

A Case Study of Variable Window Size in Linear Prediction Techniques

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Abstract: Thenormal trendin Linear Prediction (LP) techniques is fixed frame windowing. In this paper, however, dynamic window concept is introduced whereframe size is kept variable in order to achieve efficient outcome in terms of computational cost, mean square error and prediction gain. The three famous LP techniques namely Normal Equations, Levinson Durbin Algorithm (LDA) and Leroux Gueguen Algorithm (LGA) are briefly discusse dusing variable frame windowing and the above mentioned parameters are analyzed. Simulation results for the above three algorithms suggest that LDA and LGA have shown better performance than Normal Equation method based on reduced prediction error, low computational time and high prediction gain.

Key words: Linear prediction theory . system identification . internal and external prediction . Levinson Durbin algorithm . Leroux Gueguen . normal equation

INTRODUCTION

Linear Prediction (LP) is a system identification process of signal reconstructionfrom its previous samples [1-3, 17, 20]. It's the wide spread applications of LPsuch as data forecasting, speech coding, video coding, speech recognition, signal restoration, model-based spectral analysis, model-based interpolation and impulse/step input detection [4-6], which has gained the interest of researchers to work in this field. LP is used in the algorithms of speech coding in which the speech samples are modeled as a linear combination of the past output, present output and past inputs [1]. According to authors of [7] redundancy in the information can be removed with the help of LP technique. Hence there is no need for transmission of a certain amount of data which can be predicted [8]. The LP algorithms can help in predicting the stock market. Another application of LP is the estimation of a signal fundamental frequency [8]. LP can be achieved externally or internally based on the position of the predicted samples [8]. In external LinearPrediction, the signal samples are estimated outside the desired frame while in internal linear prediction the signal samples are estimated within the frame.

The basic idea of the Linear Prediction is to estimate the future data samples on the basis of the past values of the input signal within a signal frame, the weights used to compute the linear combination are

calculated by minimizing the mean square prediction error [3, 8]. In internal prediction, Linear Prediction Coefficients (LPCs) {also sometimes known as LP coding} are computed from the selected data frame by using autocorrelation concept for processing of the data window. The LPCs of external prediction are implemented for predicting the lost samples of data that means LPCs associated with external prediction are computed from the past samples of the signal [8]. The conventional Normal Equation method has been found computationally expensive [3, 6]. Alternatively, Levinson Durbin Algorithm (LDA) [16-18] considerably reduces this computational cost by avoiding the larger matrix inversions involved in the computation of LPCs. However, LDA has the drawback of a larger dynamic range in the values of LPC [8]. Another alternate isLerouxGueguen Algorithm (LGA) which eliminates the problem of dynamic range in a fixed point environment using the Schwartz inequality in computation of the LPC [8]. The LDA and LGA utilize the properties of autocorrelation matrix thereby decreasing the computational time as compared to the Normal Equation method [8].

In [9], performance optimization of the speech coding algorithms based on Linear Prediction is done through optimization of window size. This approach is based on the principle of gradient descent. The optimized window improves the system performance with no computational complexity for many cases.

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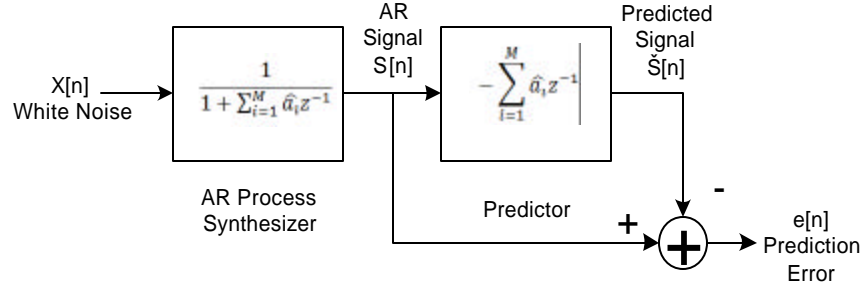


Fig. 1: Linear prediction

Khan *et al.* [10] proposes the use of Normal Linear Prediction for prediction of the lost samples to improve performance of the estimation through Kalman filter. Simulation results have shown better performance by the compensated closed loop Kalman filter with Normal Linear Prediction as compared to open loop Kalman filter. In [11], joint control algorithm and attitude estimation is discussed and used in space craft dynamical model in case of loss of observations. The lost observations are predicted by Linear Prediction techniques in order to use in the measurement update step of the Kalman filter. The robust estimation proposed in [11] outperforms conventional open loop Kalman filter.

In this paper, the three Linear Prediction techniques are studied and their performance is analyzed for both internal and external prediction of the signal. Furthermore, modification for bounded error performance in Linear Prediction techniques is discussed.

The rest of the paper is organized as follows: Section II briefly introduces Linear Prediction theory. Modified LP techniques i.e. Normal Equation, LDA and LGA are explained in section III. State Estimation is discussed in section IV. Simulation results are given in section V and paper is concluded in section VI.

LINEAR PREDICTION

Linear prediction refers to a technique in which coefficients of an auto regressive model are computed making use of the input signal [21]. The calculated coefficients are then employed to regenerate the signal. The predicted signal is given by:

$$\hat{z}[n] = \sum_{i=1}^p \alpha_i z[n-i] \quad (1)$$

where $\hat{z}[n]$ represents the predicted signal, $z[n]$ represents the input signal, α_i is the i^{th} coefficient weight of the signal. Figure 1 outlines the working principle of Linear Prediction. The operation of LP is based on minimization of mean square error. A mathematical

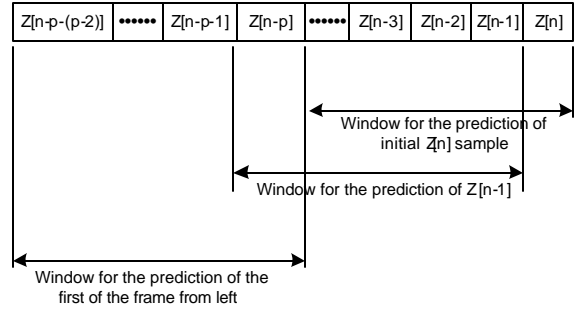


Fig. 2: Showing the sliding window concept in internal prediction

expression to compute these predictive coefficients is derived with the condition resulting in minimum mean square error. The signal error can be calculated by the following equation:

$$e[n] = z[n] - \hat{z}[n] \quad (2)$$

Linear prediction can be categorized into Internal and External Prediction, detailed below.

Internal prediction: In internal prediction, the LPCs of a certain frame of data are determined using the data inside the frame. Hence, the resulting LPCs capture the statistics of the frame accurately. A longer frame size reduces computational complexity, since the LPCs are calculated and transmitted less frequently; however, the coding delay grows larger as the system has to wait longer to collect many samples [8]. In addition, due to the changing nature of non-stationary systems, the LPCs of a long frame might not produce good prediction gain. On the other hand, a shorter data frame calls for more frequent update of the LPCs, which leads to a more accurate portrayal of the signal statistics in comparison with the longer data frame. Most internal prediction mechanisms depend on non-recursive autocorrelation estimation methods, where a finite length window is used to obtain the signal samples. In internal prediction no real prediction of the signal occurs, rather the coefficients of the given signal frame are worked out.

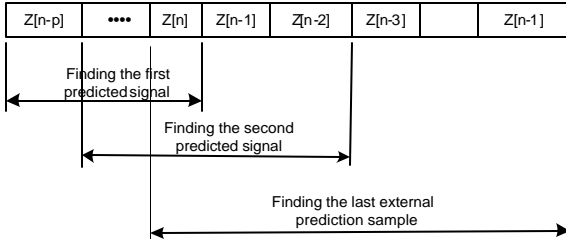


Fig. 3: Sliding window nature of the external prediction

In this paper, the sliding window concept is employed as depicted in Fig. 2. The stationary and sliding window approaches differ in the update process. In sliding window concept, the window update is in the backward direction. While in stationary window, the same window is constantly used for computing all sample values [3].

External prediction: In external prediction, the derived LPCs are used in a future frame; that is the LPCs associated with the frame are not derived from the data located inside the frame; instead they are calculated from past values of the signal [8]. External prediction is effective in case of slow variation in the signal's statistical properties. The frame size should be large enough so that in case of a problem, maximum amount of data loss is recovered. The LP filter order is determined from the following relation [9].

$$p \leq T_f * T_s$$

where T_f denotes the starting time instant of loss of measurements, T_s is the sampling time of the system and 'p' is Linear Prediction filter order or LPFO. Conceptual view of the external Linear Prediction and the sliding window used is shown in Fig. 3. In this paper, some modifications are made in the Linear Prediction algorithms in order to reduce error in the prediction process.

Prediction gain: The formula for calculating the prediction gain (PG) is given by [8]

$$PG = 10 \log_{10} \left(\frac{\sigma_z^2}{\sigma_e^2} \right) = 10 \log_{10} \left(\frac{E\{s^2[n]\}}{E\{e^2[n]\}} \right) \quad (3)$$

Equation (3) indicates that prediction gain is the ratio between the variance of the input signal and the variance of the prediction error expressed in decibel units. As such, prediction gain is a measure of the performance of a predictor. Smaller value of prediction error results in a higher value of prediction gain. So a predictor having a higher prediction gain is better than

one with lower prediction gain. One way to find out the optimum frame size of the predictor for maximized prediction gain is to plot the prediction gain as a function of the prediction order. In the resulting plot a saturation point will reach when further increase in the frame size will have no significant effect on the prediction gain.

LINEAR PREDICTION TECHNIQUES

Normal equation: The Normal Equation is based on minimum mean square error (e = original signal - predicate). In Normal Equation method the predicted signal is given by [8]:

$$\hat{z}[n] = \sum_{i=1}^p \alpha_i z[n-i] \quad (4)$$

In what follows, a mathematical approach for reduction of error between actual and predicted signals has been explained. The cost function is defined as [8].

$$J = E\{e^2[n]\} = E\left\{\left(z[n] + \sum_{i=1}^p \alpha_i z[n-i]\right)^2\right\} \quad (5)$$

'J' is the cost function which is precisely a second order function of LPCs. To get the sub-optimal value of LPC, the cost function 'J' is differentiated with respect to ' α_k ' and equated to zero, as [8].

$$\left(\frac{\partial J}{\partial \alpha_k}\right) = 2E\left\{\left(z[n] + \sum_{i=1}^p \alpha_i z[n-i]\right) z[n-k]\right\} = 0 \quad (6)$$

Rearranging Equation (6) gives:

$$E\{z[n]z[n-k]\} + \sum_{i=1}^p \alpha_i E\{z[n-i]z[n-k]\} = 0 \quad (7)$$

Now

$$R_z[i-k] = E\{z[n-i]z[n-k]\} \quad k=1, 2, 3, \dots, p \quad (8)$$

This leads to Equation (9)

$$\sum_{i=1}^p \alpha_i R_z[i-k] = -R_z[k] \quad (9)$$

The auto-correlation matrix is given by:

$$R_z = \begin{bmatrix} R_z[0] & R_z[1] & R_z[2] & \dots & R_z[p-1] \\ R_z[1] & R_z[0] & R_z[1] & \dots & R_z[p-2] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_z[p-1] & R_z[p-2] & R_z[p-3] & \dots & R_z[0] \end{bmatrix}$$

From Equations (8) and Equation (9):

$$R_z \alpha_i = -r_z \quad (10)$$

The variable r_z represents the transpose of the array formed by the elements $R_z[0]$ to $R_z[p-1]$ of the autocorrelation array. The resulting matrix in Equation (10) is a Toeplitz matrix having same diagonal elements. This property of Toeplitz matrix allows the linear equations to be solved by the Levinson-Durbin algorithm [12] or the Schur algorithm [13].

Modified levinson durbin technique: Normal Equation method has been found computationally expensive because every iteration of loss of observation involves large matrix inversion [8]. In order to avoid this problem, Levinson Durbin Algorithm (LDA) is normally employed. Levinson-Durbin algorithm (LDA) considerably reduces the computational time by using the Toeplitz symmetry property of the autocorrelation matrix [15]. The autocorrelation array found in Eq. (10) serves as the starting input of this algorithm. The coefficients that we arrive at in LDA technique are not LPCs but are in fact, Reflection Coefficients (RCs) [12]. The Reflection Coefficients found through this algorithm have one to one correspondence with LPC coefficients because the input signal is Wide Sense Stationary (WSS) in nature. For 'L' number of iterations and frame size 'P', LDA algorithm can be described as follows:

Initialization: First iteration ($L=0$), set

$J_0 = R[0]$

Threshold limit for the error $e_{th} = 0.5$

Recursion: For $L=1, 2, 3 \dots P$

Step 1: Compute the values of L^{th} RC [12]

$$k_L = (1/J_{L-1})(R[L] + \sum_{i=1}^{L-1} \alpha_i^{L-1} R[L-i])$$

Step 2: Now Calculate LPCs for the L^{th} order predictor as: [12]

$$\alpha_i^L = -K_L$$

$$\alpha_i^L = \alpha_i^{L-1} - K_L \alpha_{L-i}^{L-1}, \quad i = 1, 2, 3, 4, \dots, L-1$$

Stop: when $L=P$

Step 3: Calculating the minimum mean square prediction error as [8]

$$J_L = J_{L-1}(1 - K_L^2)$$

Check: the value of threshold error and compare with J : If $J \leq e_{th}$

No: $L \rightarrow L+1$; go to step 1

Yes: Store the corresponding values of mean square error, computation time and predicted signal.

Modified leroux gueguen technique: LDA has been found suffering from a larger dynamic range in the values of LPC. An alternate method-LerouxGueguen Algorithm (LGA) eliminates the problem related to dynamic range in a fixed-point environment by taking the application of Schwartz inequality in computation of this method [13]. This technique also reduces the computational time by avoiding large matrix inversion as happens in Normal Equation (10).

Initialize: Loop for sub-optimal size for range of frame size

For $T=10 \rightarrow 20$

Define the threshold value for error e_{th} as 0.5

Initialization: $L=0$, set

$$\varepsilon^{(0)}[k] = R[k], \quad k = -P+1, \dots, 0, \dots, P$$

Recursion: for $L=1, 2, 3 \dots P$.

Step 1: Calculate the L^{th} Reflection Coefficient [13]

$$k_L = \frac{\varepsilon^{(L-1)}[L]}{\varepsilon^{(L-1)}[0]}$$

Stop if $k=P$

Step 2: Calculate the epsilon parameters [8]

$$\varepsilon^{(L)}[k] = \varepsilon^{(L-1)}[k] - k_L \varepsilon^{(L-1)}[L-k];$$

where

$$k = -P+L+1, \dots, 0, L+1, \dots, P$$

Calculate the error 'e' as:

$$\begin{aligned} E[n] &= z[n] - \hat{z}[n] \\ &= \varepsilon_{10}^{10} z[n-1] + \varepsilon_{20}^{10} z[n-2] \\ &\quad + \varepsilon_{30}^{10} z[n-3] \dots + \varepsilon_{10}^{10} z[n-10] \end{aligned}$$

Check if $e \leq e_{th}$

No: Up $T=T+1$

Yes: Save all the required values and stop the loop.

The epsilon parameters values are used for calculating the LPC coefficients.

STATE ESTIMATION

Filtering process is used to extract information from noise contaminated data. If noise and signal are in different frequency regions, then a conventional low-pass, band-pass, band-stop or high-pass filter would be enough for extraction of information signal. But if the spectra of noise and signal overlap then a conventional filter design will be a difficult task. In such situations the useful information is obtained through estimation, prediction or smoothing [3]. Kalman filter is a best tool for the state estimation of LTI systems. Kalman filter accurately estimates the state of a system based on the noisy measurement [3]. So Kalman filter need accurate knowledge of system dynamics. Actually Kalman filter predict the system state and then updates the predicted signal by using observations, recursively.

Many control and communication systems have the problem of data loss. Some of the reasons of data loss may be sensor faults, buffer overflow, communication errors or insufficient bandwidth of the channel [10]. In case of loss of measurements, the conventional Kalman filter shows poor performance because of the unavailability of data at measurement update state. For this reason another approach called Open Loop Estimation (OLE) is used in case of data loss. In OLE only prediction step is performed. Update step cannot be performed to tune the predicted signal because of loss of measurement data. In OLE the estimation error becomes unbounded if data loss happens for an adequate time period. Recently another technique called Zero Order Hold (ZOH) is introduced. In ZOH the last data sample is stored and updated throughout the estimation. But this technique has certain shortcomings. Because if a single data sample is used for measurement update step then it may not be useful when data loss is longer. This technique also requires that the signal samples must be strictly correlated [15].

In this paper Linear Prediction techniques are used for reconstruction of the missing data in state estimation through Kalman filter. The predicted signal is then employed in the Kalman filtering at the step of measurement update.

Consider the following discrete time LTI system

$$x_k = Ax_{k-1} + Bu_{k-1} + \varepsilon_{k-1}$$

$$z_k = Cx_k + \theta_k$$

where $k \in \mathbb{N}$, , , , is the state transition matrix, is the input matrix,

is the output matrix and () are Gaussian, uncorrelated white noise sequences with mean () and covariance () respectively. The estimation through Kalman filter is summarized as follows.

1. Initialize

$$x_{0|0}, u_0, \varepsilon_0, \theta_0, P_{0|0} \text{ and } k = 1.$$

2. Prediction cycle:

$$x_{k+1|k} = Ax_{k|k} + Bu_k; \text{ State estimation}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k; \text{ Error covariance}$$

3. Time-step update: $k \rightarrow k+1$
4. Sense measurements: $z_{k+1} = Hx_{k+1} + \theta_{k+1}$
5. Innovation vector calculation:

$$r_{k+1} = z_{k+1} - Hx_{k+1|k}$$

6. Then calculate the innovation covariance matrix:

$$S_{k+1} = HP_{k+1|k}H^T + R_{k+1}$$

7. Now calculate the gain matrix:

$$K_{k+1} = P_{k+1|k}H^T S_{k+1}^{-1}$$

8. Perform update cycle:

$$x_{k+1|k+1} = x_{k+1|k} + K_{k+1}r_{k+1};$$

State estimation

$$P_{k+1|k+1} = (I - K_{k+1}H)P_{k+1|k};$$

Error covariance

It is clear from the above algorithm that update step completely relies on measurements. When the output data (z_k) is not available, KF will not result in optimal estimation. For this reason we have used Linear Prediction techniques to predict the missing output data for the update step.

SIMULATION RESULTS

Model description: For evaluation of the above analysis an example of space craft is employed which is discussed in this section. Consider a space craft accelerated with random gas bursts. It will follow an LTI system model as follows:

$$x_{k+1} = Ax_k + w_k$$

$$z_k = Cx_k + v_k$$

In eqn. x_k is comprised of position and speed of the spacecraft. Here z_k show noise contaminated observation vector and C is a square matrix. By using Kalman filter the state of the system can be estimated if the characteristics of the noise sequence are known. The above equation can also be written as:

$$\begin{bmatrix} p_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_k \\ v_k \end{bmatrix} + \begin{bmatrix} p_{w_k} \\ v_{v_k} \end{bmatrix}$$

$$\begin{bmatrix} p_k \\ v_k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_k \\ v_k \end{bmatrix} + \begin{bmatrix} p_{v_k} \\ v_{v_k} \end{bmatrix}$$

where p_k represents speed and position of the spacecraft respectively. Here p_{w_k} and v_{v_k} show the process noise sequences in the two states. Sampling frequency is selected to be 10 Hz. Acceleration and measurement noise is 0.02 and 0.1 respectively.

Optimum frame size for internal prediction techniques: The optimum frame size is selected from among a range of different frame size values. Optimum frame size is chosen based on the values of prediction error, prediction gain and computation time required for the execution of the algorithm. Table 1 shows the resulting values of computational time, prediction error and prediction gain for different frame sizes for the Normal Equation method.

Table 1 clearly shows that the frame size affects the output of the Normal Equation Linear Prediction technique. Table 1 also reveals that increasing the LPF order does not always result in smaller prediction error. Prediction gain has a direct relation with the system performance. The sub-optimum frame size for Normal Equation technique is 60 as can be seen in Table 1. Simulation results for the LDA technique for different frame sizes are shown in the Table 2.

Table 2 shows the effect of frame size on the prediction gain, prediction error and computational time and it is clear that the optimum frame size is 50. The difference in the computational time of Normal Equation scheme and LDA is not considerable but it becomes significant only when larger matrix inversion is confronted in Normal Equation technique. By comparing Table 1 and 2 it is clear that LDA has a better performance than the Normal Equation method. Simulation results for the LGA technique for different frame sizes are shown in Table 3.

Table 1: Performance of normal equation technique for internal prediction

| Frame size | Prediction gain | Error | Computational time (sec) |
|------------|-----------------|--------|--------------------------|
| 10 | -3.5498 | 1.0392 | 0.0039 |
| 20 | -5.2507 | 1.0297 | 0.0159 |
| 30 | -10.2498 | 1.7267 | 0.0187 |
| 40 | 7.8378 | 0.4245 | 0.0198 |
| 50 | 15.9393 | 0.0672 | 0.0214 |
| 60 | 14.9677 | 0.0391 | 0.0245 |
| 70 | 4.3889 | 1.3576 | 0.0278 |
| 80 | -3.6686 | 1.3371 | 0.0322 |
| 90 | -5.3421 | 1.3876 | 0.0366 |

Table 2: Performance of levinson durbin technique for internal prediction

| Frame size | Prediction gain | Error | Computational time (sec) |
|------------|-----------------|--------|--------------------------|
| 10 | -2.7505 | 1.8611 | 0.0107 |
| 20 | 5.4930 | 0.7571 | 0.0110 |
| 30 | 9.7875 | 0.5883 | 0.0141 |
| 40 | 10.9400 | 0.1690 | 0.0159 |
| 50 | 17.4751 | 0.0544 | 0.0106 |
| 60 | 5.2654 | 0.0597 | 0.0169 |
| 70 | -6.6353 | 0.6805 | 0.0170 |
| 80 | -12.7612 | 0.6515 | 0.0184 |
| 90 | -11.4513 | 0.6504 | 0.0186 |

Table 3: Performance of leroux gueguen technique for internal prediction

| Frame size | Prediction gain | Error | Computational time (sec) |
|------------|-----------------|--------|--------------------------|
| 10 | 4.9666 | 0.4321 | 0.0235 |
| 20 | 4.1984 | 0.2922 | 0.0072 |
| 30 | 5.1718 | 0.4204 | 0.0081 |
| 40 | 3.0053 | 0.4568 | 0.0085 |
| 50 | 0.7011 | 0.4932 | 0.0090 |
| 60 | -3.0359 | 0.5001 | 0.0095 |
| 70 | 8.7850 | 0.0123 | 0.0126 |
| 80 | -13.6948 | 4.1451 | 0.0129 |
| 90 | -9.8877 | 0.6910 | 0.0218 |

From Table 3 it can be seen that the optimum frame size for LGA is 70. The computational time required for the LDA technique is smaller as compared to the computational times of Normal Equation and LGA methods but the main drawback of this technique is the large dynamic range of the resulting coefficient values.

Comparison of internal prediction techniques: In this subsection, performance comparison of the three Linear Prediction techniques based on the optimum frame size has been made.

Table 4: Internal linear prediction techniques comparison for different frame sizes

| Technique used | Prediction gain | Prediction error | Computational time (sec) |
|--------------------------------|-----------------|------------------|--------------------------|
| Normal equation technique (60) | 14.9677 | 0.0391 | 0.0245 |
| Levinson Durbin technique (50) | 17.4751 | 0.0544 | 0.0106 |
| Leroux Gueguen technique (70) | 8.7850 | 0.0123 | 0.0126 |

Table 5: Performance of normal equation for external prediction

| Frame size | Prediction gain | Error | Computational time (sec) |
|------------|-----------------|--------|--------------------------|
| 10 | -16.8324 | 1.1147 | 1.0196 |
| 20 | -11.0153 | 0.7274 | 1.0169 |
| 30 | 4.1065 | 1.4197 | 1.0386 |
| 40 | 12.7607 | 0.0704 | 1.1910 |
| 50 | 8.2341 | 0.0914 | 1.4704 |
| 60 | 7.8660 | 0.6489 | 1.5291 |
| 70 | -1.2619 | 1.0570 | 1.5638 |
| 80 | -8.6143 | 0.8018 | 1.6859 |
| 90 | -14.8672 | 1.2543 | 1.8685 |

Table 6: Performance of levinso durbin technique for external prediction

| Frame size | Prediction gain | Error | Computational time (sec) |
|------------|-----------------|--------|--------------------------|
| 10 | -6.5087 | 0.6885 | 0.1032 |
| 20 | -16.0595 | 3.4087 | 0.1867 |
| 30 | 11.0452 | 1.2399 | 0.1892 |
| 40 | 11.1587 | 0.0520 | 0.1769 |
| 50 | 5.0668 | 0.6758 | 0.1851 |
| 60 | 8.7199 | 1.1707 | 0.1911 |
| 70 | -10.1204 | 1.6853 | 0.1971 |
| 80 | -8.3033 | 0.6913 | 0.2001 |
| 90 | -7.3087 | 0.7524 | 0.2186 |

Table 7: Performance of leroux gueguen technique for external prediction

| Frame size | Prediction gain | Error | Computational time (sec) |
|------------|-----------------|--------|--------------------------|
| 10 | -15.5512 | 0.3953 | 0.0918 |
| 20 | -9.4328 | 1.1821 | 0.0647 |
| 30 | 5.7453 | 1.1890 | 0.0622 |
| 40 | 7.0579 | 0.4559 | 0.0597 |
| 50 | 13.5689 | 0.0397 | 0.0634 |
| 60 | 8.3390 | 3.0770 | 0.0698 |
| 70 | -10.2531 | 1.5026 | 0.0722 |
| 80 | -21.7538 | 2.7585 | 0.0745 |
| 90 | -6.6728 | 7.8200 | 0.0815 |

From Table 4, it is clear that prediction error is small for the LGA but is high for the Normal Equation technique. The computational time is also greater for Normal Equation technique as compared to LDA and

LGA. The cause of this delay is the larger matrix inversion involved in Normal Equation technique. LDA and LGA reduce this computational time by avoiding the larger matrix inversion involved in the calculation of LPCs. It is also clear that prediction gain is greater for the LDA technique. The LGA has the minimum prediction error of all the three algorithms. And the computational time for LGA is also smaller as compared to the LDA and Normal Equation method. However the computational gain value is higher for the Levinson Durbin method as given in Table 4.

Optimum frame size for external prediction techniques:

External prediction is usually used in cases where low coding delay is the major concern [8]. Therefore in external prediction techniques smaller frame size is preferable in order to avoid larger computational delay. Table 5 shows the results of the Normal Equation technique when different frame size values are considered. Prediction error, computational time and prediction gain are shown for each value of frame size.

From Table 5 it is observed that different frame size values influence the output of the linear predictor. And by increasing the frame size computational time increases gradually. It can be noted that prediction gain shows the system performance. Greater value of prediction gain indicates higher performance of the system. The sub-optimal frame size for Normal Equation is 40. Table 6 enlists the results obtained for LDA:

From Table 6 it is obvious that the sub-optimal frame size for LDA is 40 due to smaller prediction error value and less computational time as compared to other values of frame sizes. By comparing the results in Table 5 and 6 it can be concluded that computational time for LDA is much less than that for Normal Equation method. The following table shows the values of computational time, prediction error and prediction gain for different frame sizes for LGA technique.

Table 7 shows that the sub-optimal frame size for LGA technique is 50. The computational time increases with increase in the frame size. Prediction gain gives an insight into the performance of the system.

Comparisons of linear prediction techniques for external prediction:

Table 8 depicts a comparison of

Table 8: External Linear Prediction schemes comparison for different frame sizes

| Technique used | Prediction gain | Prediction error | Computational time (sec) |
|--------------------------------|-----------------|------------------|--------------------------|
| Normal Equation Technique (40) | 12.7607 | 0.0704 | 1.1910 |
| Levinson Durbin Technique (40) | 11.1587 | 0.0520 | 0.1769 |
| LerouxGueguen Technique (50) | 13.5689 | 0.0397 | 0.0634 |

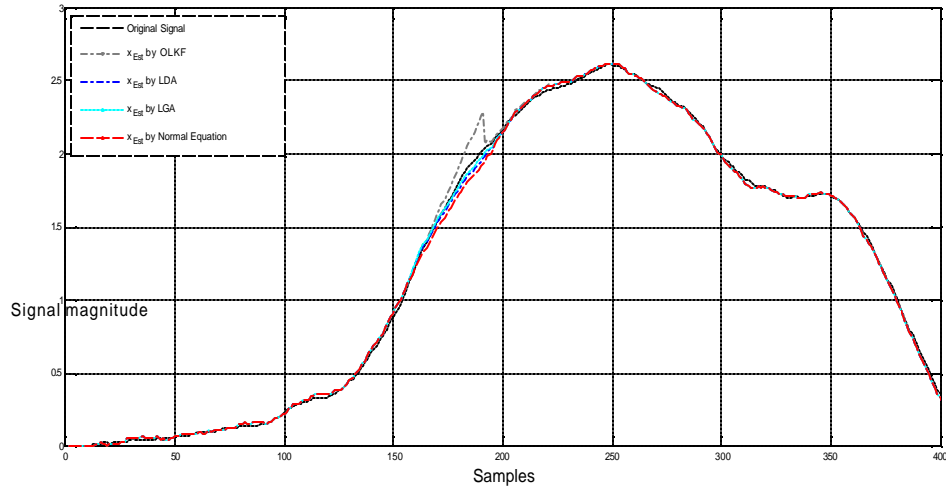


Fig. 4: Actual signal and estimated signals using linear prediction techniques (normal equation, Levinson Durbin algorithm and Leroux Gueguen algorithm)

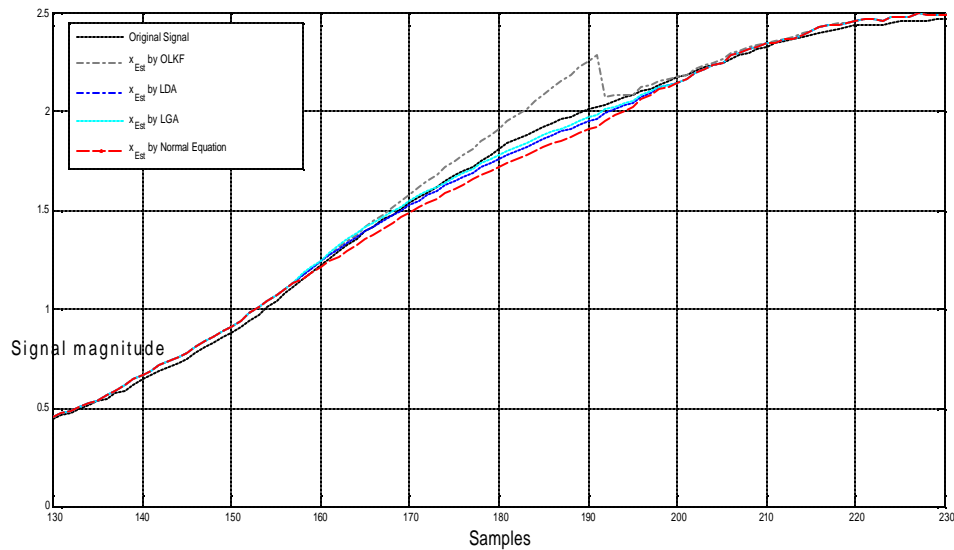


Fig. 5: Actual signal and estimated signals using linear prediction techniques (normal equation, Levinson Durbin algorithm and Leroux Gueguen algorithm)

all the three techniques for different parameters i.e. Prediction Gain, Error and Computation Time for optimum frame size of each technique.

From Table 8 it is clear that degradation in prediction error performance is in the following ascending order i.e. LGA, LDA and Normal Equation method. The performance in terms of computational time is in the following descending order (worst to best)

i.e. Normal Equation method, LDA and LGA. It is concluded that LerouxGueguen technique is the optimum prediction technique giving sub-optimal values over all the three parameters for external Linear Prediction.

In Fig. 4, the solid line represents the actual signal. The rest of the lines represent the signal estimated through Kalman filter, where the missing data (samples

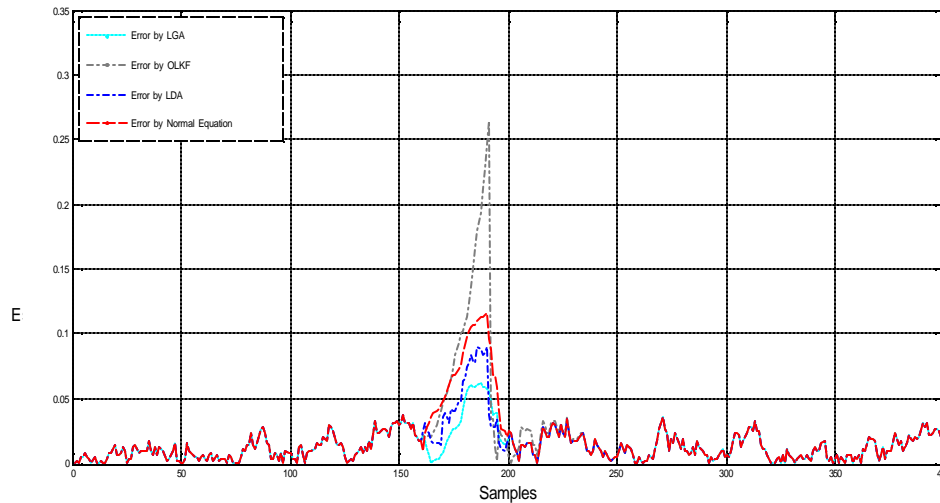


Fig. 6: Error signals by normal equation, Levinson Durbin algorithm and Leroux Gueguen algorithm

160...190) is predicted through different Linear Prediction techniques. The dotted line represents the signal estimated through Kalman filter in which the missing portion is predicted by LerouxGueguen Algorithm. The dashed line shows the estimated signal in which the missing data is predicted by Normal Equation. Similarly the signal predicted by Levinson Durbin Algorithm is given by the solid green line. The dash-dot line is the signal estimated by open loop Kalman filtering.

From Fig. 4, it is clear that the best estimation is achieved when the missing data is predicted by LerouxGueguenAlgorithm, while the worst estimation of the signal happens when open loop Kalman filtering is applied.

Figure 5 shows a clear view of the region of lost samples. It is clear that the estimated signal is more deviated in the region of lost samples (samples 160... 190) and the prediction techniques can be easily compared by looking at this region. From worst to best are OLKF, Normal Equation technique, LDA and LGA.

Figure 6 shows the estimation errors for different techniques. The dot-dash line shows the error by open loop Kalman filtering. The dashed line, dotted line and solid line represent the estimation when Normal Equation, LGA and LDA are employed in the state estimation respectively. The open loop Kalman filtering results in larger error as compared to the other techniques. Minimum estimation error occurs when LerouxGueguen technique is used for prediction of the missing samples in the process of state estimation. Normal Equation and Levinson Durbin technique show better performance than open loop Kalman filtering but they result in larger value of error than LerouxGueguen Algorithm.

CONCLUSIONS

In this paper, three different Linear Prediction techniques are presented and analyzed for both internal and external prediction of a signal. The Linear Prediction algorithms are modified by employing variable frame sizes. A threshold limit for prediction error is set in order to keep the error bounded. The effect on different performance parameters due to variable frame size is also discussed. An optimum frame size is chosen for each Linear Prediction technique based on reduced prediction error, low computational time and high prediction gain to decide efficient (optimum) Linear Prediction filter order. All the three LP algorithms are then compared for prediction gain, computational time and prediction error values at their optimum frame sizes. It is concluded that LDA and LGA show much better performance than the conventional Normal Equation method.

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