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Methods for Maximum and Minimum Cost Flow Determining in Fuzzy Conditions

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Abstract: This article considers the problems of maximum flow and minimum cost flow determining in fuzzy network. Parameters of fuzzy networks are fuzzy arc capacities in the case of maximum flow finding and transmission costs of one flow unit in the case of minimum cost flow finding. These parameters are represented as fuzzy triangular numbers. Conventional rules of operating with fuzzy triangular numbers lead to a strong "blurring" of their borders, resulting in loss of self-descriptiveness of calculations with them. The following technique of addition and subtraction of fuzzy triangular numbers is proposed in the presented paper: the centers are added (subtracted) by the conventional methods and the borders of the deviations are calculated using linear combinations of the borders of adjacent values. The fact that the limits of uncertainty of fuzzy triangular numbers should increase with the increasing of central values is taken into account. To illustrate the proposed method numerical examples are presented.

Key words: Fuzzy network · Cost flow · Uncertainty

INTRODUCTION

This article deals with flow problems arising in networks in terms of uncertainty [1]. The network corresponds to a directed graph with distinguished initial (source) and final (sink) nodes [2, 3]. Each arc has capacity determining the maximum number of flow units, which can pass along the arc. The relevance of the tasks of maximum and minimum cost flow determining lies in the fact that the researcher can effectively manage the traffic, taking into account the loaded parts of roads, redirect the traffic and choose the cheapest route.

Suppose we have a network, which arcs have capacities (q_{ij}) Formulation of the problem of maximum flow finding in the network is reduced to maximum flow determining that can be passed along arcs of the network in view of their capacities:

$$v=\sum_{x_j\in\Gamma(s)}\xi_{sj}=\sum_{x_k\in\Gamma^{-1}(t)}\xi_{kt}\to\max,$$

$$\sum_{x_j\in\Gamma(x_i)}\xi_{ij}-\sum_{x_k\in\Gamma^{-1}(x_i)}\xi_{ki}=\begin{cases}v,\ x_i=s,\\-v,\ x_i=t,\\0,\ x_i\neq s,t,\end{cases}\qquad(1)$$

$$\xi_{ij}\le q_{ij},\ \forall(x_i,x_j)\in A.$$

In (1) ξ_{ij}– the amount of flow, passing along the arcs; v – the maximum flow value in the graph, s – initial node (source); t – final node (sink); q_{ij} – maximum arc capacity. The task of minimum cost flow determining in a network can be formulated as follows: suppose we have a network, which arcs have two associated numbers: the arc capacity (q_{ij}) and transmission cost of one flow unit passing along the arc (c_{ij}). We know the maximum flow value v. The essence of this problem is to find such a flow, which doesn't exceed the maximum flow in the graph, that transmission cost of one flow unit is minimal. In mathematical terms the problem of minimum cost flow determining in the network can be represented as follows:

$$\sum_{(x_i,x_j)\in A}c_{ij}\cdot\xi_{ij}\to\min,$$

$$\sum_{x_j\in\Gamma(x_i)}\xi_{ij}-\sum_{x_k\in\Gamma^{-1}(x_i)}\xi_{ki}=\begin{cases}v,\ x_i=s,\\-v,\ x_i=t,\\0,\ x_i\neq s,t,\end{cases}\qquad(2)$$

$$\xi_{ij}\le q_{ij},\ \forall(x_i,x_j)\in A.$$

In (2) c_{ij} – transmission cost of one flow unit along the arcs.

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In practice, the arc capacities, transmission costs, the values of the flow entering the node and emanating from the node cannot be accurately measured according to their nature. Weather conditions, emergencies on the roads, traffic congestions and repairs influence arc capacities. Variations in petrol prices, tolls can either influence transmission costs. Therefore, these parameters should be presented in a fuzzy form, such as fuzzy triangular numbers [4]. Thus, we obtain a problem statement of maximum and minimum cost flow problems in fuzzy conditions.

Literature Review of the Maximum and Minimum Cost Flow Determining Tasks: The problem of the maximum flow finding in a general form was formulated by J. Danzig in 1951. L. Ford and D. Fulkerson developed famous algorithm for solving this problem, called "augmented path" algorithm in 1955 [5].

There are different versions of the Ford-Fulkerson's algorithm. Among them there is the shortest path algorithm, proposed by Edmonds and Karp in 1972, in which one can choose the shortest supplementary path from the source to the sink at each step in the residual network (assuming that each arc has unit length). The shortest path is found according to the breadth-first search. Other scientists, such as Dinic, Karzanov, Cherkasky worked at improving the running-time of the algorithm and decreasing of complexity. The Goldberg-Rao's algorithm, proposed in 1997, is the most modern modification of the Ford-Fulkerson's algorithm.

Determining the maximum flow in the transportation network in terms of uncertainty has been studied less. There are contemporary articles which solve the problem by the simplex method of linear programming [6, 7].

Many researchers have examined the task of minimum cost flow finding in crisp conditions in the literature. Methods of its solution can be divided into graph techniques and the methods of linear programming. In particular, solutions by the graph methods are considered in [8].

The methods of minimum cost flow finding in networks in fuzzy conditions can be divided into two classes. The first class involves the use of conventional flow algorithms for determining the minimum cost flow, which operate with fuzzy data instead of crisp values and require cumbersome routines with fuzzy numbers. The second class of problems implies the use of "fuzzy linear programming", which was widely reported in the literature [9].

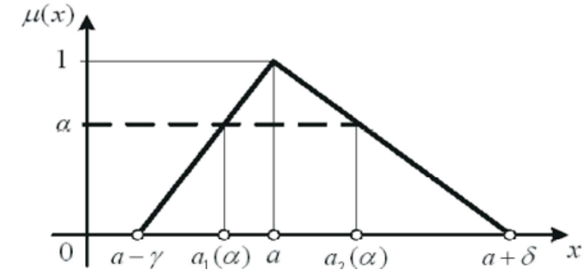

Fig. 1. Fuzzy triangular number

Authors [10] consider the tasks of "fully fuzzy linear programming". These tasks are cumbersome and can not lead to optimal solutions in the minimum cost flow determination.

Presented Method of Operating with Fuzzy Triangular Numbers: Researcher is faced with the problem of fuzziness in the network, when considering the problems of maximum and minimum cost flow finding. Arc capacities, flow values, passing along the arcs, transmission costs per unit of goods cannot be accurately measured, so we will represent these parameters as fuzzy triangular numbers.

We will represent the triangular fuzzy numbers as follows in this article: (α, γ, δ) where α – the center of the triangular number, γ – deviation to the left of the center, δ – deviation to the right of center, as shown in Fig. 1.

Conventional operations of addition and subtraction of fuzzy triangular numbers are as follows: let $\tilde{A}_1$ and $\tilde{A}_2$ be fuzzy triangular numbers, such as $\tilde{A}_1 = (a_1, \gamma_1, \delta_1)$ and $\tilde{A}_2 = (a_2, \gamma_2, \delta_2)$, where α_1 and α_2 – centers, γ_1 and γ_2 – deviations to the left, δ_1 and δ_2 – deviations to the right of fuzzy triangular numbers. Therefore, the sum of triangular numbers can be written as: $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2, \gamma_1 + \gamma_2, \delta_1 + \delta_2)$ and the difference represented as: $\tilde{A}_1 - \tilde{A}_2 = (a_1 - a_2, \gamma_1 + \delta_2, \delta_1 + \gamma_2)$ [4]. The disadvantage of the conventional methods of addition and subtraction of fuzzy triangular numbers is a strong "blurring" of the resulting number and, consequently, the loss of its self-descriptiveness. For example, when adding the same triangular number with itself, the borders of its uncertainty increase: (2,1,1) + (2,1,1) = (4, 2, 2) and (2,1,1) + (2,1,1) + (2,1,1) = (6, 3, 3). Generally, it is not true, because the center of the triangular number should increase, while its borders must remain constant. The fact that the degree of borders "blurring" of fuzzy number depends on the size of its center is not usually considered, when specifying the

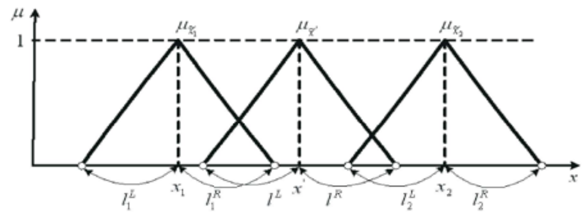

Fig. 2: Given values of arc capacities (flows or transmission costs)

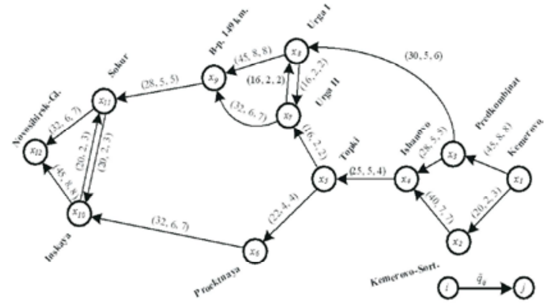

Fig. 3: Initial network

triangular fuzzy numbers. Therefore, the more the center, the more "blurred" the borders should be (while measuring 1 kg of material, we are talking "about 1 kg", implying the number "from 900 to 1100 g", but while measuring 1 t. of material, imply that "about 1 t." is the number "from 990 kg to 1110 kg").

Comparison of fuzzy triangular numbers according to various criteria is also very difficult and time-consuming. Consequently, following method is proposed to use when operating with triangular fuzzy numbers. Suppose there are the values of arc capacities, flows or transmission costs in a form of fuzzy triangular numbers on the number axis. Then when adding (subtracting) the two original triangular fuzzy numbers their centers will be added (subtracted) and to calculate the deviations it is necessary to define required value by adjacent values. Let the fuzzy arc capacity (flow or transmission cost) "near $\tilde{x}'$" is between two adjacent values "near $\tilde{x}'$" and "near $\tilde{x}_2$", $(x_1 \le x' \le x_2)$ which membership functions $\mu_{\tilde{x}_1}(x_1)$ and $\mu_{\tilde{x}_2}(x_2)$ have a triangular form, as shown in Fig. 2.

Thus, the borders of membership function of fuzzy arc capacity (flow or transmission cost) "near $\tilde{x}'$" one can set by the linear combination of left and right borders of adjacent values:

$$l^L = \frac{(x_2 - x)}{(x_2 - x_1)} \times l_1^L + \left(1 - \frac{(x_2 - x)}{(x_2 - x_1)}\right) \times l_2^L$$
$$l^R = \frac{(x_2 - x)}{(x_2 - x_1)} \times l_1^R + \left(1 - \frac{(x_2 - x)}{(x_2 - x_1)}\right) \times l_2^R \quad (3)$$

In (3) l^L is the left deviation border of required fuzzy number, l^R is the right deviation border. In the case when the central value of triangular number resulting by adding (subtracting) repeats the already marked value on the number axis, its deviation borders coincide with the deviation borders of the number marked on the number axis. If required central value is not between two numbers, but precedes the first marked value on the number axis, its deviation borders coincide with those of the first marked on the axis. The same applies to the case when the required central value follows the last marked value on the axis.

Solving the Task of Maximum Flow Finding in Fuzzy Conditions: The task of maximum flow finding in fuzzy network can be formulated as follows:

$$\nu = \sum_{x_j \in \Gamma(s)} \xi_{sj} = \sum_{x_k \in \Gamma^{-1}(t)} \xi_{kt} \to \max,$$
$$\sum_{x_j \in \Gamma(x_i)} \xi_{ij} - \sum_{x_k \in \Gamma^{-1}(x_i)} \xi_{ki} = \begin{cases} \nu, x_i = s, \\ -\nu, x_i = t, \\ 0, x_i \ne s, t, \end{cases} \quad (4)$$
$$\xi_{ij} \le \tilde{q}_{ij}, \forall (x_i, x_j) \in A.$$

In (4) $\tilde{q}_{ij}$ – maximum fuzzy arc capacity.

Let's consider an example, illustrating the solution of this problem, represented in Fig. 3. Let network, representing the part of the railway map, is given in a form of fuzzy directed graph, obtained from GIS "Object Land [11]. Let the node x_1 is a source, node x_{12} is a sink. The values of arc capacities in the form of fuzzy triangular numbers are defined above the arcs. It is necessary to calculate the maximum flow value between stations "Kemerovo" (x_1) and Novosibirsk-Gl. (x_{12}) according to Edmonds-Karp's algorithm and the method, described for operations with fuzzy triangular numbers.

Therefore, the algorithm proceeds as follows: the first iteration of the algorithm performs an augmenting chain $x_1x_3x_8x_9x_{11}x_{12}$. Push the flow, equals to (28, 5, 5) units along it. The arc (x_9,x_{11}) becomes saturated. Let's define the residual capacities of the remaining arcs of augmenting chain. The arc (x_1,x_3) has a residual capacity equals to (45, 8, 8) – (28, 5, 5). Thus, the central value of the resulting number is 17. It is located between adjacent arc capacities: (16, 2, 2) and (20, 2, 3) of the original graph as shown in Fig. 4.

Compute the left and the right deviation borders of the fuzzy triangular number with a center of 17:

$$l^L = \frac{(20-17)}{(20-16)} \times 2 + \left(1 - \frac{(20-17)}{(20-16)}\right) \times 2 = 2,$$

$$l^R = \frac{(20-17)}{(20-16)} \times 2 + \left(1 - \frac{(20-17)}{(20-16)}\right) \times 3 = 2.25.$$

Thus, we obtain a fuzzy triangular number of the form (17, 2, 2.25) units.

The residual capacity of the arc (x_8,x_9) is (30, 5, 6) – (28, 5, 5). Consequently, we obtain a fuzzy triangular number with a center of 2, located to the left of fuzzy triangular number of the form (16, 2, 2). Deviation borders of the required number coincide with deviation borders of the number (16, 2, 2). Thus, we obtain a fuzzy triangular number of a type (2, 2, 2) units, as shown in Fig. 5.

The residual capacity of the arc (x_8,x_9) equals to (17, 2, 2.25) units, similarly with the arc (x_1,x_3). Finally, the residual capacity of the arc (x_{11},x_{12}) is equal to (32, 6, 7) – (28, 5, 5), i.e. we obtain fuzzy number (4, 2, 2) units.

The second iteration of the algorithm gives the augmenting chain $x_1x_2x_4x_5x_6x_{10}x_{12}$. Push the flow equals to (20, 2, 3) units along it. The arc (x_1,x_2) becomes saturated. The residual capacity of the arc (x_2,x_4) is (40, 7, 7) – (20, 2, 3), i.e. we obtain a fuzzy triangular number (20, 2, 3) units. The residual capacity of the arc (x_4,x_5) is a difference between the numbers (25, 5, 4) and (20, 2, 3). Thus, we get a number with a center of 5, located to the left of the number (16, 2, 2), i.e. (5, 2, 2) units. The residual capacity of the arc (x_5,x_6) is equal to (22, 4, 4) – (20, 2, 3), i.e. (2, 2, 2) units. The residual capacity of the arc (x_6,x_{10}) is equal to (32, 6, 7) – (20, 2, 3), i.e. (12, 2, 2) units. The residual capacity of the arc (x_{10},x_{12}) is (45, 8, 8) – (20, 2, 3), i.e. (25, 5, 4) units.

The third iteration of the algorithm performs the augmenting chain $x_1x_3x_4x_5x_6x_{10}x_{12}$. Push the flow equals to (2, 2, 2) units along. The arc (x_5,x_6) becomes saturated. Let's define the residual capacities of the remaining arcs of the augmenting chain. The residual capacity of the arc (x_1,x_3) is (17, 2, 2.25) – (2, 2, 2), i.e. (15, 2, 2) units. The residual capacity of the arc (x_3,x_4) is equal to (28, 5, 5) – (2, 2, 2). We get the number with a center of 26, located between adjacent values (25, 5, 4) and (28, 5, 5), as shown in Fig. 6.

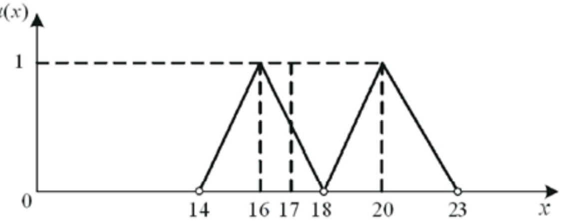

Fig. 4: Fuzzy triangular number with a center equals to 17 and its adjacent numbers

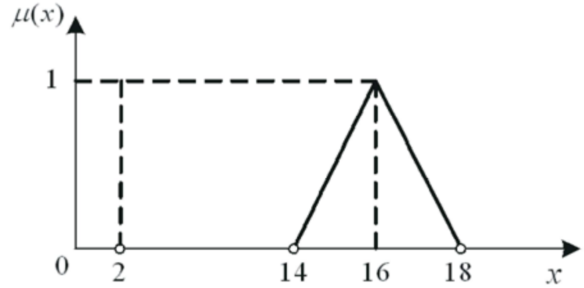

Fig. 5: Fuzzy triangular number with a center equals to 2 and its adjacent numbers

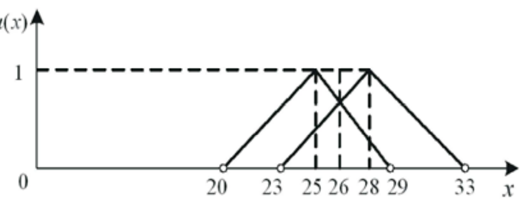

Fig. 6: Fuzzy triangular number with a center equals to 26 and its adjacent numbers

Compute the left and the right deviation borders of the fuzzy triangular number with a center of 26 according to (3). Thus, we obtain a fuzzy triangular number of the form (26, 5, 4.33) units.

The residual capacity of the arc (x_4,x_5) is equal to (5, 2, 2) – (2, 2, 2), i.e. (3, 2, 2) units. The residual capacity of the arc (x_6,x_{10}) is equal to (12, 2, 2) – (2, 2, 2), i.e. (10, 2, 2) units. The residual capacity of the arc (x_{10},x_{12}) is equal to (25, 5, 4) – (2, 2, 2), i.e. we obtain a fuzzy number with a center of 23, located between adjacent values (22, 4, 4) and (25, 5, 4), therefore, the left deviation border of the number with a center of 23 equals to 4.33, the right deviation border is 4. We obtain fizzy triangular number (23, 4.33, 4) units.

After execution of three iterations of the algorithm it is impossible to pass any single additional flow unit. The total flow is (28, 5, 5) + (20, 2, 3) + (2, 2, 2) units. Therefore, we obtain a fuzzy triangular number with a

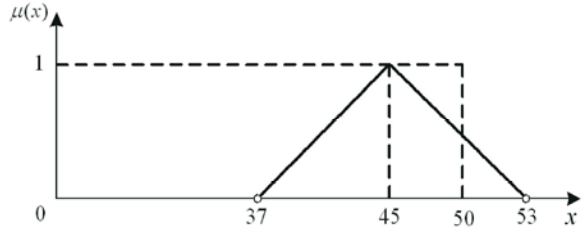

Fig. 7: Fuzzy triangular number with a center equals to 50 and its adjacent numbers

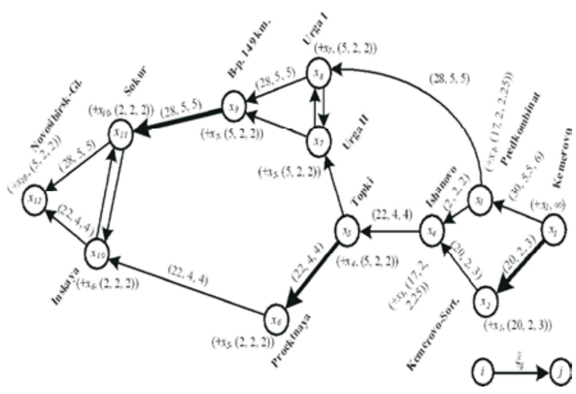

Fig. 8: Network with maximum flow

center of 50, located to the right of the number (45, 8, 8) with the borders, repeated deviations of the number 45: (50, 8, 8) units, as shown in Fig. 7.

Thus, the maximum flow value between the stations"Kemerovo" and "Novosibirsk-Gl." is (50, 8, 8) units. Let us carry out an interpretation of the results: the maximum flow between the given stations can not be less than 42 and more than 58 units, with the highest degree of confidence it will be equal to 50 units. But with changes in the environment, repairs on the roads, traffic congestions the flow is guaranteed to lie in the range from 42 to 58 units. Flow distribution along the arcs and labels of the vertices is shown in Fig. 8.

Solving the Task of Minimum Cost Flow Determining in Fuzzy Conditions: Consider the problem of minimum cost flow finding in a network according to fuzzy values of arc capacities and transmission costs of one flow unit. Let us turn to the graph, shown in Fig. 3. Fuzzy values of transmission costs in addition to fuzzy arc capacities are given in this task:

$$\tilde{c}_{x_1x_2} = (12,3,3);\ \tilde{c}_{x_1x_3} = (6,1,2);\ \tilde{c}_{x_3x_4} = (10,2,3);$$
$$\tilde{c}_{x_2x_4} = (18,4,5);\ \tilde{c}_{x_3x_8} = (4,1,1);$$
$$\tilde{c}_{x_4x_5} = (12,3,3);\ \tilde{c}_{x_5x_7} = (20,5,6);\ \tilde{c}_{x_8x_7} = (15,4,4);$$
$$\tilde{c}_{x_7x_8} = (15,4,4);\ \tilde{c}_{x_8x_9} = (21,6,7);$$
$$\tilde{c}_{x_7x_9} = (10,2,3);\ \tilde{c}_{x_5x_6} = (30,8,9);$$
$$\tilde{c}_{x_6x_{10}} = (8,2,2);\ \tilde{c}_{x_9x_{11}} = (19,5,5);$$
$$\tilde{c}_{x_{11}x_{10}} = (32,7,12);\ \tilde{c}_{x_{10}x_{11}} = (32,7,12);$$
$$\tilde{c}_{x_{11}x_{12}} = (25,7,8);\ \tilde{c}_{x_{10}x_{12}} = (20,5,6).$$

It is necessary to find a flow value of (45, 8, 8) units from the source to the sink, which has a minimal cost. Consider the Busacker-Gowen's algorithm, taking into account the fuzzy capacities and costs to solve this problem:

Step 1: Assign all arc flows and the flow rate equal to zero.

Step 2: Determine the modified arc costs $?^*_{ij}$ that depend on the value of the already found flow as follows:

$$\tilde{c}^*_{ij} = \begin{cases} \tilde{c}_{ij},\ if\ \tilde{0} \le \tilde{\xi}_{ij} \le \tilde{q}_{ij} \\ \infty,\ if\ \tilde{\xi}_{ij} = \tilde{q}_{ij} \\ -\tilde{c}_{ji},\ if\ \tilde{\xi}_{ij} > 0 \end{cases}$$

Step 3: Find the shortest chain (in our case – the chain of minimal cost) from the source to the sink taking into account arc costs $?^*_{ij}$, found in the step 1. Push the flow along this chain until it ceases to be the shortest. Receive the new flow value by adding the new flow value, passing along the considered chain, to the previous one. If the new flow value equals to v, then the end. Otherwise, go to the step 2.

Solve this problem, taking into account fuzzy arc capacities costs.

Step 1: Assign all $\tilde{\xi}_{ij} = 0$.

Step 2: Determine $\tilde{c}^*_{ij} = \tilde{c}_{ij}$.

Step 3: Find the shortest path by the Dijkstra's algorithm: $x_1x_3x_8x_9x_{11}x_{12}$ of the total cost of (75, 8, 8) standard units. Push the flow, equals to (28, 5, 5) units along this chain.

Step 2: Define the new modified arc costs:

$$\tilde{c}^*_{x_1x_3} = (6,1,2);\ \tilde{c}^*_{x_3x_1} = -(6,1,2);\ \tilde{c}^*_{x_3x_8} = (4,1,1);$$
$$\tilde{c}^*_{x_8x_3} = -(4,1,1);\ \tilde{c}^*_{x_8x_9} = (21,6,7);$$
$$\tilde{c}^*_{x_9x_8} = -(21,6,7);\ \tilde{c}^*_{x_9x_{11}} = \infty;\ \tilde{c}^*_{x_{11}x_9} = -(19,5,5);$$
$$\tilde{c}^*_{x_{11}x_{12}} = (25,7,8);\ \tilde{c}^*_{x_{12}x_{11}} = -(25,7,8).$$

Step 3: Find the shortest path using the obtained modified costs: $x_1x_3x_4x_5x_6x_{10}x_{12}$ of the total cost of (86, 8, 8) standard units. Push the flow, equals to (17, 2, 2.25) units along this chain. As a result, we obtain the total flow equals to (45, 8, 8) units, having a total transmission cost along the network, equals to (28, 5, 5) × ((75, 8, 8) + (86, 8, 8)) = (3562, 8, 8) standard units, as shown in Fig. 9.

CONCLUSION

This paper examines the problems of maximum and minimum cost flow determining in networks in terms of uncertainty, in particular, the arc capacities, as well as the transmission costs of one flow unit are represented as fuzzy triangular numbers. The technique of addition and subtraction of triangular numbers is considered. Presented technique suggests calculating the deviation borders of fuzzy triangular numbers based on the linear combinations of the deviation borders of the adjacent values. The fact that the limits of uncertainty of fuzzy triangular numbers should increase with the increasing of central values is taken into account.

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