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Numerical Method for Solving Multipoints Elliptic-Parabolic Equation for Dehydration Process

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Abstract: Drying is the oldest and efficient form of preserving fruits. This research focuses on the mathematical modeling of tropical fruits dehydration using instant controlled pressure drop (Détente Instantanée Controlée or known as DIC) technique. We proposed a modification of mathematical modeling to enhance the previous modeling from Haddad *et al*. [10]. The mathematical modeling presents the dehydration process of DIC technique which involves parameters such as pressure, water content, time dependency, dimension of region and temperature behavior. The modification of the mathematical modeling has been done by transforming the quadratic equation to partial differential equation (PDE). The simulation of the dehydration process will be illustrated through Jacobi method based on two, three and five points forward difference schemes. The sequential algorithm is developed by using Matlab 7.6.0 (R2008a) programming. The numerical analysis of finite difference schemes in terms of number of iteration, time execution, maximum error and computational cost are compared.

Key words: DIC technique . elliptic-parabolic equation . sequential algorithm . finite difference schemes

INTRODUCTION

Dehydration of fruits and vegetables is one of the most ancient and efficient preservation methods [1]. It is necessary to remove the moisture content of fruits to a certain level after harvest to prevent the growth of mould and bacterial action [2]. Recent years, the advances in dehydration techniques and the development of novel drying methods have enabled the preparation of a wide range of dehydrated fruits and vegetables in meeting the quality, stability and functional requirements coupled with economy [3]. For numerous new process, instant controlled pressure drop (DIC) technology could greatly intensify the limiting transfer phenomenon, reduce energy consumption and provide environmental friendly process [4, 5].

The DIC technique was initially developed by Allaf (1988) in the University of La Rochelle [1, 4, 6, 7]. DIC technique consists of applying instant pressure drop to modify the texture of the material and intensify functional behavior [8]. A vacuum condition is created at the beginning, followed by injection steam to the material keeping for several seconds and proceeds with the sudden pressure drop toward vacuum. The sudden pressure drop causes quick cooling of the treated material and massive evaporation of water from it. The diagrammatic layout of the DIC can be shown in Fig. 1 [9].

Many experimental studies were carried out to analyze and foresee the moisture content, steam pressure and treatment duration in foodstuff [9-11].

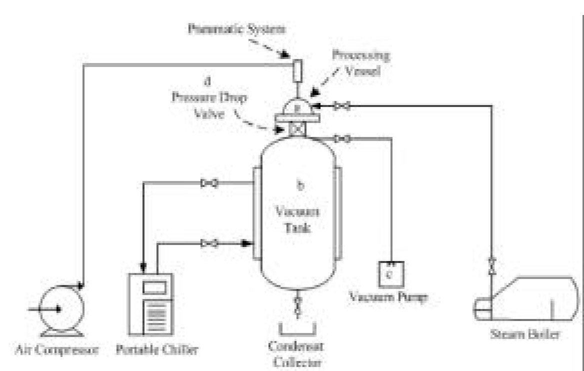

Fig. 1: Schematic diagram of the DIC reactor: (a) treatment vessel with heating jacket, (b) vacuum tank with cooling liquid jacket, (c) vacuum pump and (d) instant pressure drop valve

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However, based on the latest literature review, there are only a few mathematical modeling deals with DIC. The contribution of this paper is to select the appropriate drying model to evaluate the drying distribution which is crucial in order to get best prediction of drying behavior [12].

In this paper, a mathematical modeling based on partial differential equation (PDE) with elliptic-parabolic type is proposed to describe the drying potential of fruits to enhance the mathematical modeling from [10]. The numerical simulation of the elliptic-parabolic equation is well suited in estimating the drying rate at any moisture condition of the foodstuffs.

MATHEMATICAL SIMULATION MODELING

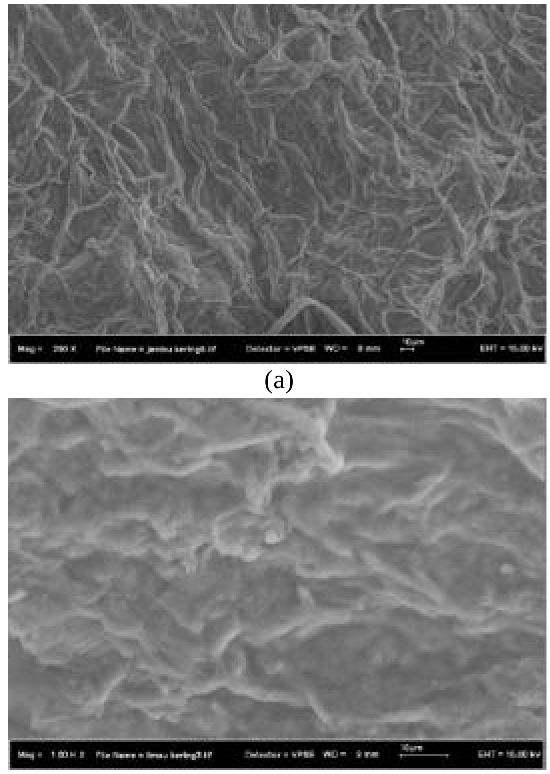
Many of the simple mathematical models are being established using DIC technique in order to predict water losses, weight reduction, dehydration rates and temperature behavior. The modification of mathematical model will be used in observing and controlling the dehydration process. Some parameters of dehydration process can range from a very simple to extremely complicated in order to upgrade the quality of dehydration technology, however, we only focus on the effect of the pressure, water content, time dependency, dimension of region and temperature behavior towards the modification of dehydration process.

An experiment of the effect of DIC treatment on the phytate content equation of Lupinus albus was done by [10]. The result showed the phytate content decreased by 16% after 1 minute of DIC treatment as compared to 30 minutes of steaming needed to reduce by 10% [13]. Based on the regression model obtained, the chosen operating parameters for an α of 5% are treatment pressure and initial water content are significant. The suggested model is as follows:

$$\begin{aligned}\theta(P,W,t) &= a + bP + ct + dW + eP^2 \\ &\quad + fPt + gPW + ht^2 + jtW + kW^2\end{aligned} \tag{1}$$

where θ is the phytate content, P is the steam pressure (Pa), t is the processing time (second), W is the initial water content (g/100g dry matter) and for constant a to k; $a = 7.02534$; $b = 0.587737$; $c = 0.132357$; $d = 0.344761$; $e = -0.0584031$; $f = -0.01375$; $g = 0.007080333$; $h = 0.000365311$; $j = -0.0028875$ and $k = -0.00272829$.

The motivation of this research started from the laboratory experimental for convective drying of the psidium guajava which is one of the guava fruit species and citrus suhueinsis as one of the sweet orange species (Fig. 2).



(a)

(b)

Fig. 2: FESEM image for (a) dried psidium guajava, (b) citrus suhueinsis

Based on the FESEM results, we proposed some important operating parameters involved in DIC process which are pressure, initial water content and treatment time [8-10]. Therefore, based on the basic equation of quadratic equation (1), it can be modified using PDE of 2 dimensional elliptic-parabolic equation. In PDE, the first order parabolic equation is used to generate the temperature behavior. The mathematical model from [10] is in quadratic function but for this paper, the PDE model is more accurate in predicting the dehydration process.

Elliptic-parabolic problems arise as a model in many applications, for example, as a model of flow through porous media [14, 15]; pressure equation in an injection moulding process [16] and also in electromagnetic field theory [17]. Thus, the elliptic-parabolic equation is chosen due to the suitability for this dehydration process.

$$\frac{\partial u}{\partial t} = \left(\frac{\partial P}{\partial x} + \frac{\partial P}{\partial y}\right) + \left(\frac{\partial W}{\partial x} + \frac{\partial W}{\partial y}\right) \tag{2}$$

where u is the dehydration process, P is the steam pressure, W is water content and x and y are directions of space variables. The initial and boundary conditions are given as:

$$u(x,y,0) = f(x,y), \quad 0 \le x \le 1, 0 \le y \le 1$$

$$P(x,y,t) = g(x,y,t), \quad 0 \le x \le 1, 0 \le y \le 1, t > 0$$

$$W(x,y,t) = h(x,y,t), \quad 0 \le x \le 1, 0 \le y \le 1, t > 0$$

where f, g and h are known functions. The equation is then discretized using forward finite difference method (FFDM). The discretization will be discussed further in Section III.

DISCRETIZATION

Discretization is a transformation of differential equations into discrete difference equations to derive the numerical method. A discrete form can be found by using FDM scheme. Equation (2) is discretized by implementing the 2 points, 3 points and 5 points forward difference at time t_k and space derivative at position x_i and y_j for first derivative; we get the recurrence equation as follows:

2 point forward difference: By using two points forward differences scheme, (2) becomes

$$\frac{u_{i,j,k+1} - u_{i,j,k}}{\Delta t} = \left(\frac{P_{i+1,j,k} - P_{i,j,k}}{\Delta x} + \frac{P_{i,j+1,k} - P_{i,j,k}}{\Delta y}\right) + \left(\frac{W_{i+1,j,k} - W_{i,j,k}}{\Delta x} + \frac{W_{i,j+1,k} - W_{i,j,k}}{\Delta x}\right)$$

Rearranging the equation, we obtain

$$u_{i,j,k+1} = \frac{\Delta t}{\Delta x}\left(P_{i+1,j,k} - W_{i+1,j,k}\right) + \frac{\Delta t}{\Delta y}\left(P_{i,j+1,k} + W_{i,j+1,k}\right) - \left(\frac{\Delta t}{\Delta x} + \frac{\Delta t}{\Delta y}\right)\left(P_{i,j,k} + W_{i,j,k}\right) + u_{i,j,k} \tag{3}$$

3 points finite difference: Based on the three points forward differences scheme, (2) becomes

$$\frac{3u_{i,j,k+1} - 4u_{i,j,k} + u_{i,j,k-1}}{2\Delta t} = \left(\begin{array}{l}\frac{3P_{i+1,j,k} - 4P_{i,j,k} + P_{i-1,j,k}}{2\Delta x} \\ + \frac{3P_{i,j+1,k} - 4P_{i,j,k} + P_{i,j-1,k}}{2\Delta y}\end{array}\right) + \left(\begin{array}{l}\frac{3W_{i+1,j,k} - 4W_{i,j,k} + W_{i-1,j,k}}{2\Delta x} \\ + \frac{3W_{i,j+1,k} - 4W_{i,j,k} + W_{i,j-1,k}}{2\Delta y}\end{array}\right)$$

Rearranging the equation, we obtain

$$u_{i,j,k+1} = \frac{1}{3}\left[\begin{array}{l}\frac{\Delta t}{\Delta x}\left(-4P_{i-1,j,k} + P_{i-2,j,k} - 4W_{i-1,j,k} + W_{i-2,j,k}\right) + \frac{\Delta t}{\Delta y}\left(-4P_{i,j-1,k} + P_{i,j-2,k} - 4W_{i,j-1,k} + W_{i,j-2,k}\right) \\ +3\left(\frac{\Delta t}{\Delta x} + \frac{\Delta t}{\Delta y}\right)\left(P_{i,j,k} + W_{i,j,k}\right) + 4u_{i,j,k} - u_{i,j,k-1}\end{array}\right] \tag{4}$$

5 points forward difference: By using two points forward differences scheme, (2) becomes

$$\frac{\begin{array}{l}-u_{i,j,k+2} + 8u_{i,j,k+1} \\ -8u_{i,j,k-1} + u_{i,j,k-2}\end{array}}{12\Delta t} = \left(\frac{-P_{i+2,j,k} + 8P_{i+1,j,k} - 8P_{i-1,j,k} + P_{i-2,j,k}}{12\Delta x} + \frac{-P_{i,j+2,k} + 8P_{i,j+1,k} - 8P_{i,j-1,k} + P_{i,j-2,k}}{12\Delta y}\right)$$
$$+ \left(\frac{-W_{i+2,j,k} + 8W_{i+1,j,k} - 8W_{i-1,j,k} + W_{i,j,k-2}}{12\Delta x} + \frac{-W_{i,j+2,k} + 8W_{i,j+1,k} - 8W_{i,j-1,k} + W_{i,j-2,k}}{12\Delta y}\right)$$

Rearranging the equation, we obtain

$$u_{i,j,k+1} = \frac{1}{8}\left[\begin{array}{l}\frac{\Delta t}{\Delta x}\left(\begin{array}{l}-P_{i+2,j,k} + 8P_{i+1,j,k} - 8P_{i-1,j,k} + P_{i-2,j,k} \\ -W_{i+2,j,k} + 8W_{i+1,j,k} - 8W_{i-1,j,k} + W_{i-2,j,k}\end{array}\right) + \frac{\Delta t}{\Delta y}\left(\begin{array}{l}-P_{i,j+2,k} + 8P_{i,j+1,k} - 8P_{i,j-1,k} + P_{i,j-2,k} \\ -W_{i,j+2,k} + 8W_{i,j+1,k} - 8W_{i,j-1,k} + W_{i,j-2,k}\end{array}\right) \\ +u_{i,j,k+2} + 8u_{i,j,k-1} - u_{i,j,k-2}\end{array}\right] \tag{5}$$

Table 1: Numerical analysis for 2 points, 3 points and 5 points finite difference schemes

Numerical analysis		2 points	3 points	5 points
Number of iteration		2	2	2
Time execution (s)		1.071589	1.094466	1.104279
Max error/abs error		0.00	0.00	0.00
Computational cost	Multiplication	14	28	28
	Additional	14	24	36

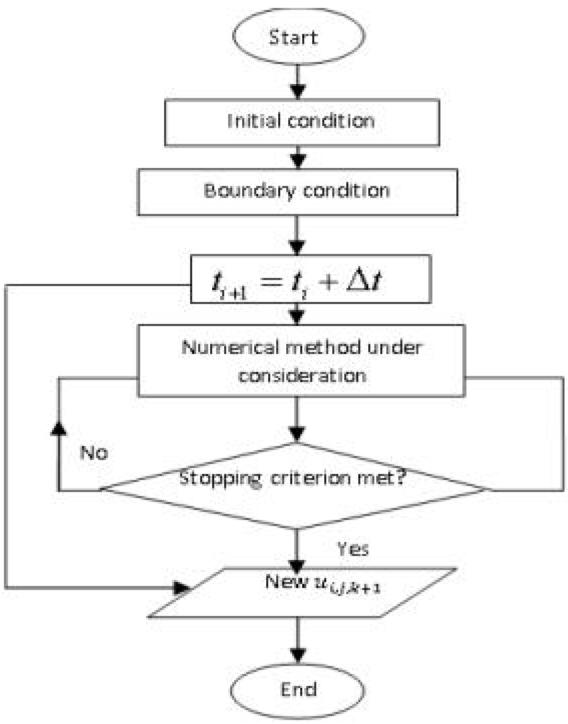


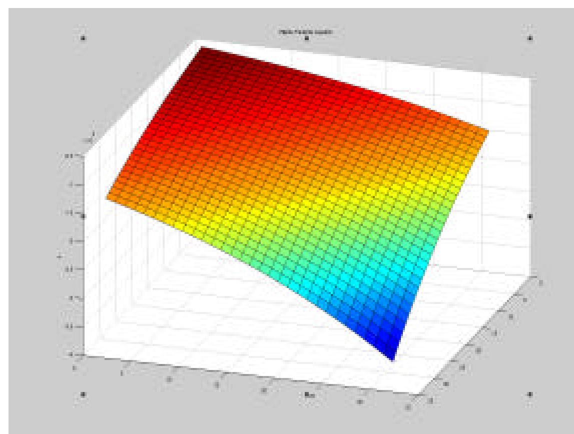
Fig. 3: The sequential algorithm

The governing equation (2) which describes the drying characteristic is solved numerically. Jacobi method is the selected scheme for solving the multipoint discretization of elliptic-parabolic equation. The transformation of the simultaneous linear system of equations into matrix form is used to solve (3-5). Jacobi Iterative method is a simple and fundamental iterative method. Jacobi method computes the value of u for each component respect to x and y.

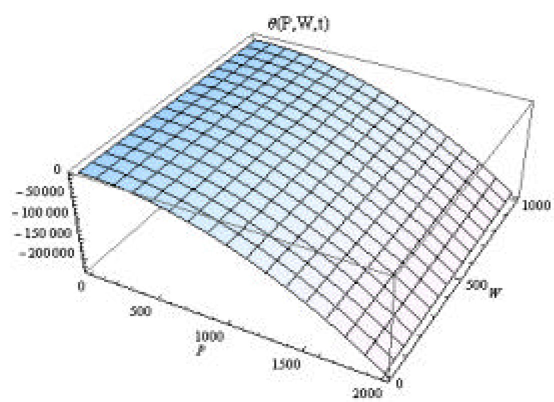
$$u_i^{(k+1)} = \left(b_i - \sum a_{ij} u_j^{(k)} \right) \Big/ a_{ii}, \quad i = 1,2,3,\ldots,m$$

The method is repeated until it reaches the stopping criterion such that $\left| u_i^{(k+1)} - u_i^{(k)} \right| \le \varepsilon$ where ε is the convergence criterion.

Sequential algorithm: Figure 3 shows the sequential algorithm for numerical simulation of the elliptic-parabolic equation.



(a)



(b)

Fig. 4: The visualization of dehydration process based on: (a) elliptic-parabolic equation using Matlab software (b) exact solution of (1) by using Mathematica software

NUMERICAL ANALYSIS

The numerical analysis of the three forward difference schemes is illustrated in this section. The analysis is based on the number of iteration, time execution, maximum error and computational cost. Matlab 7.6.0 (R2008a) software used to solve the PDE. The CPU is supported by Intel®Core^{TM} based on Quadcore processors.

Table 1 shows the two points forward difference scheme is faster than three and five points. This is due to the lowest amount of the arithmetic operations and time execution in two points FD compared to three and five points. However, the convergence of these three schemes contributes to the similar number of the iteration. This is because the values of P and W are constant.

Figure 4 shows the dehydration process of the approximation solution using elliptic-parabolic equation and the exact solution using Haddad *et al*. (2007). The figure shows as the pressure and the water content decreased respect to the increasing of time. By comparing Fig. 4 (a) and (b), the graphs show the similarity in the form of the quadratic shape and curve. Based on PDE concept and the behavior of the temperature-pressure, elliptic-parabolic graph is not a linear line at the boundary. However, Haddad's model visualized the linear line almost for all domains. Thus, the elliptic-parabolic equation can be used as an alternative model to represent the dehydration process using DIC technique.

CONCLUSION

A mathematical modeling based on PDE with elliptic-parabolic type is used to present the dehydration process which involves parameters such as pressure, water content, time dependency, dimension of region and temperature behavior. Three finite difference schemes which are two, three and five points forward difference schemes are solved using Jacobi method. The result of numerical analysis is compared based on the number of iteration, time execution, computational cost and maximum error. The visualization of the mathematical modeling using Matlab 7.6.0 (R2008a) has shown that the elliptic-parabolic equation is the alternative model to optimize the prediction of the dehydration process using DIC technique.

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