World Applied Sciences Journal 21 (12): 1870-1877, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.21.12.1991

Soft Interior Ideals of Abel Grassmann's Groupoids Related to Fuzzy Sets

Asghar Khan and Sultan Hussain

Department of Mathematics, COMSATS Institute of Information Technology,
Abbottabad-22062, KPK, Pakistan

Abstract: The concepts of $\in$-soft sets and q-soft sets in AG-groupoids are introduced and some related properties are investigated. In particular, we describe the relationships among ordinary fuzzy interior ideals and soft interior ideals over an AG-groupoid S. Moreover, we study the relation-ships among $(\in, \in \vee q)$-fuzzy interior ideals, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy interior ideals, $(\alpha; \beta)$-fuzzy interior ideals and soft interior ideals over AG-groupoid S.

Key words: Fuzzy interior ideals . (α, β)-fuzzy interior ideals . soft interior ideals over AG-groupoids

INTRODUCTION

In [25], Zhan and Jun introduced the notion of $\in$-soft set and q-soft set based on a fuzzy set and investigated characterizations for $\in$-soft set and q-soft set to be (filteristic) soft BL-algebras. In [3], Aktas, introduced the concept of soft sets in groups and studied the related properties of soft groups. Jun [8] applied the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras and introduced the notion of soft BCK/BCI-algebras and soft subalgebras and then derived their basic properties. Jun and Park [9] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory.

On the other hand, uncertainties can not be handled using traditional mathematical tools but may be deal with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [14]. Maji *et al*. [15] and Molodtsov [14] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [14] introduced the concept of a soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Maji *et al*. [15] described the application of soft set theory to a decision making problem. Chen *et al*. [6] presented a new definition of soft set parameterization reduction and compared this definition to the related concept of attributes reduction in rough set theory. The notion of a fuzzy set introduced by Zadeh [24] as a method for representing uncertainty. Since then it has become an important area of research in different areas such as medical science, signal processing, pattern recognition etc. The study of fuzzy sets in algebraic structures are carried out by several authors [1,2,4,5,7,11,13,20-23]. In [12], Khan *et al*. introduced the concept of a generalized fuzzy interior ideal in AG-groupoids and discussed some of their related properties for this structure by using the notion of a generalized fuzzy interior ideal. In [12], the concept of a strongly fuzzy interior ideal in an AG-groupoid was introduced and some of the related properties were studied. Also, the authors studied some characterizations of AG-groupoids by using different types of (α, β)-fuzzy interior ideals.

In this paper, the concepts of an $\in$-soft set and q-soft set are introduced in AG-groupoids and some of their related properties are investigated. In particular, we describe the relationships among ordinary fuzzy interior ideals and soft interior ideal over an AG-groupoid S. Moreover, we study the relationships among $(\in, \in \vee q)$-fuzzy interior ideals, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy interior ideals, (α, β)-fuzzy interior ideals and soft interior ideals over AG-groupoids S.

PRELIMINARIES

An Abel Grassmann's groupoid, abbreviated as AG-groupoid, is a groupoid S whose elements satisfy the left invertive law: $(ab)c = (cb)a$ for all $a, b, c \in S$ [19]. An AG-groupoid is the midway structure between

Corresponding Author: Asghar Khan, Department of Mathematics, COMSATS Institute of Information Technology, Abbottabad-22062, KPK, Pakistan

a commutative semigroup and a groupoid [17]. It is a useful non-associative structure with wide applications in theory of flocks [18]. In an AG-groupoid the medial law, (ab)(cd) = (ac)(bd) holds for all a, b, c, d$\in$S [17]. If there exists an element e in an AG-groupoid S such that ex = x for all x$\in$S then S is called an AG-groupoid with left identity e and the following laws hold:

$$(ab)(cd) = (dc)(ba) \text{ and } a(bc) = b(ac)$$

for all a, b, c, d$\in$S. If an AG-groupoid S has the right identity then S is a commutative monoid.
For subsets A, B of an AG-groupoid S, we denote by

$$AB = \{ab \mid a \in A, b \in B\}$$

A nonempty subset A of an AG-groupoid S is called an AG-subgroupoid of S if $A^2 \subseteq A$. A is called an interior ideal [13] of S if (i) $A^2 \subseteq A$ and (ii) $(SA)S \subseteq A$.

Throughout this paper S will denote an AG-groupoid unless stated otherwise.

Now we recall some fuzzy logic concepts.

By a fuzzy subset μ of S, we mean a mapping, $\mu : S \to [0,1]$.

Definition 2.1: [13]. A fuzzy subset μ of S is called a fuzzy subsemigroup of S if it satisfies the following condition:

$$(\forall x, y \in S)\left(\mu(xy) \geq \min\{\mu(x), \mu(y)\}\right)$$

Definition 2.2: [13]. A fuzzy subset μ of S is called a fuzzy interior ideal of S, if it satisfies the following conditions:

(i) $(\forall x, y \in S)\left(\mu(xy) \geq \min\{\mu(x), \mu(y)\}\right)$

(ii) $(\forall x, y, a \in S)\left(\mu((xa)y) \geq \mu(a)\right)$

Lemma 2.3: [13]. A non-empty subset A of S is an interior ideal of S if and only if χ_A (the characteristic function of A) is a fuzzy interior ideal of S.

Let S be an AG-groupoid and μ a fuzzy subset of S. Then for every $\lambda \in (0,1]$ the set

$$U(\mu;\lambda) := \{x \mid x \in S \text{ and } \mu(x) \geq \lambda\}$$

is called a level set of μ:

Lemma 2.4: [12]. A fuzzy subset μ of S is a fuzzy interior ideal of S if and only if $U(\mu;\lambda)(\neq \phi)$ is an interior ideal of S for every $\lambda \in (0,1]$.

In what follows let S denote an AG-groupoid and let α, β denote any one of $\in$, q, $\in\vee q$ or $\in\wedge q$.
A fuzzy subset μ of the form

$$\mu(y) := \begin{cases} \lambda\ (\neq 0) & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$$

is called a fuzzy point with support x and value λ and is denoted by $[x;\lambda]$.

A fuzzy point $[x;\lambda]$ is said to belong to (resp. quasi-coincidence with) a fuzzy set μ, written as $[x;\lambda] \in \mu$ (resp. $[x;\lambda] q\mu$) if $\mu(x) \geq \lambda$ (resp. $\mu(x) + \lambda \geq 1$). If $[x;\lambda] \in \mu$ or $[x;\lambda] q\mu$, then $[x;\lambda] \in\vee q\mu$. The symbol $\overline{\in\vee q}$ mean $\in\vee q$ does not hold.

Theorem 2.5: [12]. For any fuzzy subset μ of S, the conditions (i) and (ii) of Definition 2.2, are equivalent to the following:

(iii) $(\forall x, y \in S)\ (\forall \lambda_1, \lambda_2 \in (0,1])$
$\left([x;\lambda_1] \in \mu, [y,\lambda_2] \in \mu \Rightarrow [xy; \min\{\lambda_1, \lambda_2\}] \in \mu\right)$.

(iv) $(\forall x, y, a \in S)\ (\forall \lambda \in (0,1])\ \left([a;\lambda] \in \mu \Rightarrow [(xa)y;\lambda] \in \mu\right)$

Definition 2.6: [12]. A fuzzy subset μ of S is called an (α, β)-fuzzy interior ideal of S; where $\alpha \neq \in\wedge q$, if it satisfies the following conditions:

(i) $(\forall x, y \in S)\ (\forall \lambda_1, \lambda_2 \in (0,1])$
$\left([x;\lambda_1] \alpha\mu, [y,\lambda_2] \alpha\mu \Rightarrow [xy; \min\{\lambda_1, \lambda_2\}] \beta\mu\right)$

(ii) $(\forall x, y, a \in S)\ (\forall \lambda \in (0,1])$
$\left([a;\lambda] \alpha\mu \Rightarrow [(xa)y;\lambda] \beta\mu\right)$

Proposition 2.7: [12]. Let μ be a fuzzy subset of S. If $\alpha = \in$ and $\beta = \in\vee q$ in Definition 2.6. Then (i) and (ii) respectively, of Definition 2.6, are equivalent to the following conditions:

(i) $(\forall x, y \in S)\left(\mu(xy) \geq \min\{\mu(x), \mu(y), 0.5\}\right)$.

(ii) $(\forall x, y, a \in S)\left(\mu((xy)a) \geq \min\{\mu(a), 0.5\}\right)$.

Definition 2.8: A fuzzy subset μ of an AG-groupoid S is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy interior ideal of S if it satisfies the following conditions:

(i) $(\forall x, y \in S)\ (\forall \lambda_1, \lambda_2 \in (0,1])$
$\left([xy; \min\{\lambda_1, \lambda_2\}] \overline{\in} \mu \Rightarrow [x;\lambda_1] \overline{\in} \vee \overline{q}\mu \text{ or } [y;\lambda_2] \overline{\in} \vee \overline{q}\mu\right)$

(ii) $(\forall x, y \in S)\ (\forall \lambda \in (0,1])\ \left([(xa)y;\lambda] \overline{\in}\mu \Rightarrow [a;\lambda] \overline{\in} \vee \overline{q}\mu\right)$

Example 2.9: Let S = {a,b,c,d,e} with the following multiplication

Table:

.	a	b	c	d	E
a	a	a	a	a	A
b	a	a	a	a	A
c	a	a	e	c	D
d	a	a	d	e	C
e	a	a	c	d	E

Then (S,.) is an AG-groupoid. The interior ideals of S are {a} and {a,c,d,e}.
Define a fuzzy subset μ of S as follows:

$$\mu : S \to [0, 1], x \to \begin{cases} 0.8 & \text{if } x = a \\ 0.6 & \text{if } x = e \\ 0.5 & \text{if } x = d \\ 0.4 & \text{if } x = c \\ 0.2 & \text{if } x = b \end{cases}$$

By routine calculations we know that μ is an $(\in, \in \vee \bar{q})$-fuzzy interior ideal of S.

Theorem 2.10: A fuzzy subset μ of S is an $(\bar{\in}, \bar{\in} \vee \bar{q})$-fuzzy interior ideal of S if and only if it satisfies the following assertions:

(i) $(\forall x, y \in S)$ $(\max\{\mu(xy), 0.5\} \geq \min\{\mu(x), \mu(y)\})$

(ii) $(\forall x, y, a \in S)$ $(\max\{\mu((xa)y), 0.5\} \geq \mu(a))$

Proof: Suppose that μ is an $(\bar{\in}, \bar{\in} \vee \bar{q})$-fuzzy interior ideal of S. If there exist $x_0, y_0 \in S$ such that

$$(\max\{\mu(x_0y_0), 0.5\} < \min\{\mu(x_0), \mu(y_0)\}) = \lambda_0$$

then $0.5 < \lambda_0 \leq 1$; $[x_0; \lambda_0] \in \mu$, $[y_0; \lambda_0] \in \mu$ and $[x_0y_0; \lambda_0] \bar{\in} \mu$, By (i) of Definition 2.8, $[x_0; \lambda_0] \bar{q} \mu$ and $[y_0; \lambda_0] \bar{q} \mu$. Then

$$(\lambda_0 \leq \mu(x) \text{ and } \lambda_0 + \mu(x_0) < 1)$$

or

$$(\lambda_0 \leq \mu(y) \text{ and } \lambda_0 + \mu(y_0) < 1)$$

Thus $\lambda_0 \leq 0.5$, a contradiction. Hence (i) valid. If there exists $x_0, a_0, y_0 \in S$ such that

$$\max\{\mu((x_0a_0)y_0), 0.5\} < \mu(a_0) = \lambda_1$$

then $0.5 < \lambda_0 \leq 1$ and $[a_0; \lambda_1] \in \mu$ but $[(x_0a_0)y_0; \lambda_1] < \bar{\in} \mu$. By (ii) of Definition 2.8, we have $[a_0; \lambda_1] \bar{q} \mu$. Then $(\lambda_1 \leq \mu(a) \text{ and } \lambda_1 + \mu(a_0) < 1)$. Thus $\lambda_1 \leq 0.5$, contradiction. Hence (ii) valid.

Conversely, assume that (i) and (ii) hold. Let $x, y \in S$ and $\lambda_1, \lambda_2 \in [0,1]$ be such that $[xy; \min\{\lambda_1, \lambda_2\}] \bar{\in} \mu$, then $\mu(xy) < \min\{\lambda_1, \lambda_2\}$. If

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}$$

then

$$\min\{\mu(x), \mu(y) < \min\{\lambda_1, \lambda_2\}\}.$$

Thus $(\mu(x) < \lambda_1 \text{ or } \mu(y) < \lambda_2)$ i.e., $[x; \lambda_1] \bar{\in} \mu$, or $[y; \lambda_2] \bar{\in} \mu$, thus $[x; \lambda_1] \bar{\in} \vee \bar{q} \mu$, or $[y; \lambda_2] \bar{\in} \vee \bar{q} \mu$. If $\mu(xy) \leq \min\{\mu(x), \mu(y)\}$, then $\min\{\mu(x), \mu(y) \leq 0.5\}$ by (i).

Now, assume that $[x; \lambda_1] \in \mu$, and $[y; \lambda_2] \in \mu$ then $(\lambda_1 + \mu(x) \leq 2\mu(x) \leq 1 \text{ or } \lambda_2 + \mu(y) \leq 2\mu(y) \leq 1)$, that is, $[x; \lambda_1] \bar{q} \mu$ or $[y; \lambda_2] \bar{q} \mu$. Therefore $[x; \lambda_1] \bar{\in} \vee \bar{q} \mu$, or $[y; \lambda_2] \bar{\in} \vee \bar{q} \mu$. Let $x, a, y \in S$ and $\lambda \in [0,1]$ be such that $[(xa)y; \lambda] \bar{\in} \mu$, Then $\mu((xa)y) < \lambda$. If $\mu((xa)y) \geq \mu(a)$ then $\mu(a) < \lambda$ i.e., $[a; \lambda] \bar{\in} \mu$, and hence, $[a; \lambda] \bar{\in} \vee \bar{q} \mu$. If $\mu((xa)y) < \mu(a)$ then $\mu(a) \leq 0.5$ by (ii).

Let $[a; \lambda] \in \mu$, then $\lambda + \mu(a) \leq 2\mu(a) \leq 1$ i.e., $[a; \lambda] \bar{q} \mu$. Therefore $[a; \lambda] \bar{\in} \vee \bar{q} \mu$. Consequently, μ is an $(\bar{\in}, \bar{\in} \vee \bar{q})$-fuzzy interior ideal of S.

Remark 2.11: A fuzzy subset μ of an AG-groupoid S is an $(\bar{\in}, \bar{\in} \vee \bar{q})$-fuzzy interior ideal of S if and only if it satisfies conditions (i) and (ii) of Theorem 2.10.

INTERIOR IDEALISTIC SOFT AG-GROUPOIDS WITH FUZZY POINTS

Molodtsov [14] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let ξ(U) denote the power set of U and $A \subset U$.

A pair (ϑ, A) is called a soft set over U, where ϑ is a mapping given by $\vartheta : A \to \xi(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A, \vartheta(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F,A).

Definition 3.1: Let (ϑ, A) be a soft set over S. Then (ϑ, A) is called a soft interior ideal over S if $\vartheta(x)$ is an interior ideal of S for all $x \in A$.

Example 3.2: Let S = {a,b,c,d,e} with the following multiplication table:

.	a	b	c	D	e
a	a	a	a	A	a
b	a	a	a	A	a
c	a	a	e	C	d
d	a	a	d	E	c
e	a	a	c	D	e

Then (S,.) is an AG-groupoid. The interior ideals of S are {a} and {a,c,d,e}. Let (ϑ, A) be a soft set over S, where A = (0,1] and $\vartheta: A \to \xi(U)$ is a set-valued function defined by

$$\vartheta(x) := \begin{cases} S \text{ if } 0 < \lambda \le 0.1 \\ \{a,c,d,e\} \text{ if } 0.1 < \lambda \le 0.2 \\ \{a\} \text{ if } 0.6 < \lambda \le 1 \\ \phi \text{ if } 0.8 < \lambda \le 1 \end{cases}$$

Thus $\vartheta(x)$ is an interior ideal of S for all $x \in A$ and so (ϑ, A) is soft interior ideal over S.

Given a fuzzy subset μ of an AG-groupoid S and $A \subseteq [0,1]$ consider two set-valued functions

$$\vartheta: A \to \xi(S),\ \lambda \to \{x \in S \mid [x;t] \in \mu\} = \{x \in S \mid \mu(x) \ge 0\}$$

$$\vartheta_q: A \to \xi(S),\ \lambda \to \{x \in S \mid [x;t] q\mu\} = \{x \in S \mid \mu(x) + \lambda > 1\}$$

Then (ϑ, A) and (ϑ_q, A) are called ε-soft set and q-soft set over S, respectively.

Theorem 3.3: Let μ be a fuzzy subset of S and (ϑ, A) an ε-soft set over S with A = (0,1]. Then (ϑ, A) is a soft interior ideal over S if and only if μ is a fuzzy interior ideal of S.

Proof: Suppose that μ is a fuzzy interior ideal of S and $t \in A$. Let $x, y \in \vartheta(\lambda)$.

Then $[x,\lambda] \in \mu$ and $[y,\lambda] \in \mu$ and $[xy; \min\{\lambda,\lambda\}] = [xy;\lambda] \in \mu$ i.e., $xy \in \vartheta(\lambda)$.

Let $a \in \vartheta(\lambda)$. Then $[a,\lambda] \in \mu$ and hence $[(xa)y,\lambda] \in \mu$. Thus $(xa)y \in \vartheta(\lambda)$.

Hence (ϑ, A) is a soft interior ideal over S.

Conversely, assume that (ϑ, A) is a soft interior ideal over S. If there exist $a,b \in S$ such that

$$\mu(ab) < \min\{\mu(a), \mu(b)\} = \lambda_0$$

then $\lambda_0 \in A$ and $[a,\lambda_0] \in \mu$, $[b,\lambda_0] \in \mu$, that is, $a,b \in \vartheta(\lambda_0)$ and so $ab \in \vartheta(\lambda_0)$. Thus $[ab,\lambda_0] \in \mu$, a contradiction.

Hence $\mu(xy) \ge \min\{\mu(x), \mu(y)\}$ for all $x,y \in S$. If there exist $x_0, a, y_0 \in S$ such that $\min\{\mu(a), \mu(b)\} < \mu(ab) = \lambda_1$ then $[ab,\lambda] \in \mu$ i.e., $ab \in \vartheta(\lambda_1)$. It follows that $a,b \in \vartheta(\lambda_1)$. This is a contradiction. Thus $\min\{\mu(x), \mu(y)\} \ge \mu(xy)$ for all $x,y \in S$. Therefore μ is a fuzzy interior ideal of S.

Theorem 3.4: Let μ be a fuzzy subset of an AG-groupoid S and (ϑ_q, A) a q-soft set over S with A = (0,1]. Then the following are equivalent:

(i) μ is a fuzzy interior ideal of S,
(ii) $(\forall \lambda \in A)$ $(\vartheta_q(\lambda)) \ne \phi \Rightarrow \vartheta_q(\lambda)$ is an interior ideal of S.

Proof: (i)$\Rightarrow$(ii). Assume that μ is a fuzzy interior ideal of S and let $\lambda \in A$ such that $\vartheta_q(\lambda) \ne \phi$. Let $x,y \in S$ be such that $x \in \vartheta_q(\lambda)$ and $y \in \vartheta_q(\lambda)$. Then $[x;\lambda] q\mu$ and $[y;\lambda] q\mu$ or equivalently, $(\mu(x) + \lambda > 1 \text{ and } \mu(y) + \lambda > 1)$. Thus

$$\mu(xy) + \lambda \ge \min\{\mu(x), \mu(y)\} + \lambda$$
$$= \min\{\mu(x) + \lambda, \mu(y) + \lambda\} > 1$$

and so $[xy;\lambda] q\mu$ i.e., $xy \in \vartheta_q(\lambda)$. Let $x,a,y \in S$ be such that $a \in \vartheta_q(\lambda)$.
Then $[a;\lambda] q\mu$ or equivalently, $\mu(a) + \lambda! > 1$. Thus

$$\mu((xa)y) + \lambda \ge \mu(a) + \lambda > 1$$

and so $[(xa)y;\lambda] q\mu$ i.e., $(xa)y \in \vartheta_q(\lambda)$. Thus $\vartheta_q(\lambda)$ is an interior ideal of S.

(ii) $\Rightarrow$ (i). Assume that for all $\lambda \in A$, the set $\vartheta_q(\lambda) \neq \phi$, is an interior ideal of S. If there exist $a, b \in S$ such that $\mu(ab) < \min\{\mu(a), \mu(b)\}$. Then

$$\mu(ab) + \lambda \leq 1 < \min\{\mu(a), \mu(b)\} + \lambda$$
$$= \min\{\mu(a) + \lambda, \mu(b) + \lambda\}$$

for $\lambda \in A$. Thus $\mu(a) + \lambda > 1$ and $\mu(b) + \lambda > 1$. It follows that $[a;\lambda]q\mu$ and $[b;\lambda]q\mu$. Hence, $a, b \in \vartheta_q(\lambda)$ and we have $ab \in \vartheta_q(\lambda)$. Then, $[ab;\lambda]q\mu$ i.e., $\mu(ab) + \lambda > 1$ a contradiction. Hence, $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in S$.

If there exist, $x_0, a_0, y_0 \in S$ such that, $\mu((x_0 a_0) y_0) < \mu(a_0)$. Then

$$\mu((x_0 a_0) y_0) + \lambda \leq 1 < \mu(a_0) + \lambda$$

for $\lambda \in A$. Thus $\mu(a_0) + \lambda > 1$. It follows that $[a_0;\lambda]q\mu$. Hence $a_0 \in \vartheta_q(\lambda)$ and we have $(x_0 a_0) y_0 \in \vartheta_q(\lambda)$. Then, $[(x_0 a_0) y_0;\lambda]q\mu$ i.e., $\mu((x_0 a_0) y_0) + \lambda > 1$, a contradiction. Hence, $\mu((xa)y) \geq \mu(0)$ for all $x, a, y \in S$. Thus, μ is a fuzzy interior ideal of S.

Theorem 3.5: Let μ be a fuzzy subset of S and (ϑ, A) an $\in$-soft set over S with A = (0, 0.5]. Then the following are equivalent:

(i) μ is an $(\in, \in \vee q)$-fuzzy interior ideal of S,
(ii) (ϑ, A) is a soft interior ideal over S.

Proof: (i)$\Rightarrow$(ii). Suppose that μ is an $(\in, \in \vee q)$-fuzzy interior ideal of S and let $\lambda \in A$. If $x, y \in \vartheta(\lambda)$ then $[x,\lambda] \in \mu$ and $[y,\lambda] \in \mu$ that is, $\mu(x) \geq \lambda$ and $\mu(y) \geq \lambda$. By Proposition 2.7 (i), we have

$$\mu(xy) \geq \min\{\mu(x), \mu(y), 0.5\} \geq \min\{\lambda, 0.5\} = \lambda$$

so $[xy;\lambda] \in \mu$ implies that $xy \in \vartheta(\lambda)$. If $a \in \vartheta(\lambda)$ then, $[a,\lambda] \in \mu$ that is, $\mu(a) \geq \lambda$ By Proposition 2.7 (ii), we have

$$\mu((xa)y) \geq \min\{\mu(a), 0.5\} \geq \min\{\lambda, 0.5\} = \lambda$$

so $[(xa)y;\lambda] \in \mu$ implies that $(xa)y \in \vartheta(\lambda)$. Hence, (ϑ, A) is a soft interior ideal over S. (ii)$\Rightarrow$(i). Assume that (ϑ, A) is a soft interior ideal over S. If there exist, $a, b \in S$ such that $\mu(ab) < \min\{\mu(a), \mu(b), 0.5\}$. Taking $\lambda = \frac{1}{2}(\mu(ab) + \min\{(a), \mu(b), 0.5\})$ we have $\lambda \in A$ and $\mu(ab) < \lambda < \min\{\mu(a), \mu(b), 0.5\}$, which implies $[a;\lambda] \in \mu$ and $[b;\lambda] \in \mu$ and $[ab;\lambda] \overline{\in} \mu$ i.e., $ab \in \vartheta(\lambda)$ but $ab \notin \vartheta(\lambda)$ a contradiction. Hence $\mu(xy) \geq \min\{\mu(x), \mu(y), 0.5\}$ for all $x, y \in S$. If there exist, $a, b, c \in S$ such that

$$\mu((ab)c) < \min\{\mu(b), 0.5\}$$

Taking

$$\lambda = \frac{1}{2}(\mu((ab)c + \min\{\mu(b), 0.5\})$$

we have $\lambda \in A$ and $\mu((ab)c) < \lambda < \min\{\mu(b), 0.5\}$, which implies $[b;\lambda] \in \mu$ and hence, $[(ab)c;\lambda] \overline{\in} \mu$ i.e., $b \in \vartheta(\lambda)$ and $(ab)c \notin \vartheta(\lambda)$ a contradiction. Hence $\mu((xa)y) \geq \min\{\mu(a), 0.5\}$ for all $x, a, y \in S$.
Therefore, μ is a fuzzy interior ideal of S.

Theorem 3.7: Let μ be a fuzzy subset of S and (ϑ, A) an $\in$-soft set over S with A = (0.5,1]. Then the following are equivalent:

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy interior ideal of S.
(ii) (ϑ, A) is a soft interior ideal over S.

Proof: (i) $\Rightarrow$ (ii). Suppose that μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy interior ideal of S. For any $\lambda \in A$, let $x, y \in S$ be such that $x, y \in \vartheta(\lambda)$, then $[x;\lambda] \in \mu$ and $[y;\lambda] \in \mu$ that is, $\mu(x) \geq \lambda$ and $\mu(y) \geq \lambda$. It follows from Theorem 2.10 (i), that

$$\lambda \leq \min\{\mu(x), \mu(y)\} \leq \max\{\mu(xy), 0.5\} = \mu(xy)$$

It follows that, $[x;\lambda] \in \mu$ i.e $xy \in \vartheta(\lambda)$. Let $x, a, y \in S$ be such that $a \in \vartheta(\lambda)$ then $[a;\lambda] \in \mu$, that is, $\mu(a) \geq \lambda$. It follows from Theorem 2.10 (ii), that

$$\lambda \leq \mu(a) \leq \max\{\mu((xa)y), 0.5\} = \mu((xa)y)$$

which implies that $[(xa)y;\lambda] \in \mu$ i.e., $(xa)y \in \vartheta(\lambda)$. Therefore, (ϑ, A) is a soft interior over S.
(ii) $\Rightarrow$ (i). Assume that (ϑ, A) is a soft interior ideal over S. If there exist, $a, b \in S$ such that

$$\min\{\mu(a),\mu(b)\} \geq \lambda > \max\{\mu(ab),0.5\}$$

for some $\lambda \in A$.

Thus, $[a;\lambda] \in \mu$ and $[b;\lambda] \in \mu$ but $[ab;\lambda] \overline{\in} \mu$ that is, $a,b \in \vartheta(\lambda)$ but $ab \notin \vartheta(\lambda)$ a contradition. Hence, $\max\{\mu(xy),0.5\} \geq \min\{\mu(x),\mu(y)\}$ for all $x,y \in S$. If there exist $a,b,c \in S$ such that $\mu((ab)c) \geq \lambda > \min\{\mu(b),0.5\}$ for some $\lambda \in A$.

Thus, $[(ab)c;\lambda] \in \mu$ but $[b;\lambda] \overline{\in} \mu$, that is, $b \in \vartheta(\lambda)$ and so $(ab)c \notin \vartheta(\lambda)$ a contradition. Hence, $\max\{\mu((xa)y),0.5\} \geq \mu(a)$ for all $x,a,y \in S$. Thus, μ is an $(\overline{\in},\overline{\in} \vee \overline{q})$ -fuzzy interior ideal of S.

Definition 3.17: Let S be an AG-groupoid and let ε, $\beta \in [0,1]$ with $\alpha < \beta$, let μ be a fuzzy subset of S. Then μ is called a fuzzy interior ideal with thresholds (α, β) of S if it satisfies the following conditions:

(i) $(\forall x,y \in S)\ (\max\{\mu(xy),\alpha\} \geq \min\{\mu(x),\mu(y),\beta\})$

(ii) $(\forall x,y,a \in S)\ (\max\{\mu((xa)y),\alpha\} \geq \min\{\mu(a),\beta\})$

Theorem 3.18: A fuzzy subset μ of an AG-groupoid S is an interior ideal with thresholds of S if and only if $U(\mu,\lambda)(\neq \phi)$ is an interior ideal of S, for all $\lambda \in [\alpha, \beta]$.

Proof: Let μ be a fuzzy interior ideal with thresholds (α, β) of S. Let $x,y \in U(\mu,\lambda)$, then $\mu(x) \geq \lambda$ and $\mu(y) \geq \lambda$ and from (i) of Definition 3.17, it follows that

$$\max\{\mu(xy),\alpha\} \geq \min\{\mu(x),\mu(y),\beta\} \geq \min\{\lambda,\beta\} = \lambda > \alpha$$

so $\mu(xy) \geq \lambda$ i.e., $xy \in U(\mu;\lambda)$. Let $x,a,y \in S$ be such that $a \in U(\mu;\lambda)$ then $\mu(a) \geq \lambda$ and by (ii) of Definition 3.17, it follows that

$$\max\{\mu((xa)y),\alpha\} \geq \min\{\mu(a),\beta\} \geq \min\{\lambda,\beta\} = \lambda > \alpha$$

and so $(xa)y \in U(\mu;\lambda)$

Conversely, let μ be a fuzzy subset of S such that $U(\mu;\lambda)(\neq \phi)$ is an interior ideal of S. If there exist $x_0, y_0 \in S$ such that

$$\max\{\mu(x_0y_0),\alpha\} < \min\{\mu(x_0),\mu(y_0),\beta\} = \lambda_0$$

$\alpha < \lambda_0 \leq \beta$, $x_0, y_0 \in U(\mu;\lambda_0)$ and $\mu(x_0y_0) < \lambda_0$. Since $U(\mu;\lambda_0)$ is an interior ideal so we have, $x_0y_0 \in U(\mu;\lambda_0)$, then $\mu(x_0y_0) \geq \lambda_0$, a contradiction. Hence,

$$\max\{\mu(xy),\alpha\} \geq \min\{\mu(x),\mu(y),\beta\},\ \text{for all}\ x,y \in S$$

If there exist $x_0, a_0, y_0 \in S$ such that

$$\max\{\mu((x_0a_0)y_0),\alpha\} < \min\{\mu(a_0),\beta\} = \lambda_1,$$

then $\alpha < \lambda_1 \leq \beta$, $\mu((x_0a_0)y_0) < \lambda_1$ and $a_0 \in U(\mu;\lambda_1)$. Then $(x_0a_0)y_0 \in U(\mu;\lambda_1)$ and hence $\mu((x_0a_0)y_0) \geq \lambda_1$, a contradiction. Hence,

$$\max\{\mu((xa)y),\alpha\} \geq \min\{\mu(a),\beta\},\ \text{for all}\ x,a,y \in S$$

Therefore, μ is an (α, β)-fuzzy interior ideal with thresholds (α, β) of S.

Theorem 3.19: Given $\alpha,\beta \in [0,1]$ and $\alpha < \beta$ let μ be a fuzzy subset of S and (ϑ, A) an $\in$-soft set over S with $A = (\alpha,\beta]$. Then the following are equivalent:

(i) μ is a fuzzy interior ideal with thresholds $(\alpha, \beta]$ of S
(ii) (ϑ, A) is a soft interior ideal over S.

Proof: (i) $\Longrightarrow$ (ii). Suppose that μ is a fuzzy interior ideal with thresholds $(\alpha, \beta]$ of S and let $\lambda \in A$. If $x,y \in \vartheta(\lambda)$ for any $\lambda \in (\alpha,\beta]$, then $[x;\lambda] \in \mu$ and $[y;\lambda] \in \mu$ and so $\mu(x) \geq \lambda$ and $\mu(y) \geq \lambda$. By Definition 3.17 (i), we have

$$\max\{\mu(xy),\alpha\} \geq \min\{\mu(x),\mu(y),\beta\} \geq \min\{\lambda,\beta\} = \lambda > \alpha$$

and so $\mu(xy) \geq \lambda$, that is, $[xy;\lambda] \in \mu$. It follows that $xy \in \vartheta(\lambda)$. If a $a \in \vartheta(\lambda)$ for any $\lambda \in (\alpha,\beta]$, then $[a;\lambda] \in \mu$ and so $\mu(a) \geq \lambda$. By Definition 3.17 (ii), we have

$$\max\{\mu((xa)y),\alpha\} \geq \min\{\mu(a),\beta\} \geq \min\{\lambda,\beta\} = \lambda > \alpha$$

and so $\mu((xa)y) \geq \lambda$, that is $[(xa)y;\lambda] \in \mu$. It follows that $xy \in \vartheta(\lambda)$. Thus (ϑ, A) is a soft interior ideal soft over S.

(ii) $\Longrightarrow$ (i). Assume that (ϑ, A) is a soft interior ideal soft over S. If there exist $a,b \in S$ such that

$$\max\{\mu(ab),\alpha\} < \min\{\mu(a),\mu(b),\beta\}$$

then

$$\max\{\mu(ab)\} < \lambda \leq \min\{\mu(a),\mu(b),\beta\}$$

for $\lambda \in (\alpha,\beta]$ and so $[a;\lambda],[b;\lambda] \in \mu$ implies that $[ab;\lambda]\overline{\in}\mu$. Since $a,b \in \vartheta(\lambda)$, we have $ab \in \vartheta(\lambda)$, then $[ab;\lambda]\overline{\in}\mu$, which is a contradiction. Hence,

$$\max\{\mu(xy),\alpha\} \geq \min\{\mu(x),\mu(y),\beta\}$$

for all $x,y \in S$.

Finally, if there exist $a,b,c \in S$ such that

$$\max\{\mu((ab)c),\alpha\} < \min\{\mu(b),\beta\}.$$

Then

$$\max\{\mu((ab)c);\alpha\} < \lambda_0 \leq \min\{\mu(b),\beta\}$$

for $\lambda_0 \in (\alpha,\beta]$ and so we have, $[b;\lambda_0] \in \mu$ and $[(ab)c;\lambda]\overline{\in q}\mu$. Since, $b \in \vartheta(\lambda)$, we have $(ab)c \in \vartheta(\lambda)$, then $[(ab)c;\lambda]\overline{\in q}\mu$, a contradiction. Hence

$$\max\{\mu((xa)y),\alpha\} \geq \min\{\mu(a),\beta\}$$

for all $x,a,y \in S$.

CONCLUDING REMARKS

In this paper, the concepts of $\in$-soft sets and q-soft sets in AG-groupoids are introduced and some of their related properties are investigated. In our future work, we will try to define fuzzy soft left (right) ideals, fuzzy soft bi-ideals etc over an AG-groupoid and investigate the structural properties of the so called AG-groupoids.

Hopefully, our research in this direction will continue and will make a plate form for other algebraic structures.

ACKNOWLEDGEMENT

We would like to thank the referees for their valuable comments and suggestions for the improvement of this paper.

REFERENCES

1. Akram, M. and W.A. Dudek, 2009. Interval-valued Intuitionistic fuzzy Lie ideals of Lie algebras. World Applied Sciences Journal, 7 (7): 812-819.
2. Akram, M., K.H. Dar, B.L. Meng, Y.L. Liu, 2009. Redefined Fuzzy K-algebras. World Applied Sciences Journal, 7 (7): 805-811.
3. Aktas, H. and N. Cagman, 2007. Soft sets and soft groups. Inform. Sci., 177: 2726-2735.
4. Bhakat, S.K. and P. Das, 1996. ($\in$, $\in \vee q$)-fuzzy subgroup. Fuzzy Sets and Systems, 80: 359-368.
5. Davvaz, B. and A. Khan, 2011. Characterizations of regular ordered semigroups in terms of (α, β)-fuzzy generalized bi-ideals. Inform. Sci., 181 (9): 1759-1770.
6. Chen, D., E.C.C. Tsang, D.S. Yeung and X. Wang, 2005. The parameterization reduction of soft sets and its applications. Comput. Math. Appl., 49: 757-763.
7. Chang, C.C., 1958. Algebraic analysis of many valued logics. Trans. Amer. Math. Soc., 88: 467-490.
8. Jun, Y.B., 2008. Soft BCK/BCI-algebras. Comput. Math. Appl., 56: 1408-1413.
9. Jun, Y.B. and C.H. Park, 2008. Applications of soft sets in ideal theory of BCK/BCI-algebras. Inform. Sci., 178: 2466-2475.
10. Kazim, M.A. and M. Naseerudin, 1972. On almost semigroups. The Alig. Bulletin of Mathematics, 2: 1-7.
11. Khan, A. and M. Shabir, 2009. (α, β)-fuzzy interior ideals in ordered semigroups. Lobachevskii Journal of Mathematics, 30 (1): 30-39.
12. Khan, A., Y.B. Jun and T. Mahmood, 2010. Genealized fuzzy interior ideals in Abel Grassmann's groupoids. Inter. J. Math. Math. Sci., pp: 14, Article ID 838392, doi:10.1155/2010/838 392.
13. Khan, M. and M. Nouman Aslam Khan, Fuzzy Abel Grassmann's groupoids, (to appear) Journal Advances in Fuzzy Mathematics (AFM).
14. Molodtsov, D., 1999. Soft set theory First results. Comput Math Applic, 37: 19-31.
15. Maji, P.K., A.R. Roy and R. Biswas, 2002. An application of soft sets in a decision making problem. Comput. Math. Appl., 44: 1077-1083.
16. Maji, P.K., R. Biswas and A.R. Roy, 2003. Soft set theory. Comput. Math. Appl., 45: 555-562.
17. Mushtaq, Q. and S.M. Yousuf, 1978. On LA-semigroups. The Alig. Bull. Math., 8: 65-70.
18. Naseeruddin, M., 1970. Some studies in almost semigroups and flocks. Ph.D., Thesis, Alig. Muslim University, Alig., India.
19. Protic, P.V. and N. Stevanovic, 1995. AG-test and some general properties of Abel-Grassmannís groupoids. PU. M. A., 4 (6): 371-383.

20. Pu, P.M. and Y.M. Liu, 1980. Fuzzy topology I, Neighborhood structure of a fuzzy point and Moore Smith convergence. J. Math. Anal. Appl., 76: 571-599.
21. Shabir, M. and A. Khan, 2008. Fuzzy filters in ordered semigroups. Lobachevskii Journal of Mathematics, 29 (2): 82-89.
22. Saeid, A.B., 2009. Bipolar-valued fuzzy BCK/BCI-algebras. World Applied Sciences Journal, 7 (11): 1404-1411.
23. Saeid, A.B., A. Namdar and R.A. Borzooei, 2009. Ideal theory of BCH-algebras. World Applied Sciences Journal, 7 (11): 1446-1455.
24. Zadeh, L.A., 1965. Fuzzy sets. Inform. Control, 8: 338-353.
25. Zhan, J. and Y.B. Jun, 2009. Soft BL-algebras based on fuzzy sets. Comput. Math. Applic., doi:10.1016/j.camwa.2009.12.008.