

Estimation of Stability of Dynamic Systems Containing Inertial Links

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Abstract: Development of auto-oscillations may occur in the dynamic systems containing frictional contact. For research purposes, a model of locomotive traction drive in wheelslip mode is considered here as an example of such system. This paper proposes an estimation method for dynamic system stability in relation to frictional auto-oscillations. The method is based on the balance of power input into the system and dissipated with auto-oscillations. This method makes it possible to determine the system parameters whereby the system remains on stability boundary. It is demonstrated how the said method can be applied for estimation of stability of system containing aperiodic links. The auto-oscillations caused by electromagnetic characteristics of the traction electric motor are examined therein. It is also shown that system stability depends on time response of the electrical machine. And where the loss in stability can be either aperiodic or accompanied by auto-oscillations. Time response values of the electrical machine, with which the system maintains its stability, are determined within this research.

Key words: Stability • Traction drive • Frictional auto-oscillations • Energy balance • Natural frequencies

INTRODUCTION

Different multi-link mechanical systems driven by electric motor and containing friction transmissions or couplings are well known and extensively used in mechanical engineering.

If an excessive slippage occurs in frictional contact, then auto-oscillations, accompanied with high (and sometimes extreme) dynamic loads in construction elements, can thereby develop in the system.

Upon taking into consideration the variety of possible variants of construction and designations of such systems, it would be reasonable to present all relevant theoretical principles and methods by providing a concrete example, assuming that principal conditions can be applied to a wide range of electromechanical systems containing frictional contact.

A traction electric motor (TM) of rail transport in wheel slip state can be regarded as the most characteristic example of such system, when rotation of wheel pair is also accompanied by its slippage on rails. Since such slippage of wheels happens quite frequently in real

operating conditions, then parameters of traction drive should ensure its stability in relation to frictional auto-oscillations.

This paper proposes the method which allows, with predefined inertial and stiffness parameters of drive, to obtain such relations between its dissipative parameters and characteristics of traction electrical motor (TM), with which any development of frictional auto-oscillations is eliminated.

Research Analysis and Publications: Different methods and criteria of dynamic systems stability estimation (i.e. algebraic, root, frequency criteria), that are known and widely utilized in engineering at present, require that all parameters of the system are to be preset. Then it will be possible to estimate the stability of such dynamic system by the sign of the real part of characteristic polynomial roots. However, in this case, it is rather difficult to determine what parameters and to what degree actually affect the said stability. In order to establish this, it will be necessary to search and go through their values within defined limits.

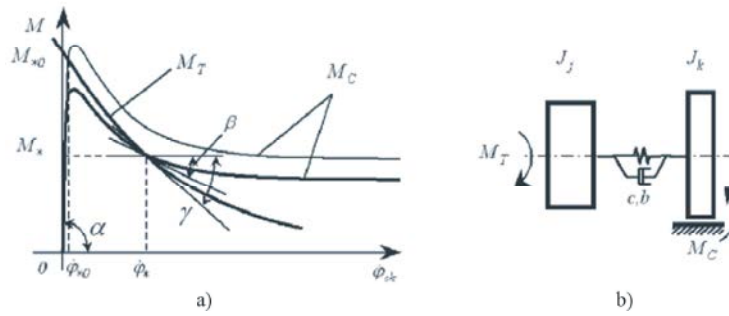


Fig. 1: Relative position of wheel grip characteristic and traction characteristic in wheelslip state - a. Block diagram of traction drive - b.

Certain algebraic criteria (Hurwitz's criterion, for example) allow system parameters to be set in symbol form, however as the system dimensionality increases, they result in development of complex and hard-to-observe analytical expressions. And it is not clear what frequencies and what mode auto-oscillations can develop in cases when the system turns unstable.

An estimation method of stability of the dynamic systems with small dissipation was proposed in research paper [1] and it is based on oscillation energy balance of each mode. The advantage of this method is that stability conditions are recorded in dissipative parameters of dynamic system. In spite of being simple and ostensive as it is, it can not be readily applied in the form as it was interpreted in the paper [1], to research stability of the systems that, apart from express oscillators, also contain aperiodic links describing electromechanical characteristics of system, for instance. Therefore, in calculation models of locomotive traction drives, stability of which was evaluated with this method, TM electromagnetic processes were either excluded from consideration altogether or they were examined separately.

The research paper [2] describes an attempt to take into account the electromagnetic transients in TM circuits within the dynamic model of traction drive by introducing an additional oscillator. However, since certain randomness and loss of physical meaning are allowed in selection of parameters for that oscillator, it can not be accepted as a correct approach.

Therefore, appropriate research and development of the approximate method of system stability estimation, with accuracy sufficient for engineering calculations, remains a topical problem at present.

Objective of the Research: The objective of the current research is to establish possibility of application of the power balance method, by using traction drive model in

wheelslip state, for estimation of stability of dynamic oscillation systems with small dissipation and containing aperiodic inertial links.

Materials and Results of the Research: Wheel slip usually occurs when traction moment M_T , applied to a wheelpair in traction mode M_{s0} , turns out to be larger than the maximum resulting moment of wheel grip M_C with rails; and as a result of it, the equilibrium state $[M_s, \phi_s]$ is moved to descending area of wheel grip characteristic – Fig. 1.

Model of traction drive can be presented in the form of two-mass torsion system (Fig. 1b), the motion of which is described by the system of differential equations:

$$\left. \begin{aligned} TM + M &= M_T(\phi_j); \\ J_j \ddot{\phi}_j + b\dot{\Delta} + c\Delta &= M; \\ J_k \ddot{\phi}_k - b\dot{\Delta} - c\Delta &= -M_C(\phi_k). \end{aligned} \right\} \quad (1)$$

where $\Delta = \phi_j - \phi_k$; ϕ_j and ϕ_k – are angular coordinates of TM armature and wheel pair respectively;

J_j and J_k – inertia moment, applied to wheel pair axis, of TM armature and wheel pair relative to axis of rotation;

c and b – are elastic and dissipative characteristics of connection between TM armature and wheel pair;

$M_T(\phi_j)$ – quasi-static, traction characteristic of TM applied to wheel pair axis;

$M_C(\phi_k)$ – friction characteristic;

T – time response of TM.

Wheel slip is assumed to occur in motion of locomotive with a small speed, when traction moment can reach its maximum value and the influence of forced oscillation of carriage on torsion system of the drive can be ignored.

The first equation of system (1) shows electromagnetic processes in TM, which corresponds to recommendations provided in the paper [3] and it is regarded to be a conventional approach in cases when main emphasis of entire research is actually put on the mechanical part of a traction drive rather than its electric part.

Building of existence areas of frictional auto-oscillations in the space of traction drive parameters is reduced to finding such damping coefficient b , with which the dynamic system stays on stability boundary.

Consider the system stability "in the small", i.e. in the neighbourhood of equilibrium state $[M_*, \dot{\phi}_*]$ – Fig. 1a.

In order to do so we linearize system (1) near the specified state and we will move to the new co-ordinates that eliminate the constant components of moments and angular velocities. The linearized system (1) will become:

$$\left. \begin{aligned} TM + M &= \gamma \dot{\phi}_j; \\ J_j \ddot{\phi}_j + b \dot{\Delta} + c \Delta &= M; \\ J_k \ddot{\phi}_k - b \dot{\Delta} - c \Delta &= \beta \dot{\phi}_k. \end{aligned} \right\} \quad (2)$$

where γ and β – are ruggedness of traction characteristic and wheel grip characteristic near the state $[M_*, \dot{\phi}_*]$, respectively.

And if the system remains on stability boundary then, from the condition of power balance it follows that the power, average for the period, which is input into the system by auto-oscillations E^+ is equal to the average power dissipated with auto-oscillations E^- – [1].

Auto-oscillations are close to harmonic in the systems with small dissipation and they occur with frequencies close to natural frequencies of system oscillations ω_i [1, 4]. Since own modes of oscillations are orthogonal, then it appears possible to derive relations of power balance separately for each oscillation mode shape.

In this case, with $T = 0$ we have the following:

$$E^+ = \frac{1}{2} \beta \dot{\phi}_k^2; E^- = \frac{1}{2} [b(\dot{\phi}_j - \dot{\phi}_k)^2 + \gamma \dot{\phi}_j^2]. \quad (3)$$

In general, the condition of equality of powers $E_i^+ = E_i^-$ can be presented using oscillation coefficients μ_{qi} with natural frequency ω_i :

$$\beta \mu_{ki}^2 = b(\mu_{ji} - \mu_{ki})^2 + \gamma \mu_{ji}^2. \quad (4)$$

The system examined here (Fig. 1b) has the only non-zero natural frequency, $\omega = \sqrt{c \left(\frac{1}{J_j} + \frac{1}{J_k} \right)}$, for which, while assuming $\mu_k = 1$, we will define $\mu_j = -\frac{J_k}{J_j}$.

Then, from the equality (4) we will get the expression to define boundary damping coefficient \bar{b} with $T = 0$:

$$\bar{b} = \frac{\beta - \gamma \mu_j^2}{(1 - \mu_j)^2}. \quad (5)$$

With $T \neq 0$ the expression (5) will look as follows:

$$b = \frac{\beta - \gamma_* \mu_j^2}{(1 - \mu_j)^2}, \quad (6)$$

where γ_* – the coefficient depending on time response T and frequency ω .

And it is coefficient γ_* that should be defined for estimating stability of the dynamic system considering TM electromagnetic processes.

Block diagrams of system simulations (2) in the program complex PC MVTU [5] for variants $T \neq 0$ and $T = 0$ are shown on Fig. 2.

From Fig. 2 it follows that the diagram considering electromagnetic transients ($T \neq 0$, Fig. 2a) differs from analogous by having flexible (frequency-dependent) feedback to transfer function $W(s) = \frac{k}{T_s + 1}$ presented by

inertial link of the first order.

On frequencies, much larger than break frequencies $\omega_c = 1/T$, the first order inertial link changes the input signal phase by value up to $\psi = -\pi/2$. With assumption that dissipative coefficients have minor influence on natural frequencies and mode shape coefficients, the phase shift has no fundamental importance. However the phase angle ψ affects the value of dissipated power. Therefore, there is a relation between coefficients γ and γ_* :

$$\begin{aligned} \gamma_* &= \gamma |W(s)| \cos \psi = \gamma \frac{1}{\sqrt{(T\omega_i)^2 + 1}} \cos[\arctg(T\omega_i)] = \\ & \gamma |W(s)|^2 = \gamma \frac{1}{(T\omega_i)^2 + 1}, \end{aligned} \quad (7)$$

Expression (7) reflects the dependence of electromagnetic damping coefficient γ_* on auto-oscillations frequency ω_i , on electric machine time response T and on ruggedness of the quasi-static traction characteristic γ , corresponding to equilibrium state.

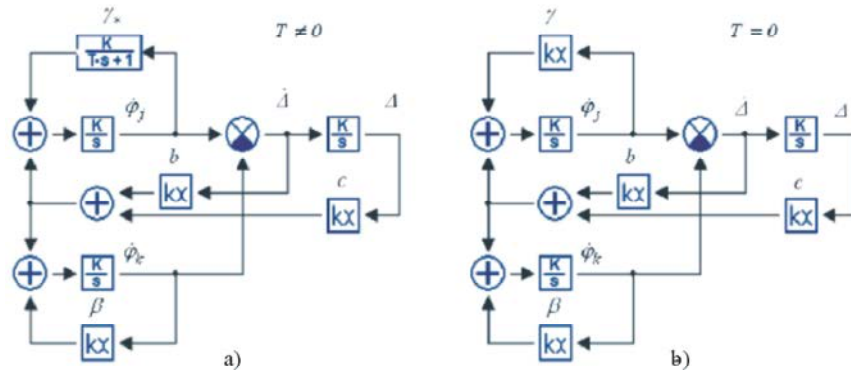


Fig. 2: Block diagrams of system simulation (2).

Relations (6) and (7) allow us to calculate boundary damping coefficient with auto-oscillations for every natural frequency ω_i of multi-mass dynamic system.

If $\omega_i \ll \omega_e$, then it is possible to accept $\gamma_* = \gamma$. And if $\omega_i \ll \omega_e$, then it is possible to accept $\gamma_* = 0$. If frequency ω_i is commensurable with frequency ω_e , then it is reasonable to use relations (6) and (7). Thus, with $\omega_i = \omega_e$ we get: $\gamma_* = 0,5 \gamma$.

The calculations of boundary damping, that were executed using the relations (6) and (7), completely confirm the patterns received from direct simulation and described in research paper [3].

Value difference between boundary damping received through calculations and direct simulation does not exceed 2 % for real values of elastic-dissipative inertial parameters of locomotive traction drives and TM time response $T = 0,001 \dots 0,25 \text{ sec}$.

One more kind of auto-oscillations caused exclusively by electromagnetic transients in TM [3] can occur in traction drive. These auto-oscillations have frequency ω_e , which is considerably less than the frequencies of own oscillations of mechanical system.

In this case all components of traction drive operate in-phase and its mechanical part can be considered as a single body of rotation. Then, taking that $\dot{\phi}_j = \dot{\phi}_k = \dot{\phi}$ and $J = J_j + J_k$ and adding the second and third equations in the system of equations (2), we get the following:

$$\begin{cases} T\dot{M} + M = -\gamma\dot{\phi}; \\ J\ddot{\phi} = M + \beta. \end{cases} \quad (8)$$

Then the characteristic equation of the system of equations (8) will be:

$$Tjp^2 + (J - T\beta)p + (\gamma - \beta) = 0. \quad (9)$$

And frequency of oscillations of the model described by the system of equations (8) can be determined from this expression:

$$\omega_e = \sqrt{\frac{(\gamma - \beta)}{TJ} - \left(\frac{J - T\beta}{2TJ}\right)^2}. \quad (10)$$

It follows from expression (10) that oscillations are possible to occur under certain condition:

$$(J - T\beta)^2 - 4TJ(\gamma - \beta) < 0, \quad (11)$$

and from whence we can obtain the values of T_k , which make the oscillatory process possible.

Thus for the parameters of drive model: $J_j = 1,2 \cdot 10^6 \text{ kgm}^2$, $J_k = 0,4 \cdot 10^6 \text{ kgm}^2$, ($J = 1,6 \cdot 10^6 \text{ kgm}^2$), $c = 10^3 \dots 10^4 \text{ kNm}$, $\beta = 6 \text{ kNmsec}$, $\gamma = 10 \text{ kNmsec}$, from an inequality (11) we find that: $0,06 \text{ sec} < T_k < 1,184 \text{ sec}$. On the boundaries of specified range T_{ek} the frequency $\omega_e = 0$ and within this range the frequency has a maximum which we can determine by using the method of derivatives: $\omega_e^{\max} = 3,94 \text{ sec}^{-1} = 0,63 \text{ Hz}$ with

$$T_k^{\max} = 0,11 \text{ sec}.$$

Found frequency ω_e^{\max} is one order less than the oscillation frequency ω of torsion system of the drive ($\omega = 57,7 \dots 182,6 \text{ sec}^{-1}$) and mutual influence of oscillations with frequencies ω_e and ω can be ignored. Therefore, representation of drive model in the form of a single solid body of rotation for studying infra-low-frequency oscillations is quite justified.

Stability condition for the system of equations (8) follows from the equation (9):

$$J - T\beta > 0,$$

whence we find the minimum value of time response T_e , the exceeding of which leads to development of auto-oscillations:

$$T^{\min} \leq \frac{J}{\beta} \quad (12)$$

For the accepted parameters of the model we get the value: $T^{\min} = 0,267\text{sec}$.

Thus, we have the following:

- at $T < 0,06\text{sec}$ - the oscillations caused by electromagnetic properties of TM are absent;
- at $0,06\text{sec} < T < 0,267\text{sec}$ - the oscillations with frequency ω_e have attenuating character;
- at $0,267\text{sec} < T < 1,184\text{sec}$ - the auto-oscillations with frequency ω_e start developing in the drive;
- at $T > 1,184\text{sec}$ - an aperiodic instability occurs in the drive.

We should note that the time response value lies in the range of $T = 0,047...0,37\text{sec}$ – [3] for real designs of locomotive TMs. It means that there is objectively a narrow range of values of time response $T = 0,267...0,37\text{sec}$, at which auto-oscillations with infra-low frequency $\omega_e < 3,94\text{sec}^{-1}$ may develop in traction drive. To eliminate such auto-oscillations only by means of dissipative parameters of the mechanical part of drive is not possible; and with $T > T^{\min}$ the expressions (5) and (6) lose their meaning (in this case $T^{\min} = 0,267\text{sec}$).

Therefore, when using the described method of power balance, it is necessary to establish an application range of this method for an actual dynamic system by defining the boundary value T^{\min} from the expression (12).

It is possible to exclude auto-oscillations with the infra-low frequency, caused by electromagnetic properties of TM, but only by making artificial modification of traction characteristic (the increase of dynamic stiffness) through appropriate choice of structure and parameters of drive control system.

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CONCLUSIONS

- On the basis of analysis of the block diagram of dynamic system simulation, we received the expression for definition of electromagnetic damping coefficient in TM circuits, determined by the parameters of electric machine.
- This received expression reflects the connection between TM time response, its quasistatic characteristic and natural frequencies of dynamic system.
- The mathematics obtained in this research allow to apply the power balance method to dynamic systems containing electric subsystems.
- The results received with the method proposed in this paper conform with the results of direct simulation, with exactitude acceptable for engineering calculations.

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