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Method of Solution and Computational Algorithm for Mixed Thermo-Mechanics Problem

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Abstract: In this paper we developed a method and a corresponding software system modeling steady-state thermal stress bearing elements of construction, working under the simultaneous effect of local temperatures, heat flow, heat transfer and thermal insulation. In addition, full-scale takes into account the dependence of the thermal expansion coefficient of the temperature. Impose strict energy conservation, minimization methods and the convergence of the Gauss solution of linear algebraic equations and experimental data to establish the relationship of the coefficient of thermal expansion of materials bearing structural elements of the field temperature distribution. Elaborated computational algorithm and approach is relatively universal in the sense of possibility of computational solution of steady problems of thermostressed condition of construction bearing components, which operate in condition of simultaneous existence of local temperature, thermal flows, heat exchange and heat conservation.

Key words: Thermal flows · Heat exchange · Law of energy conservation

INTRODUCTION

In many technological lines of processing industries and in modern gas turbine plants, hydrogen engines and steam generators of nuclear power stations, some supporting elements undergo local heat streams changing by coordinate following special sinusoidal law at having simultaneous availability of lateral heat insulation and heat exchange processes through cross section areas at both ends. To study regularities of thermomechanical state of such supporting elements let's consider a horizontal rod of a limited length $L(cm)$. Herewith let's designate the cross section area as $F(cm^2)$ and assume that it's constant by its length. The rod material thermal expansion coefficient and thermal conductivity coefficient as α(1/°C) and $K_{xx}(W/(cm.°C))$ accordingly. Let's direct axis Ox from the left to the right. It coincides with the rod axis. Let's assume the left end of the rod under consideration is rigidly fixed whereas the right end is loose. Heat exchange with the environment takes place across the area of cross sections of the rod left end. Herewith environment temperature $T_{oc1}(°C)$, whereas heat exchange coefficient $h_1(W/(cm^2.°C))$. Similarly heat exchange takes place through areas of cross section of the rod right end. Here environment temperature $T_{oc2}(°C)$, whereas heat exchange coefficient $h_2(W/(cm^2.°C))$. Besides lateral area of the rod sections $(0 \le x \le x_a)$ и $(x_b \le x \le L)$ is heat-insulated. The rod lateral surface area within section $(x_a \le x \le x_b)$ is vented by heat flow $q(W/cm^2)$ changing by coordinate according to the sine law [1].

$q = -A \sin \frac{\pi(x - x_a)}{(x_b - x_a)}$, $x_a \le x \le x_b$. where $A = const > 0$, $x_a < x_b$.

Computational model of the problem under study is shown in Figure 1. Now it's required to determine temperature distribution law $T = T(x)$ throughout the tested rod length taking into account simultaneously presence of local heat exchange processes, heat insulation and heat flow changing its flow by coordinate according to the sine law. Moreover based on found temperature distribution law throughout the tested rod length as well as the rod material heat expansion coefficient value α we need to identify the tested rod elongation value due to heat expansion effect. At the same time we need to consider that α may be a constant value or it may subject to temperature that is $\alpha = \alpha(T(x))$. This dependency shall be determined for each individual material through by an experimental approach. To find out temperature distribution law throughout the tested rod

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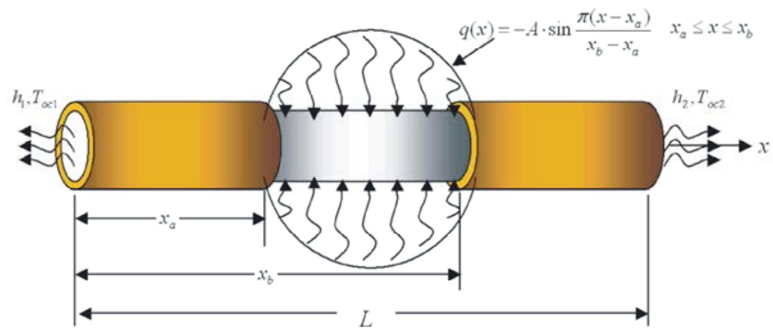


Fig. 1: Problem computational model

length let's apply universal energy conservation law. When using this law it's guaranteed that we will be able to acquire adequate solutions with high accuracy. For these purposes let's discretise the rod under study by n- quadratic elements with the same length $\ell = \frac{L}{n}$. Herewith within the rod section ($0 \le x \le x_a$) number of elements will be $n_a = \frac{x_a \times n}{L}$. Within the rod section ($x_b \le x \le x_b$) it will be $n_{ab} = \frac{(x_b - x_a) \times n}{L}$ elements. Then within the rod last section ($x_b \le x \le L$) number of elements will be equal to $n_{bL} = \frac{(L - x_b) \times n}{L}$. Hence one can actually prove that throughout the entire rod length the number of discrete elements will be equal to

$$n_a + n_{ab} + n_{bL} = \frac{x_a \cdot n}{L} + \frac{(x_b - x_a) \cdot n}{L} + \frac{(L - x_b) \cdot n}{L} = \frac{x_a \cdot n + x_b \cdot n - x_a \cdot n + L \cdot n - x_b \cdot n}{L} = n$$

Samples and Analytical Methods: As for the first discrete element (on the left) the functional formula which specifies its total heat energy will have the following appearance [1]

$$J_1 = \int\limits_{V^{(1)}} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV + \int\limits_{S^{(1)}_{(x=0)}} \frac{h_1}{2} (T - T_{oc1})^2 dS \tag{1}$$

where V_1 - volume of 1st discrete element.

As for the rest discrete elements (n_a – 1) within the section ($0 < x \le x_a$) analogical functional formula will have the following appearance

$$J_{i1} = \int\limits_{V_{i1}} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV \tag{2}$$

where $i1 = (2 \div n_a)$.

Now we shall look into the rod section ($x_a \le x \le x_b$) where the lateral area is vented by heat flow changing its flow by coordinate according to the sine law. For discrete elements within this section the corresponding functional formula will have the following appearance

$$J_{i1} = \int\limits_{V_{i2}} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV + \int\limits_{S_{i2}} qTdS \tag{3}$$

where $i2 = ((n_a + 1) \div n_{ab})$

where

$$\int_{S_{i2}} qTdS = 2\pi r \int_{(i2-1)}^{i2} qTdx \tag{4}$$

Now let's consider the rod last section ($x_a \le x \le L$). The lateral area of this section is heat-insulated. But heat exchange with surrounding environment takes place through the cross sections of the rod right end ($x = L$). But this element will be the last n-th discrete element. Then for the internal elements within this section the corresponding functional formula will have the following appearance

$$J_{i3} = \int_{V_{i3}} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV \tag{5}$$

where $i3(n_{ab} + 1) \div (n-1)$.

Finally for the last n discrete element the corresponding functional formula will have the following appearance

$$J_n = \int_{V_n} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV + \int_{S_{(x=L)}} \frac{h_2}{2}(T - T_{oc2})^2 dS \tag{6}$$

Then functional formula for the tested rod on the whole which specifies its total heat energy will look like this:

$$J = J_1 + \sum_{i1=2}^{n_a} J_{i1} + \sum_{i1=(n_a+1)}^{n_{ab}} J_{i2} + \sum_{i3=(n_b+1)}^{(n-1)} J_{i3} + J_n \tag{7}$$

Now let's approximate temperature distribution field within each discrete element length using a complete polynomial of the second kind that is

$$T(x) = ax^2 + bx + c,\; a, b, c = const \tag{8}$$

Or using terms of finite elements method for a discrete element we have [2]

$$T(x) = \varphi_i(x) \cdot T_i + \varphi_j(x) \cdot T_j + \varphi_k(x) \cdot T_k,\; 0 \le x \le \ell \tag{9}$$

where $\varphi_i(x)$, $\varphi_j(x)$ and $\varphi_k(x)$ are form functions for quadratic discrete elements with three knots. They have the following appearance [2]

$$\varphi_i(x) = \frac{\ell^2 - 3\ell x + 2x^2}{\ell^2};\; \varphi_j(x) = \frac{4\ell x - 4x^2}{\ell^2};\;\; \varphi_k(x) = \frac{2x^2 - \ell x}{\ell^2}; \tag{10}$$

Then within the length of each element the temperature gradient shall be identified as follows:

$$\frac{\partial T}{\partial x} = \frac{\partial \varphi_i(x)}{\partial x} T_i + \frac{\partial \varphi_j(x)}{\partial x} T_j + \frac{\partial \varphi_k(x)}{\partial x} T_k = \frac{\ell^2 - 3\ell x + 2x^2}{\ell^2} T_i + \frac{4\ell x - 4x^2}{\ell^2} T_j + \frac{2x^2 - \ell x}{\ell^2} T_k \tag{11}$$

where *i,j,k*- local numbers of one discrete element. Herewith their local coordinates x_i=0, $x_j = \frac{\ell}{2}$, $x_k=\ell$. Now substituting (11) instead of (7) and minimizing J by $T_i(i = 1 \div (2n + 1))$ us will acquire a resolving system in the form of linear algebraic equations:

$$\frac{\partial J}{\partial T_i} = 0,\ i = 1 \div (2n+1) \tag{12}$$

Upon resolving the system (9) by Gauss method the temperature pivotal values may be found $T_i(i = 1 \div (2n+1))$. Based on them temperature distribution law $T = T(x)$ throughout the tested rod length is built. Whereupon based on heat insulation laws [1] the rod elongation value due to heat expansion may be determined [3].

$$\Delta\ell_T = \int_0^L \alpha \cdot T(x)dx \tag{13}$$

There may be two cases:

1) $\alpha = const$. Then the rod elongation value will be determined by the following formula:

$$\Delta\ell_{T1} = \int_0^L \alpha \cdot T(x)dx \tag{14}$$

2) $\alpha = \alpha\,(T(x))$. Then the rod elongation value will be determined by the following formula:

$$\Delta\ell_{T2} = \int_0^L \alpha(T(x)) \cdot T(x)dx \tag{15}$$

Having correlations (9-11) we shall analytically integrate integral by volume using expressions (1-3) and (5-6).

$$\int_V \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV = \frac{K_{xx} \cdot F}{2\ell}\left[\frac{7}{3}T_i^2 - \frac{16}{3}T_i \cdot T_j + \frac{2}{3}T_i \cdot T_k - \frac{16}{3}T_j \cdot T_k + \frac{16}{3}T_j^2 + \frac{7}{3}T_k{}^2\right] \tag{16}$$

Here it's worth to mention that the sum of coefficients before temperature pivot values $T_i = T(x_i) = T(0)$; $T_i = T(x_j) = T(\frac{\ell}{2})$; $T_к = T(x_к) = T(\ell)$ will be equal to zero. Indeed $\left(\frac{7}{3} + \frac{16}{3} + \frac{2}{3} - \frac{16}{3} + \frac{16}{3} + \frac{7}{3}\right) = 0$. In our opinion this equation is indication on energy conservation law.

Now we shall similarly consider integrals throughout the cross section of the rod left and right ends.

$$\int_{S_{(x=0)}} \frac{h_1}{2}(T - T_{oc})^2 dS = Fh_1(T_1 - T_{oc})^2 \tag{17}$$

where $T(x = 0) = T_i$. In this expression the sum of coefficients before T_i and T_{oc} also will be equal to zero, that is (1-1) =0. Now let's likewise consider the next integral throughout the lateral area for one discrete element.

$$\int_{S_{пвп}} qT(x)dS = \int_{S_{пвп}} -A\sin\frac{\pi(x-x_a)}{(x_в - x_a)}\cdot\left[\varphi_i(x)\cdot T_i + \varphi_j(x)\cdot T_j + \varphi_k(x)\cdot T_k\right]dS$$
$$= -\frac{PA}{\ell^2}\int_0^{\ell}\sin\frac{\pi x}{\ell}\left[\left(\ell^2 - 3\ell x + 2x^2\right)\cdot T_i + \left(4\ell x - 4x^2\right)\cdot T_j + \left(2x^2 - \ell x\right)\cdot T_k\right]dx$$
$$= -\frac{PA\ell}{\pi^3}\left[\left(\pi^2 - 8\right)\cdot T_i + 16\cdot T_j + \left(\pi^2 - 8\right)\cdot T_k\right] \qquad (18)$$

In this expression for all discrete elements within the rod section $x_a \le x \le x_b$ the sum of coefficients before pivotal values will be equal to $2\pi^2$[2].

RESULTS AND DISCUSSION

For approbation of the foregoing model let's assume the following as reference data: $L = 30(c)$, n=300 discrete elements, $\ell = \frac{L}{300} = 0{,}1\,(cm)$ length of each discrete element, m, $F = \pi r^2 = \pi(cm^2)$, $K_{xx} = 72(W/(cm°C))$, $h_1 = 10(W(cm^2.°C))$, $T_{co1} = 20(°C)$, $h_2 = 10(W/(cm^2.°C))$, $T_{co2} = 20(°C)$, $x_b = 20\,(cm\;\; q = -100\sin\frac{\pi(x-x_a)}{x_b - x_a}\,(W/cm^2)$, $(x_a \le x \le x_b)$ $\alpha = 125\times10^{-7}\,(1/°C)$.

In this case the number of knots will be equal to 2n+1=2*300+1=601. So the number of equations in the system of resolving equations will also be 601. Temperature distribution field for this problem is shown in Figure 2.

Under effect of such temperature distribution law across the tested rod length it elongates due to heat expansion to the extent of

$$\Delta\ell_T = \int_0^L \alpha\cdot T(x)dx = \alpha\int_0^L T(x)dx = 0{,}0645785\,cm$$

Now let's assume that both ends of tested rod are firmly fixed. Then naturally the rod under study is not able to elongate. Due to heat expansion in this case a compressive force R(kg) as well as corresponding field of strain and stress distribution appear. In this case to find the appeared field of shifting, strains and stress let's use minimization technique of elastic deformation potential energy at having available temperature field by shift pivotal values $u = u(x)$. Having said so let's take shifting field within each discrete element as:

$$u(x) = \varphi_i(x)\cdot u_i + \varphi_j(x)\cdot u_j + \varphi_k(x)\cdot u_k,\; 0 \le x \le \ell \qquad (19)$$

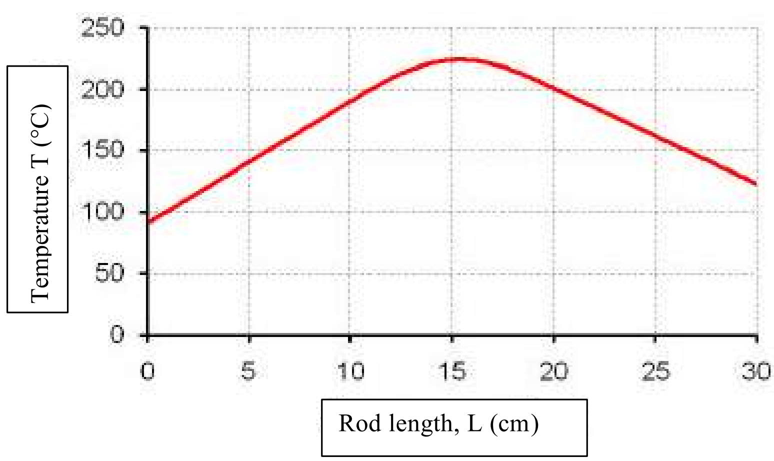


Fig. 2: Temperature distribution field across the rod length

where

$$u_i = u(x = 0);\ u_j = u(x = \frac{\ell}{2});\ u_k = u(x = \ell) \tag{20}$$

Formula for specifying the foregoing elastic deformation potential energy at having available temperature field will look like this [3].

$$\Pi = \int_V \frac{\sigma_x \cdot \varepsilon_x}{2} \cdot dV - \int_V \alpha E T(x) \cdot \varepsilon_x dV \tag{21}$$

where $\varepsilon_x = \frac{\partial u}{\partial x}$ – elastic component of strain;

$\sigma_x = E \cdot \varepsilon_x = E \cdot \frac{\partial u}{\partial x}$ – Elastic component of stress;

For one discrete element the integrated type of functional (21) will have the following form:

$$\Pi_r = \frac{EF}{2}\left[\frac{7}{3\ell} \cdot u_i^2 - \frac{16}{3\ell} u_i \cdot u_j + \frac{2}{3\ell} u_i \cdot u_k + \frac{16}{3\ell} u_j^2 - \frac{16}{3\ell} u_j \cdot u_k + \frac{7}{3\ell} u_k^2\right]$$
$$-\alpha EF\left[-\frac{1}{2} T_i \cdot u_i + \frac{2}{3} T_i \cdot u_j - \frac{1}{6} T_i \cdot u_k - \frac{2}{3} T_j \cdot u_i + \frac{2}{3} T_j \cdot u_k + \frac{1}{6} T_k \cdot u_i - \frac{2}{3} T_k \cdot u_j + \frac{1}{2} T_k \cdot u_k\right] \tag{22}$$

where $r = 1 \div n$; n - total number of discrete elements in the rod under study. Here it's worth to mention that for the 1st discrete element i=1; j=2; k=3; for the 2nd discrete element i=3; j=4; k=3 and so on. For n-th discrete element i=2n–1; j=2n; k=2n+1. Now using (22) let's write down formula for potential energy of elastic deformations taking into account availability of temperature field for the rod under study on the whole which will have the following appearance:

$$\Pi = \sum_{r=1}^{n} \Pi_r \tag{23}$$

Now minimizing Π by shift pivot values u_r(r = 1÷(2n+1)) we will acquire a resolving system of equations in the form of linear algebraic equations relative to

$$\frac{\partial \Pi}{\partial u_r} = 0,\ r = 1 \div (2n+1) \tag{24}$$

Resolving system (24) shift pivot value u may be found and using (19) shifting field across the rod length is restored. After this using formula $\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial \varphi_i}{\partial x} \cdot u_i + \frac{\partial \varphi_j}{\partial x} \cdot u_j + \frac{\partial \varphi_k}{\partial x} \cdot u_k$ within each discrete element field of elastic component of strain shall be drawn. Using formula $\varepsilon_x = -\alpha \cdot T(x) = -\alpha\left[\varphi_i(x) \cdot T_i + \varphi_j(x) \cdot T_j + \varphi_k(x) \cdot T_k\right]$ field of temperature component of strain is built within each composing element limits. Then distribution field of thermoelastic component of strain across the length of each element may be identified using the following formula:

$$\varepsilon = \varepsilon_x + \varepsilon_T$$

Analogically distribution field of components of stress may be determined across the lelngth of each discrete element: $\sigma_x = \mathrm{E} \cdot \varepsilon_x$, $\sigma_T = \mathrm{E} \cdot \varepsilon_T$ and $\sigma = \sigma_x + \sigma_T$.

Now that we have the foregoing reference data for the rod under study we might as well determine shift distribution field $u = u(x)$ and corresponding components of strain and stress.

First of all let's vary the rod length value in the problem under research according to the following pattern:

$L \in$ [25;26;27;28;29;30]cm. Table -1 show that if one end of the rod is retained and the other is free, then due to heat expansion the rod elongation value $\Delta\ell_T$ increases as the

Table 1:

No.	L (cm)	$\Delta\ell_T$	$\varepsilon = \varepsilon_x + \varepsilon_T$	$\sigma = \sigma_x + \sigma_T$	R
1	25	0,0428818987	-0,0017152759	-3430,55189425	10777,3966342
2	26	0,0453363138	-0,0017437044	-3487,40875218	10956,0177211
3	27	0,0478459774	-0,0017720732	-3544,14647241	11134,2645261
4	28	0,0504108895	-0,0018003889	-3600,77781967	11312,1771510
5	29	0,0530310501	-0,0018286569	-3657,31379806	11489,7901659
6	30	0,0557064592	-0,001856882	-3713,76394448	11667,1335320

a)

b)

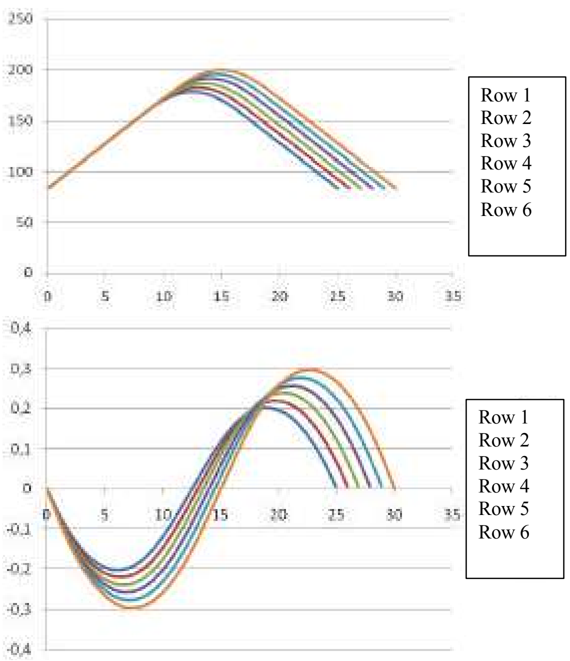


Fig. 3: Temperature distribution and shift field across the tested rod length depending on its length, $u;(x)x100$;

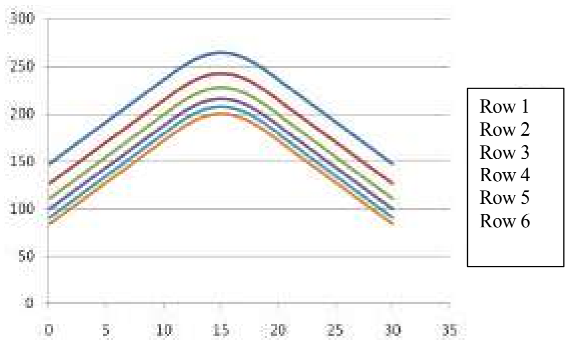
rod length is increased. As you can see from this Table in the problem under consideration at increasing the rod length for 20% the elongation value correspondingly increases for 29,9%. This process is motivated by the fact that more quantity of heat will concentrate in a longer rod in comparison with that in a shorter one. Because at increasing the rod length its cross section remains a constant value. Apart from this, at any length of the rod a part $\frac{2L}{3}$ of its lateral area will be heat-insulated. If both ends of the tested rod are firmly fixed then a compressive force R (kg) and field of components of strain and stress appear. Here it should be noted that all of them have compressive nature. As you can see from Table -1 at increasing the tested rod length for 20% the value R of compressive force, thermoelastic components of strain ε and stress σ tend to increase to the same extent by 8,25%. Figure -3 α) shows temperature distribution field T = T(x) across the tested rod length for length values range $L \in [25;26;27;28;29;30]$cm. As you can see from this Figure that regardless increasing the rod length in all 6 variants the temperature values within the rod section $0 \le x \le \left(\frac{L}{3} = \frac{25}{3}\right)$ will be almost the same. But as the rod length increases the temperature value at the rod middle point $\left(x = \frac{L}{2}\right)$ will grow. Apart from this as the rod length naturally increases the temperature amplitudes also shift to the right. Figure -3 b) shows distribution field of the rod sections' shifting. Since both rod ends are firmly fixed, those ends do not shift. In all 6 variants the rod lengths, cross sections located within its first quarter that is $\left(x = \frac{L}{4}\right)$ shift to the left, i.e. against the Ox axis direction. The rod central cross section $\left(x = \frac{L}{2}\right)$ doesn't shift anywhere in all 6 variants that is in all 6 variants $u = \left(x = \frac{L}{2}\right) = 0$. Due to symmetry of cross section in the problem under research this section $\left(x = \frac{3L}{4}\right)$ of the rod shifts to the right so far as the rod section $\left(x = \frac{L}{4}\right)$ shift to the left;

Apart from this, amplitudes of shifting these two sections tend to increase as the rod length increases. Naturally this process is preconditioned by the reason that in the problem under research the rod volume increases as its length increases. In its turn as the volume increases the quantity of heat in this volume also increases.

Now at fixed values of the rest parameters let's study influence of heat exchange coefficient value h on the tested rod thermomechanical state. Table – 2 shows influence of heat exchange coefficient value h on the values of elongation, compressive force and components of strain and stress. In case if one end of the rod is firmly fixed whereas the other one is free, then as heat exchange coefficient value h increases the rod elongation is decreasing due to the heat expansion effect. This process

Table 2:

No.	$H(W/(cm^2.°C))$	$\Delta\ell_T$	$\varepsilon = \varepsilon_x + \varepsilon_T$	$\sigma = \sigma_x + \sigma_T$	R
1	5	0,0795738106	-0,0026524604	-5304,92070863	16665,8999199
2	6	0,0716180268	-0,0023872676	-4774,53512058	14999,6444572
3	7	0,0659353241	-0,0021978441	-4395,68827198	13809,4619869
4	8	0,0616732970	-0,0020557766	-4111,55313552	12916,8251185
5	9	0,0583583871	-0,0019452796	-3890,55914050	12222,5520125
6	10	0,0557064592	-0,001856882	-3713,76394448	11667,1335320

a)


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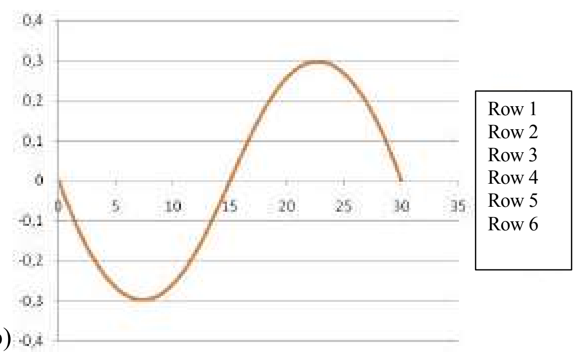

Fig. 4: Temperature distribution and shift field across the tested rod length depending on heat exchange coefficient, u(x) ×100.

is preconditioned by the reason that as heat exchange coefficient value *h* increases more heat is lost in the rod. As you can see from Table -2 when increasing heat exchange coefficient twice as much the tested rod elongation value decreases by 30% due to the heat expansion effect. This Table also shows that as heat exchange coefficient value increases the compressive force value R also decreases. For instance at increasing heat exchange coefficient value *h* twice as much R value decreases by 30%. Figure 4 a) shows temperature distribution field across the tested rod length at different temperature values. As you can see from this Figure in all 6 variants temperature distribution field is strictly symmetrical and maximum temperature value corresponds to the rod middle point. At the same time temperature maximum amplitude corresponds to the first variant, i.e. for the case $h = 5\frac{??}{cm^2 \cdot {}^{\circ}?}$. As *h* value increases, temperature amplitude tends to decrease at the middle of the rod.

Figure – 4 α) shows temperature distribution field across the tested rod length depending on heat exchange coefficient value. As you can see from this Figure the less *h* value the more temperature amplitude. Naturally maximum value corresponds to the middle point of the rod. But Figure – 4 b) show that change of heat exchange coefficient has no significant effect on shift distribution field.

CONCLUSIONS

Here it's worth to mention that this developed model based on energy conservation law is universal in term of resolving the whole class of analogical complicated problems. Moreover acquired numerical solutions are characterized with high accuracy.

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