies for nonlinear partial differential equations have attracted much attention in mathematical physics and play a crucial role in applied mathematics. Furthermore, when an original nonlinear equation is directly calculated, the solution will preserve the actual physical characters of solutions [3]. Explicit solutions to the nonlinear equations are of fundamental importance. Also different methods for acquiring explicit solutions to nonlinear evolution equations have been suggested. Many explicit exact methods have been established in [4-32]. We may list them such as generalized Miura transformation, Darboux Transformation, Cole-Hopf Transformation, Hirota's dependent variable Transformation, the inverse scattering Transform and the Backlund Transformation, tanh method, sine-cosine method, Painlevé method, homogeneous balance method, similarity reduction method, Kudryashov method,etc. The author presented a powerful and effective method for obtaining exact solutions of nonlinear ordinary differential equations in [13]. Our aim is to find exact solutions of nonlinear PDE's with the final version of unified rational expansion method in [13].

In next section we give the analyse of the method given in [13]. In the following section, we apply the method given in [13] to the CRWP equation and (1+1)-dimensional dispersive long wave equation. In the last section, we give the conclusion.

SUMMARY OF THE RATIONAL EXPANSION METHOD

In the following we would like to outline the main steps of our method:

Step 1: Given a system of polynomial NLEEs with constant coefficients, with some physical fields

$$u(x,t) = u(\xi)$$

$$\psi(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0 \qquad (2.1)$$

use the wave transformation ξ = kx-wt where k, l and k are constants to be determined later. Then the nonlinear partial differential system (2.1) is reduced to a nonlinear ordinary differential equation (ODE)

$$\psi(u, -wu', ku', u'', u''', \ldots) = 0 \qquad (2.2)$$

Step 2: Ansatz in terms of finite rational formal expansion in the following forms:

$$u(\xi) = a0 + \sum_{j=1}^{m} \frac{\sum_{r_1+r_2=j} a^{j}_{r_1 r_2} F^{r_1}(\xi) G^{r_2}(\xi)}{\left(\mu_1 F(\xi) + \mu_2 G(\xi) + 1\right)^{j}} \qquad (2.3)$$

where $a_0, a^{j}_{r_1 r_2}, \mu_1$ ve $\mu_2 (i = 1,2,\ldots)$ are constants to be determined later and the new variables

$$F = F(\xi) \text{ ve } G = G(\xi)$$

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satisfy

$$\frac{dF}{d\xi} = K_1(F,G)\,,\ \frac{dG}{d\xi} = K_2(F,G)$$

here K_1 and K_2 are polynomial of F and G.

Step 3: Determine the m of the rational formal polynomial solutions (2.3) by respectively balancing the highest nonlinear terms and the highest-order partial derivative terms in the given system equations [7-12] and then give the formal solutions.

Step 4: Substitute (2.3) into (2.2) and then set all coefficients of $F^p(\xi)G^q(\xi)$, (p = 1,2,... and q = 0,1) of the resulting systems numerator to be zero to get an over-determined system of nonlinear algebraic equations with respect to k, $a_0, a^j_{r_1r_2}, \mu_1$ ve μ_2

Step 5: By solving the over-determined system of nonlinear algebraic equations by use of symbolic computation system Maple or Matematica, we end up with the explicit expressions for k, $a_0, a^j_{r_1r_2}, \mu_1$ ve μ_2

Step 6: According to the general solutions of system and the conclusions in Step 5, we can obtain rational formal exact solutions of system (2.1) [13].

JACOBI ELLIPTIC FUNCTION RATIONAL EXPANSION METHOD

In this section we would like to apply our method to obtain rational formal Jacobi elliptic function solutions of NLEEs, i.e., restricting F and G in Jacobi elliptic functions.

Here sn, cn, dn, sc, cs, nc, nd, sd and ns are the Jacobian elliptic sine function, the Jacobian elliptic cosine function and the Jacobian elliptic function of the third kind and other Jacobian functions which is denoted by Glaishers symbols and are generated by these three kinds of functions, namely [19],

$$ns(\xi) = \frac{1}{sn(\xi)}, nc(\xi) = \frac{1}{cn(\xi)}, nd(\xi) = \frac{1}{dn(\xi)}, sd(\xi) = \frac{sn(\xi)}{dn(\xi)}$$

$$sc(\xi) = \frac{sn(\xi)}{cn(\xi)}, cs(\xi) = \frac{cn(\xi)}{sn(\xi)}, ds(\xi) = \frac{dn(\xi)}{sn(\xi)}$$

which are double periodic and posses the following properties

1. Properties of triangular function

$$sn(\xi)^2 + cn^2(\xi) = 1$$

$$m^2 sn(\xi)^2 + dn^2(\xi) = 1$$

2. Derivatives of the Jacobi elliptic functions

$$\frac{d}{d\xi}\left(sn(\xi)\right) = cn(\xi)dn(\xi)$$

$$\frac{d}{d\xi}\left(cn(\xi)\right) = -sn(\xi)dn(\xi)$$

$$\frac{d}{d\xi}\left(dn(\xi)\right) = -m^2 sn(\xi)cn(\xi)$$

where m is a modulus.

3. Properties of limit

$$sn(\xi,0) = \sin(\xi)$$

$$sn(\xi,1) = \tanh(\xi)$$

$$cn(\xi,0) = \cos(\xi)$$

$$cn(\xi,1) = \mathrm{sech}(\xi)$$

$$dn(\xi,0) = 1$$

$$dn(\xi,1) = \mathrm{sech}(\xi)$$

The Jacobi-Glaisher functions for elliptic function can be found in Ref. [19].

Example 1: The six main steps of the Jacobi elliptic function (here just consider the condition F(ξ) = sn(ξ) and G(ξ) = cn(ξ)) rational expansion method are illustrated with Clannish random walkers parabolic equation (CRWP),

$$u_t - u_x + 2uu_x - u_{xx} = 0 \qquad (3.1)$$

According to the Step 1 in Section 2, we make the following travelling wave transformation

$$u(x,t) = y(\xi)\,, \xi = x - ct$$

where c is constant to be determined later and thus (3.1) becomes

$$-cu' - u' + 2uu' - u'' = 0 \qquad (3.2)$$

According to Step 2 in Section 2, we expand the solution of equation (3.2) in the form for m→0

$$u(\xi) = a0 + \sum_{i=1}^{m} \frac{a_i \sin^i(\xi) + b_i \sin^{i-1}(\xi)\cos(\xi)}{\left(\mu_1 \sin(\xi) + \mu_2 \cos(\xi) + 1\right)^i} \tag{3.3}$$

According to Step 3 in Section 2, by balancing uu' and u'' in equation (3.2) we can obtain that m = 1. So we have

$$u(\xi) = a_0 + \frac{a_1 \sin(\xi) + b_1 \cos(\xi)}{\mu_1 \sin(\xi) + \mu_2 \cos(\xi) + 1} \tag{3.4}$$

According to Step 4 in Section 2, with the aid of Maple, substituting (3.4) into (3.2), yields a set of algebraic equations for $\sin^i(\xi)\cos^i(\xi)$ (i = 0,1,2,…) Setting the coefficients of these terms $\sin^i(\xi)\cos^i(\xi)$ of the resulting equation numerator to be zero yields a set of over-determined algebraic equations with respect to a_0, a_1, b_1, μ_1 and μ_2.

According to Step 5 in Section 2, by use of the Matematica, solving the over-determined algebraic equations, we get explicit expressions for a0, a_0, a_1, b_1, μ_1.

According to Step 6 in Section 2, we get following algebraic equations system of CRWP equation

$$b_1(1 - 2b_1\mu_1 - 2\mu_1^2 + (1 - 2a_0 + c)\mu_1\mu_2) - a_1(1 + c - 2b_1\mu_2 - 2\mu_1\mu_2 + \mu_2^2 + c\mu_2^2 - 2a_0(1 + \mu_2^2)) = 0$$

$$\frac{1}{2}(3a_1\mu_1 + 3b_1\mu_1 - 6a_0b_1\mu_1 + 3b_1c\mu_1 - 3a_1\mu_2 + 6a_0a_1\mu_2 + 3b_1\mu_2 - 3a_1c\mu_2) + \frac{1}{2}(b_1((-1 + 2a_0 - c)\mu_1 - \mu_2) + a_1(4b_1 + \mu_1 + (-1 + 2a_0 - c)\mu_2))) = 0 \tag{3.5}$$

$$2a_1 + 2b_1 - 4a_0b_1 + 2b_1c - 4a_1b_1\mu_1 + 2b_1\mu_1^2 - 4a_0b_1\mu_1^2 + 2b_1c\mu_1^2 + 4a_1^2\mu_2 - 2a_1\mu_1\mu_2 + 4a_0a_1\mu_1\mu_2 + 4b_1\mu_1\mu_2 - 2a_1c\mu_1\mu_2 - 4a_1\mu_2^2 = 0$$

$$2a_1^2 - 2b_1^2 - a_1\mu_1 + 2a_0a_1\mu_1 - b_1\mu_1 - a_1c\mu_1 - a_1\mu_2 + b_1\mu_2 - 2a_0b_1\mu_2 + b_1c\mu_2 = 0$$

From the solution of the algebraic equations (3.5) we have

1. Family

$$a_0 = \pm\frac{1}{4}\left(2 + \sqrt{2} + 2c\right),\ b_1 = \pm\frac{1}{4}\sqrt{9 - 16a_1^2},\ \mu_2 = \frac{4\left(2a_1^2 - b_1^2\right)\left(-1 + 2a_0 - c\right)}{3(-2a_1 + 4a_0a_1 + b_1 - 2a_1c)}$$
$$a_1 \neq 0, \mu_1 = -\frac{4}{3}\left(-a_1 + 2a_0a_1 + b_1 - a_1c\right), -2a_1 + 4a_0a_1 + b_1 - 2a_1c \neq 0 \tag{3.6}$$

2. Family

$$a_0 = \pm\frac{1}{4}\left(2 - \sqrt{2} + 2c\right),\ a_1 = 0, b_1 = \pm\frac{3}{4}, \mu_1 = -\frac{4b1}{3},\ \mu_2 = \frac{4}{3}\left(b_1 - 2a_0b_1 + b_1c\right) \tag{3.7}$$

3. Family

$$a_1 = 0, b_1 = \pm\frac{1}{2}\left(-2 + 4a_0 - 4a_0^2 - 2c + 4a_0c - c^2\right), b_1 \neq 0, \mu_1 = 1,$$
$$\mu_2 = \left(-1 + 2a_0 - c\right), 1 - 8a0 + 8a0^2 + 4c - 8a0c + 2c^2 \neq 0. \tag{3.8}$$

4. Family

$$a_0 = \pm\frac{1}{4}\left(2 - \sqrt{2} + 2c\right),\ b_1 = \pm\frac{1}{4}\sqrt{9 - 16a_1^2}, a_1 \neq 0, \mu_1 = \frac{1}{3}\left(2\sqrt{2}a_1 + \sqrt{9 - 16a_1^2}\right)$$
$$-2a_1 + 4a_0a_1 + b_1 - 2a_1c \neq 0, \mu_2 = \frac{-3 + 16a_1^2}{8a_1 + \sqrt{18 - 32a_1^2}} \tag{3.9}$$

and from these coefficients we have the following solutions respectively.

Solution

$$u(x,t)=\frac{1}{4}\left(2-\sqrt{2}+2c\right)+\frac{a_1\sin(x-ct)-b_1\cos(x-ct)}{-\frac{4}{3}(-a_1+2a_0a_1-b_1-a_1c)\sin(x-ct)+\frac{4(2a_1^2-(-b_1)^2)(-1+2a_0-c)}{3(-2a_1+4a_0a_1+-\frac{1}{4}b_1-2a_1c)}\cos(x-ct)+1} \tag{3.10}$$

Solution

$$u(x,t)=\frac{\frac{1}{4}(2-\sqrt{2}+2c)-\frac{3}{4}\cos(x-ct)}{\sin(x-ct)+\frac{4}{3}(-\frac{3}{4}+\frac{3}{8}(2-\sqrt{2}+2c)-\frac{3}{4}c)\cos(x-ct)+1} \tag{3.11}$$

Solution

$$u(x,t)=a_0+\frac{\frac{1}{2}(-2+4a_0-4a_0^2-2c+4a_0c-c^2)\cos(x-ct)}{\sin(x-ct)+(-1+2a_0-c)\cos(x-ct)+1} \tag{3.12}$$

Solution

$$u(x,t)=\frac{1}{4}(2-\sqrt{2}+2c)+\frac{a_1\sin(x-ct)-b_1\cos(x-ct)}{\frac{1}{3}(2\sqrt{2}a_1+4b_1)\sin(x-ct)+(\frac{-3+16a_1^2}{8a_1+\sqrt{18-32a_1^2}})\cos(x-ct)+1} \tag{3.13}$$

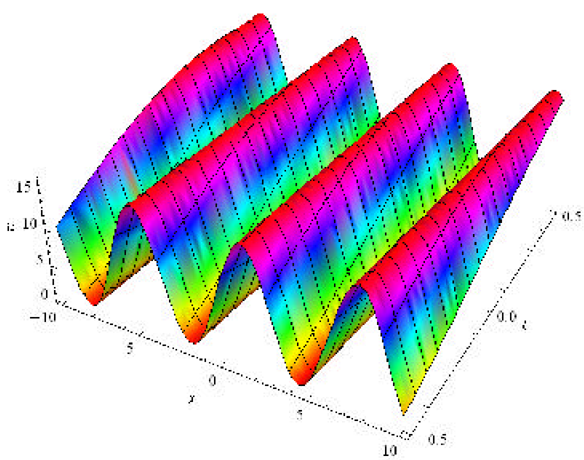

Fig. 1: Graph of the reel part of (3.13) for 4. u(x,t) corresponding to the values a_1 = i and c = 2

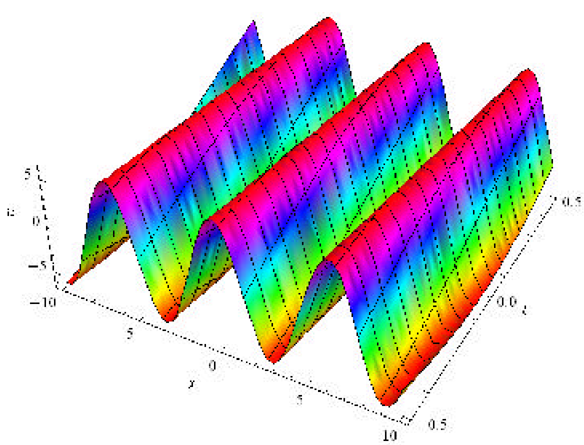

Fig. 2: Graph of the imaginary part of (3.13) for 4. u(x,t) corresponding to the values a_1 = i and c = 2

Example 2: The (1+1)-dimensional dispersive long wave equation

$$u_t+uu_x+v_x=0, v_t+vu_x+uv_x+\frac{1}{3}u_{xxx}=0 \tag{3.14}$$

According to the Step 1 in Section 2, we make the following travelling wave transformation

$$u(x,t)=y(\xi), \xi=x-ct$$

where c is constant to be determined later and thus (3.14) becomes

$$-cu'+uu'+v'=0 \quad -cv'+vu'+uv'+\frac{1}{3}u'''=0 \tag{3.15}$$

Integrating the second equation of equations (3.15) once with regard to ξ, we obtain

$$-cu+\frac{1}{2}u^2+v=0, -cv+uv+\frac{1}{3}u''=0 \tag{3.16}$$

with the integration constants taken to be zero. According to Step 2 in Section 2, we expand the solution of equation (3.16) in the form for m→0

$$u(\xi)=a0+\sum_{i=1}^{m}\frac{a_i\sin^i(\xi)+b_i\sin^{i-1}(\xi)\cos(\xi)}{\left(\mu_1\sin(\xi)+\mu_2\cos(\xi)+1\right)^i} \tag{3.17}$$

According to Step 3 in Section 2, by balancing uv and u'' in equation (3.16) we can obtain that m = 1 and by balancing v and u^2 in equation (3.16) we can obtain that n = 2. So we have

$$
\begin{aligned}
u(\xi) &= h_0 + \frac{h_1\sin(\xi) + k_1\cos(\xi)}{\mu_1\sin(\xi) + \mu_2\cos(\xi) + 1} \\
v(\xi) &= a_0 + \frac{a_1\sin(\xi) + b_1\cos(\xi)}{\mu_1\sin(\xi) + \mu_2\cos(\xi) + 1} + \frac{a_2\sin(\xi)^2 + b_2\sin(\xi)\cos(\xi)}{(\mu_1\sin(\xi) + \mu_2\cos(\xi) + 1)^2}
\end{aligned}
\tag{3.18}
$$

According to Step 4 in Section 2, with the aid of Matemarica, substituting (3.18) into (3.16), yields a set of algebraic equations for $\sin^i(\xi)\cos^i(\xi)$ (i = 0,1,2,…) Setting the coefficients of these terms $\sin^i(\xi)\cos^i(\xi)$ of the resulting equation numerator to be zero yields a set of over-determined algebraic equations with respect to h_0, h_1, k_1, μ_1, μ_2, a_0, a_1, a_2, b_1 and b_2.

According to Step 5 in Section 2, by using of the Matematica and solving the over-determined algebraic equations, we get explicit expressions for h_0, h_1, k_1, μ_1, μ_2, a_0, a_1, a_2, b_1 and b_2.

According to Step 6 in Section 2, we get following algebraic equations system of (1+1)-dimensional dispersive long wave equation

$$
\begin{aligned}
&-24b_1h_0\mu_1 - 6b_2h_1\mu_1 + 8k_1\mu_1 - 24a_0k_1\mu_1 - 6a_2k_1\mu_1 + 24ck_1\mu_1 - 6b_2h_0\mu_1^2 - 6b_1h_1\mu_1^2 - 6a_1k_1\mu_1^2 - 6b_1h_0\mu_1^3 \\
&-16k_1\mu_1^3 - 6a_0k_1\mu_1^3 + 6ck_1\mu_1^3 + 24a_1h_0\mu_2 - 8h_1\mu_2 + 24a_0h_1\mu_2 + 18a_2h_1\mu_2 - 24ch_1\mu_2 + 6b_2k_1\mu_2 + 12a_2h_0\mu_1\mu_2 \\
&+12a_1h_1\mu_1\mu_2 - 12b_1k_1\mu_1\mu_2 + 6a_1h_0\mu_1^2\mu_2 + 16h_1\mu_1^2\mu_2 + 6a_0h_1\mu_1^2\mu_2 - 6ch_1\mu_1^2\mu_2 + 6b_2h_0\mu_2^2 + 6b_1h_1\mu_2^2 + 6a_1k_1\mu_2^2 \\
&-6b_1h_0\mu_1\mu_2^2 - 16k_1\mu_1\mu_2^2 - 6a_0k_1\mu_1\mu_2^2 + 6ck_1\mu_1\mu_2^2 + 6a_1h_0\mu_2^3 + 16h_1\mu_2^3 + 6a_0h_1\mu_2^3 - 6ch_1\mu_2^3 = 0
\end{aligned}
$$

$$
\begin{aligned}
&3b_2k_1 + 6a_2h_0\mu_1 - 30b_1k_1\mu_1 + 18b_2h_0\mu_2 - 30b_1h_0\mu_1\mu_2 + 10k_1\mu_1\mu_2 - 30a_0k_1\mu_1\mu_2 + 30ck_1\mu_1\mu_2 + h_1(-4 + 9a_2 - 12c \\
&+23\mu_1^2 - 3c\mu_1^2 + 18b_1\mu_2 + 13\mu_2^2 - 33c\mu_2^2 + 3a_0(4 + \mu_1^2 + 11\mu_2^2)) + 3a_1(2h_1\mu_1 + 6k_1\mu_2 + h_0(4 + \mu_1^2 + 11\mu_2^2))) = 0
\end{aligned}
$$

$$
\begin{aligned}
&6a_1k_1 + 4k_1\mu_1 + 6a_0k_1\mu_1 + 3a_2k_1\mu_1 - 6ck_1\mu_1 + 3a_1k_1\mu_1^2 - 4k_1\mu_1^3 + 3a_0k_1\mu_1^3 - 3ck_1\mu_1^3 + 6a_1h_0\mu_2 + 4h_1\mu_2 \\
&+6a_0h_1\mu_2 - 9a_2h_1\mu_2 - 6ch_1\mu_2 - 6a_2h_0\mu_1\mu_2 - 6a_1h_1\mu_1\mu_2 - 3a_1h_0\mu_1^2\mu_2 + 4h_1\mu_1^2\mu_2 - 3a_0h_1\mu_1^2\mu_2 + 3ch_1\mu_1^2\mu_2 \\
&+3a_1k_1\mu_2^2 + 4k_1\mu_1\mu_2^2 - 3a0k_1\mu_1\mu_2^2 + 3ck_1\mu_1\mu_2^2 + 3a_1h_0\mu_2^3 - 4h_1\mu_2^3 + 3a_0h_1\mu_2^3 - 3ch_1\mu_2^3 + 3b_2(h_1\mu_1 + k_1\mu_2 \\
&+h_0(2 + \mu_1^2 + \mu_2^2)) + 3b_1(h_1(2 + \mu_1^2 + \mu_2^2) + \mu_1(-2k_1\mu_2 + h_0(2 + \mu_1^2 - \mu_2^2))) = 0
\end{aligned}
$$

$$
\begin{aligned}
&-9a_2h_1 + 9b_2k_1 - 6a_2h_0\mu_1 - 6a_1h_1\mu_1 + 6b_1k_1\mu_1 - 3a_1h_0\mu_1^2 + h_1\mu_1^2 - 3a_0h_1\mu_1^2 + 3ch\mu1_{2} + 6b_2h_0\mu_2 + 6b_1h_1\mu_2 \\
&+6a_1k_1\mu_2 + 6b_1h_0\mu_1\mu_2 - 2k_1\mu_1\mu_2 + 6a_0k_1\mu_1\mu_2 - 6ck_1\mu_1\mu_2 + 3a_1h_0\mu_2^2 - h_1\mu_2^2 + 3a_0h_1\mu_2^2 - 3ch_1\mu_2^2 = 0
\end{aligned}
$$

$$
\begin{aligned}
&-12b_1h_0 - 3b_2h_1 + 4k_1 - 12a_0k_1 - 3a_2k_1 + 12ck_1 - 18b_2h_0\mu_1 - 18b_1h_1\mu_1 - 18a_1k_1\mu_1 - 33b_1h_0\mu_1^2 - 13k_1\mu_1^2 - 33a_0k_1\mu_1^2 + 33ck_1\mu_1^2 \\
&+30a_2h_0\mu_2 + 30a_1h_1\mu_2 - 6b_1k_1\mu_2 + 30a1h_0\mu_1\mu_2 - 10h1\mu_1\mu_2 + 30a_0h1\mu_1\mu_2 - 30ch1\mu_1\mu_2 - 3b_1h_0\mu_2^2 - 23k_1\mu_2^2 - 3a_0k_1\mu_2^2 + 3ck_1\mu_2^2 = 0
\end{aligned}
$$

$$
\begin{aligned}
&12a_2h_0 + 12a_1h_1 - 12b_1k_1 + 12a_1h_0\mu_1 + 8h_1\mu_1 + 12a_0h_1\mu_1 - 12ch_1\mu_1 - 12b_2k_1\mu_1 - 12b_1k_1\mu_1^2 - 12b_1h_0\mu_2 + 12b_2h_1\mu_2 \\
&-8k_1\mu_2 - 12a_0k_1\mu_2 + 12a_2k_1\mu_2 + 12ck_1\mu_2 - 12b_1h_0\mu_1^2\mu_2 + 16k_1\mu_1^2\mu_2 - 12a_0k_1\mu_1^2\mu_2 + 12ck_1\mu_1^2\mu_2 + 12a_2h_0\mu_2^2 + 12a_1h_1\mu_2^2 \\
&+12a_1h_0\mu_1\mu_2^2 - 16h_1\mu_1\mu_2^2 + 12a_0h_1\mu_1\mu_2^2 - 12ch_1\mu_1\mu_2^2 = 0
\end{aligned}
$$

$$
-3b1\mu1 + 3ck1\mu1 - 3h0k1\mu1 + 3a1\mu2 - 3ch1\mu2 + 3h0h1\mu2 = 0
$$

$$
\begin{aligned}
&(2b_2 + 2h_1k_1 + b_1\mu_1 - ck_1\mu_1 + h_0k_1\mu_1 + a_1\mu_2 - ch_1\mu_2 + h_0h_1\mu_2) = 0 \\
&(-2b_1 + 2ck_1 - 2h_0k_1 - 2b_2\mu_1 - 2h_1k_1\mu_1 - 2b_1\mu_1^2 + 2ck_1\mu_1^2 - 2h_0k_1\mu_1^2 \\
&+4a_2\mu_2 + 2h_1^2\mu_2 + 2a_1\mu_1\mu_2 - 2ch_1\mu_1\mu_2 + 2h_0h_1\mu_1\mu_2) = 0 \\
&2a_2 + h_1^2 - k_1^2 + a_1\mu_1 - ch_1\mu_1 + h_0h_1\mu_1 - b_1\mu_2 + ck_1\mu_2 - h_0k_1\mu_2 = 0
\end{aligned}
\tag{3.19}
$$

From the solution of the algebraic equations (3.19) we have

Family

$$\mu_1 = 1, \mu_2 = -1, a_2 = -\frac{2}{3}, a_0 = \frac{1}{6}\left(-4 - 3b_2 + 6c - 6ch_0 + 6h_0^{\,2}\right), c - h_0 \neq 0$$
$$h_1 = \frac{3 \pm \sqrt{3}\sqrt{3 + 8ch_0 - 8h_0^{\,2}}}{6(c - h0)}, k_1 = 0, a_1 = b_2 + ch_1 - h_0 h_1, b_1 = -b_2, h_0 \neq 0 \tag{3.20}$$

Family

$$\mu_1 = 1, \mu_2 = -1, c = h_0, a_2 = -\frac{2}{3}, a_0 = \frac{1}{6}\left(-4 - 3b_2 + 6h_0\right)$$
$$h_1 = -\frac{2h_0}{3}, k_1 = 0, a_1 = b_2, b_1 = -b_2, h_0 \neq 0 \tag{3.21}$$

Family

$$\mu_1 = 1, \mu_2 = -1, h_0 = 0, a_2 = -\frac{2}{3}, a_0 = \frac{1}{6}\left(-4 - 3b_2 + 6c\right)$$
$$c \neq 0, h_1 = \frac{1}{c}, k_1 = 0, a_1 = b_2 + ch_1, b_1 = -b_2 \tag{3.22}$$

Family

$$\mu_1 = 1, \mu_2 = -1, a_2 = -\frac{2}{3}, a_0 = \frac{1}{6}\left(-4 - 3b_2 + 6c - 6ch_0 + 6h_0^{\,2}\right)$$
$$c - h_0 \neq 0, h_1 = \frac{3 \pm \sqrt{3}\sqrt{3 + 8ch_0 - 8h_0^{\,2}}}{6\left(c - h_0\right)}, b_1 = -b_2, k_1 = 0, a_1 = b_2 + ch_1 - h_0 h_1 \tag{3.23}$$

From these coefficients we get following solutions of (1+1)-dimensional dispersive long wave equation with respectively.

Solution

$$u\left(x,t\right) = h_0 + \frac{\frac{3 \pm \sqrt{3}\sqrt{3 + 8ch_0 - 8h_0^{\,2}}}{6\left(c - h_0\right)}\sin[x - ct]}{\sin[x - ct] - \cos[x - ct] + 1} \tag{3.24}$$

$$v\left(x,t\right) = \frac{1}{6}\left(-4 - 3b_2 + 6c - 6ch_0 + 6h_0^{\,2}\right) + \frac{-\frac{2}{3}\sin[x - ct]^2 + b_2\sin[x - ct]\cos[x - ct]}{(\sin[x - ct] - \cos[x - ct] + 1)^2}$$
$$+ \frac{(b_2 + \frac{1}{6}(3 - \sqrt{3}\sqrt{3 + 8ch_0 - 8h_0^{\,2}}))\sin[x - ct] - b_2\cos[x - ct]}{\sin[x - ct] - \cos[x - ct] + 1} \tag{3.25}$$

Solution

$$u\left(x,t\right) = c + \frac{-\frac{2c}{3}\sin[x - ct]}{\sin[x - ct] - \cos[x - ct] + 1} \tag{3.26}$$

$$v\left(x,t\right) = \frac{1}{6}(-4 - 3b_2 + 6c) + \frac{b_2\sin[x - ct] - b_2\cos[x - ct]}{\sin[x - ct] - \cos[x - ct] + 1} + \frac{-\frac{2}{3}\sin[x - ct]^2 + b_2\sin[x - ct]\cos[x - ct]}{(\sin[x - ct] - \cos[x - ct] + 1)^2} \tag{3.27}$$

Solution

$$u\left(x,t\right) = \frac{\frac{1}{c}\sin[x - ct]}{\mu_1\sin[x - ct] + \mu_2\cos[x - ct] + 1} \tag{3.28}$$

$$v\left(x,t\right) = \frac{1}{6}(-4 - 3b_2 + 6c) + \frac{(b_2 + 1)\sin[x - ct] - b_2\cos[x - ct]}{\sin[x - ct] - \cos[x - ct] + 1} + \frac{-\frac{2}{3}\sin[x - ct]^2 + b_2\sin[x - ct]\cos[x - ct]}{(\sin[x - ct] - \cos[x - ct] + 1)^2} \tag{3.29}$$

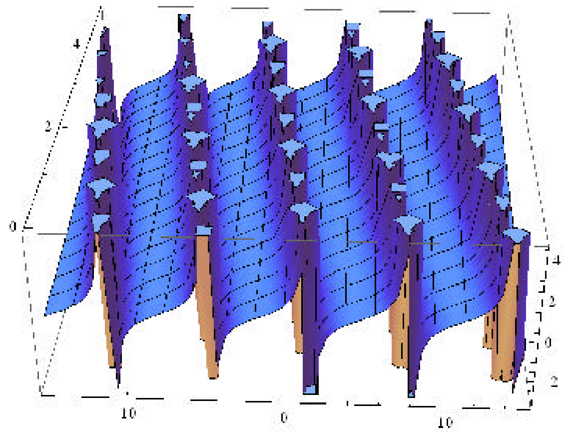

Fig. 3: Graph of the (3.30) for u(x,t) corresponding to the values $b_2 = 1$, $h_0 = \frac{\sqrt{3}}{2}$ and $c = \frac{\sqrt{3}}{4}$

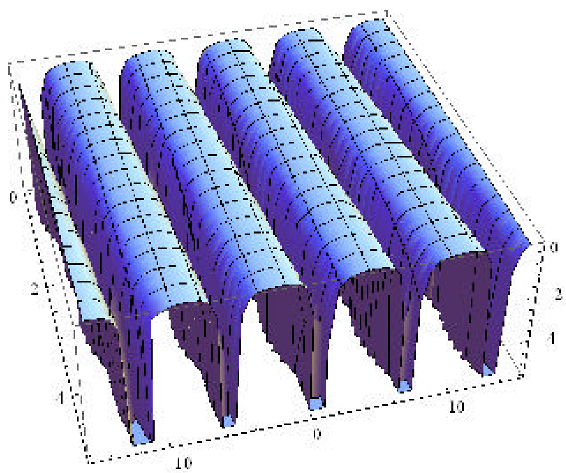

Fig. 4: Graph of the (3.31) for v(x,t) corresponding to the values $b_2 = 1$, $h_0 = \frac{\sqrt{3}}{2}$ and $c = \frac{\sqrt{3}}{4}$

Solution

$$u(\xi) = h_0 + \frac{\left(3 \pm \sqrt{3}\sqrt{3 + 8ch_0 - 8h_0^2}\right)\sin(\xi)}{6(c - h_0)\left(\sin(\xi) - \cos(\xi) + 1\right)} \tag{3.30}$$

$$v(\xi) = a_0 + \frac{(b_2 + ch_1 - h_0h_1)\sin(\xi) - b_2\cos(\xi)}{\sin(\xi) - \cos(\xi) + 1} + \frac{-\frac{2}{3}\sin(\xi)^2 + b_2\sin(\xi)\cos(\xi)}{\left(\sin(\xi) - \cos(\xi) + 1\right)^2} \tag{3.31}$$

CONCLUSION

In this paper, we apply the Jacobi elliptic function rational expansion method [13] to show that this method is efficient and practically well suited to use in finding exact travelling wave solutions for the CRWP equation and system of (1+1)-dimensional dispersive long wave equation. By using Mathematica, we have provided the correctness of the obtained solutions by putting them back into the original equation. These solutions will be useful for further studies in applied sciences.

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