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Prediction of Radial-Ply Tire Deflection Based on Section Width, Overall Unloaded Diameter, Inflation Pressure and Vertical Load

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Abstract: Tire deflection is a key parameter and many equations have been developed based on it to evaluate the tractive performance of bias-ply and radial-ply tires. As deflections for a given tire size, inflation pressure and vertical load are significantly different between bias-ply and radial-ply tires, this study was conducted to predict deflection (δ) of radial-ply tire based on section width (b), overall unloaded diameter (d), inflation pressure (P) and vertical load (W). For this purpose, deflection of four radial-ply tires with different section width and/or overall unloaded diameter were measured at five levels of inflation pressure and five levels of vertical load. Results of deflection measurement for radial-ply tires No. 1, 2 and 3 were utilized to determine multiple variables regression model and results of deflection measurement for radial-ply tire No. 4 were used to verify selected model. The paired samples t-test results showed that the difference between the deflection values predicted by model and measured by test apparatus were not statistically significant and to predict deflection of radial-ply tire based on section width, overall unloaded diameter, inflation pressure and vertical load the multiple variables regression model $\delta = 75.67 + 0.104\ b - 0.107\ d - 0.758\ P + 3.519\ W$ with $R^2 = 0.986$ can be strongly recommended.

Key words: Radial-ply tire · Deflection · Section width · Overall unloaded diameter · Inflation pressure · Vertical load · Prediction

INTRODUCTION

In the case of tracked vehicles, the contact area between machine and ground surface is relatively constant for varying sinkage in the soil and is calculated as the length of track on hard ground times track width. However, a flexible tire has a smaller contact area on hard surface than it dose on soft ground. A rule of thumb which can be used for estimation of tire contact area is shown by equation 1 [1]:

$$A = bL \tag{1}$$

where:
A = Contact area (m^2)
b = Section width (m)
L = Contact length (m)

Wong [2] and Bekker [3] gave an approximate method for calculating contact length as equation 2:

$$L = 2(d\delta - \delta^2)^{0.5} \tag{2}$$

where:
d = Overall unloaded diameter (m)
δ = Deflection (m)

Deflection is a key parameter and many equations have been developed based on it to evaluate the tractive performance of bias-ply and radial-ply tires operating in cohesive-frictional soils. Gross traction, motion resistance, net traction and tractive efficiency are predicted as a function of soil strength, tire load, tire slip, tire size and tire deflection [4]. The most widely used dimensional analysis approach for predicting off-road traction makes use of the following ratios [4-6]:

$$C_n = \frac{CI \cdot b \cdot d}{W} \tag{3}$$

$$WD = \frac{b}{d} \tag{4}$$

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$$DR = \frac{\delta}{h} \quad (5)$$

where:

C_n = Wheel numeric (dimensionless)
CI = Cone index (kPa or kNm^{-2})
W = Vertical load (kN)
WD = Section width to overall unloaded diameter ratio (dimensionless)
DR = Deflection ratio (dimensionless)
h = Section height (m)

Fig. 1 shows the tire dimensions (b, d, δ and h) used. The tire dimensions can be obtained from tire data book or by measuring the tire [4]. The section width (b) is the first number in a tire size designation (i.e., nominally 18.4 inches for an 18.4-38 tire). The overall unloaded diameter (d) can be obtained from the tire data handbooks available from off-road tire manufacturers. The tire deflection (δ) on a hard surface is equal to d/2 minus the measured static loaded radius. The static loaded radius for the tire's rated load and inflation pressure is also standard tire data from the tire data handbooks. It can also be obtained by measuring the tire. The section height (h) is equal to half the difference between the overall unloaded diameter and the rim diameter. The rim diameter can in turn be estimated by adding 50 mm to the nominal rim diameter, which is the second number in a tire size designation, i.e. 38 inches for an 18.4-38 tire [4, 5].

To further simplify the prediction equations, Brixius [4] combined above three dimensionless ratios into a single product termed the mobility number, which is given by equation 6 [5-7]:

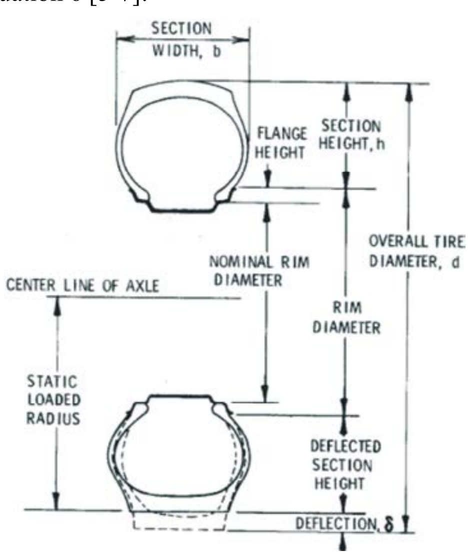

Fig. 1: Tire dimensions, adapted from Brixius [4]

$$B_n = \frac{CI \cdot b \cdot d}{W}\left(\frac{1+5\frac{\delta}{h}}{1+3\frac{b}{d}}\right) \quad (6)$$

where:

B_n = Mobility number (dimensionless)

The empirical model developed by Brixius [4] is widely used for prediction of off-road tire performance. It has also been adopted in ASAE standard D497.4 [8] for predicting tractor performance. In this model, soil condition is represented by the cone index value, which is the average force per unit area required to force a cone-shaped probe vertically into the soil at a steady rate. The average before-traffic cone index for the top 150 mm layer of soil is used in the prediction equations that follow [5, 7]. ASAE standards S313.3 [9] and EP542 [10] describe the soil cone penetrometer and procedures for its use. An average of several cone index values obtained at a test site often yields a representative measure of soil strength [11].

In addition, the coefficient of gross traction is dependent on the mobility number and the tire slip and is given by equation 7 [4-7]:

$$\mu_{GT} = 0.88(1-e^{-0.1B_n})(1-e^{-7.5S}) + 0.04 \quad (7)$$

where:

μ_{GT} = Coefficient of gross traction (dimensionless)
S = Tire slip (decimal).
e = Napier's constant (the numerical value of e truncated to 30 decimal places is 2.71828 18284 59045 23536 02874 71353).

Additionally, the coefficient of motion resistance depends on the mobility number and the tire slip and is given by equation 8 [4-7]:

$$\rho = \frac{1}{B_n} + 0.04 + \frac{0.5S}{\sqrt{B_n}} \quad (8)$$

where:

ρ = Coefficient of motion resistance (dimensionless)

By combining equations 7 and 8, we obtain equation 9 for the coefficient of net traction [4-7]:

$$\mu_{NT} = 0.88(1-e^{-0.1B_n})(1-e^{-7.5S}) - (\frac{1}{B_n} + \frac{0.5S}{\sqrt{B_n}}) \quad (9)$$

where:
μ_{NT} = Coefficient of net traction (dimensionless)

The empirical models of Brixius [4] were originally developed for pneumatic bias-ply tires with deflection ratio (δ/h) ranging from 0.1 to 0.3. Although extrapolation of empirical equations may be not valid, the use of Brixius models with a rigid wheel is an extrapolation of these equations, where δ/h = 0 [7]. Also, as radial-ply tire usage has expanded, the need for radial-ply tire prediction equations has increased. Radial-ply tire equations should be similar to the equations for bias-ply tires, with appropriate adjustment in several of the equation constants. The constant 7.5 in equations 7 and 9 should be increased to 8.5-10.5. This term accounts for the significant improvement in gripping the soil. The constant 0.04 should be reduced to 0.030-0.035, which accounts for the reduced motion resistance on a hard surface. The $0.1B_n$ term, combined with 0.88 controls the maximum torque that can be applied at high tire slip. This value seems to be the same for bias-ply and radial-ply tires. Further analysis may show a slightly higher value is needed, but no lower. The $1/B_n$ term in equations 8 and 9 should be changed to $0.9/B_n$. This reflects lower motion resistance due to less soil compaction and tire sinkage for radial-ply tires. Further investigation is needed regarding the effect of tire deflection. The constant controlling deflection in equation 6 possibly needs revision [4, 5]. Therefore, the following modified forms of the above three equations are suggested for radial-ply tires [5, 6]:

$$\mu_{GT} = 0.88(1-e^{-0.1B_n})(1-e^{-9.5S}) + 0.0325 \quad (10)$$

$$\rho = \frac{0.9}{B_n} + 0.0325 + \frac{0.5S}{\sqrt{B_n}} \quad (11)$$

$$\mu_{NT} = 0.88(1-e^{-0.1B_n})(1-e^{-9.5S}) - (\frac{0.9}{B_n} + \frac{0.5S}{\sqrt{B_n}}) \quad (12)$$

As deflections for a given tire size, inflation pressure and vertical load are significantly different between bias-ply and radial-ply tires [4], this study was conducted to predict deflection (δ) of radial-ply tire based on section width (b), overall unloaded diameter (d), inflation pressure (P) and vertical load (W).

MATERIALS AND METHODS

Tire Deflection Test Apparatus: A tire deflection test apparatus (Fig. 2) was designed and constructed to measure deflection of tires with different sizes at diverse

Fig. 2: Tire deflection test apparatus

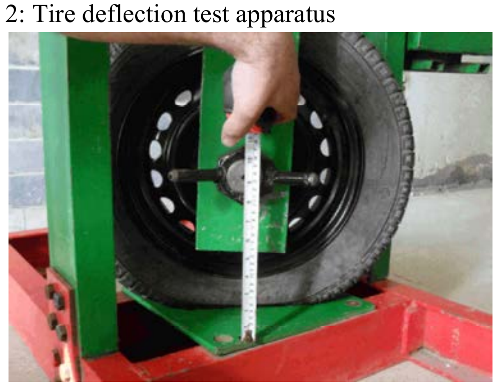

Fig. 3: Measuring static loaded radius

levels of inflation pressure and vertical load. As deflection on a hard surface is equal to d/2 minus the measured static loaded radius [4, 5], the static loaded radius was obtained by measuring as shown in Fig. 3.

Experimental Procedure: For this purpose, deflection of four radial-ply tires with different dimensions were measured at five levels of inflation pressure and five levels of vertical load. The dimensions of four radial-ply tires are given in Table 1. Results of deflection measurement for radial-ply tires No. 1, 2 and 3 (Tables 2, 3 and 4) were utilized to determine multiple variables regression models and results of deflection measurement for radial-ply tire No. 4 (Table 5) were used to verify selected model.

Regression Model: A typical multiple variables regression model is shown in equation 13:

$$Y = C_0 + C_1X_1 + C_2X_2 + \ldots + C_nX_n \quad (13)$$

Table 1: Dimensions of the four radial-ply tires used in this study

Tire No.	Tire size designation	Section width b (mm)	Overall unloaded diameter d (mm)
1	R13-165/65	165	535
2	R14-185/65	185	580
3	R15-185/65	185	610
4	R16-216/60	216	650

Table 2: Section width, overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 1

Tire No.	Section width b (mm)	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
1	165	535	30	5.8690	31.0
				7.8250	39.0
				9.7810	47.5
				11.738	55.0
				13.694	62.0
			32	5.8690	28.5
				7.8250	38.0
				9.7810	47.0
				11.738	53.0
				13.694	60.0
			34	5.8690	29.0
				7.8250	36.5
				9.7810	44.5
				11.738	51.5
				13.694	58.0
			36	5.8690	27.5
				7.8250	36.0
				9.7810	43.0
				11.738	49.0
				13.694	55.0
			38	5.8690	26.5
				7.8250	35.0
				9.7810	42.5
				11.738	49.0
				13.694	55.0

Table 3: Section width, overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 2

Tire No.	Section width b (mm)	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
2	185	580	30	5.8690	29.5
				7.8250	38.0
				9.7810	44.5
				11.738	50.5
				13.694	58.0
			32	5.8690	28.5
				7.8250	35.5
				9.7810	43.0
				11.738	48.0
				13.694	55.0
			34	5.8690	28.0
				7.8250	35.0
				9.7810	41.5
				11.738	47.5
				13.694	54.0
			36	5.8690	26.5
				7.8250	33.0
				9.7810	44.5
				11.738	46.0
				13.694	51.5
			38	5.8690	26.0
				7.8250	31.5
				9.7810	40.5
				11.738	43.5
				13.694	50.5

Table 4: Section width, overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 3

Tire No.	Section width b (mm)	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
3	185	610	30	5.8690	26.0
				7.8250	35.0
				9.7810	42.0
				11.738	48.0
				13.694	54.5
			32	5.8690	28.0
				7.8250	35.0
				9.7810	40.5
				11.738	47.5
				13.694	53.5
			34	5.8690	22.5
				7.8250	31.5
				9.7810	37.0
				11.738	45.0
				13.694	52.0
			36	5.8690	22.0
				7.8250	30.5
				9.7810	36.0
				11.738	42.5
				13.694	49.5
			38	5.8690	21.0
				7.8250	26.5
				9.7810	34.5
				11.738	41.5
				13.694	47.5

Table 5: Section width, overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 4

Tire No.	Section width b (mm)	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Deflection δ (mm)
4	216	650	30	5.8690	26.0
				7.8250	33.5
				9.7810	40.0
				11.738	46.0
				13.694	52.0
			32	5.8690	25.0
				7.8250	32.5
				9.7810	38.0
				11.738	44.0
				13.694	50.5
			34	5.8690	24.0
				7.8250	31.5
				9.7810	37.5
				11.738	42.5
				13.694	50.0
			36	5.8690	23.0
				7.8250	30.5
				9.7810	35.0
				11.738	42.0
				13.694	48.5
			38	5.8690	23.0
				7.8250	29.0
				9.7810	34.5
				11.738	40.5
				13.694	46.0

Table 6: Seven multiple variables regression models and their relations

Model No.	Model	Relation
1	$\delta = C_0 + C_1\, b + C_2\, d + C_3\, P + C_4\, W$	δ = 75.67 + 0.104 b - 0.107 d - 0.758 P + 3.519 W
2	$\delta = C_0 + C_1\, b + C_2\, P + C_3\, W$	δ = 71.38 - 0.219 b - 0.758 P + 3.519 W
3	$\delta = C_0 + C_1\, d + C_2\, P + C_3\, W$	δ = 77.43 - 0.078 d - 0.758 P + 3.519 W
4	$\delta = C_0 + C_1\, (bd) + C_2\, P + C_4\, W$	δ = 54.83 - 0.0002 (bd) - 0.758 P + 3.519 W
5	$\delta = C_0 + C_1\, (b/d) + C_2\, P + C_3\, W$	δ = - 9.675 + 135.7 (b/d) - 0.758 P + 3.519 W
6	$\delta = C_0 + C_1\, (d/b) + C_2\, P + C_3\, W$	δ = 76.20 - 13.58 (d/b) - 0.758 P + 3.519 W
7	$\delta = C_0 + C_1\, (bd)^{0.5} + C_2\, P + C_3\, W$	$\delta = 76.28 - 0.137\, (bd)^{0.5} - 0.758\, P + 3.519\, W$

where:

Y = Dependent variable, for example deflection of radial-ply tire

$X_1, X_2, \ldots, X_n$ = Independent variables, for example section width, overall unloaded diameter, inflation pressure and vertical load

$C_0, C_1, C_2, \ldots, C_n$ = Regression coefficients

In order to predict deflection of radial-ply tire from section width, overall unloaded diameter, inflation pressure and vertical load, seven multiple variables regression models were suggested and all the data were subjected to regression analysis using the Microsoft Excel 2007. All the multiple variables regression models and their relations are shown in Table 6.

Statistical Analysis: A paired samples t-test and the mean difference confidence interval approach were used to compare the deflection values predicted by selected model with the deflection values measured by test apparatus. The Bland-Altman approach [12] was also used to plot the agreement between the deflection values measured by test apparatus with the deflection values predicted by selected model. The statistical analyses were also performed using Microsoft Excel 2007.

RESULTS AND DISCUSSION

The p-value of independent variables and coefficient of determination (R^2) for the seven multiple variables regression models are shown in Table 7. Among the seven models, model No. 1 had the highest R^2 value (0.986). Moreover, this model totally had the lowest p-value of independent variables among the seven models. Based on the statistical results model No. 1 was selected as the best model, which is given by equation 14:

$$\delta = 75.67 + 0.104\, b - 0.107\, d - 0.758\, P + 3.519\, W \quad (14)$$

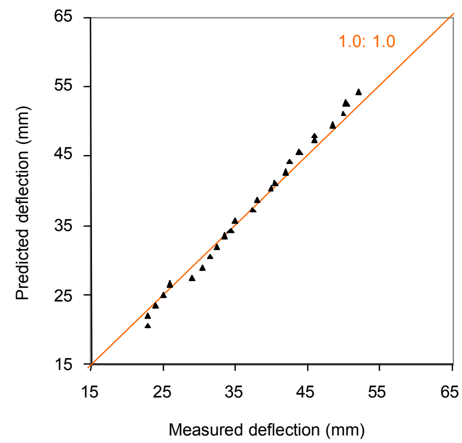

Fig. 4: Measured deflection using test apparatus and predicted deflection using model No. 1 for radial-ply tire No. 4 with the line of equality (1.0: 1.0)

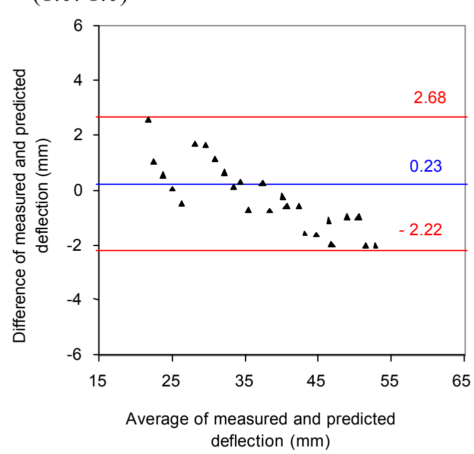

Fig. 5: Bland-Altman plot for the comparison of measured deflection using test apparatus and predicted deflection using model No. 1 for radial-ply tire No. 4; the outer lines indicate the 95% limits of agreement (-2.22, 2.68) and the center line shows the average difference (0.23)

Deflection of radial-ply tire No. 4 was then predicted at five levels of inflation pressure and five levels of vertical load using the multiple variables regression model

Table 7: The p-value of independent variables and coefficient of determination (R^2) for the seven multiple variable regression models

	p-value								
Model No.	b	d	bd	b/d	d/b	$(bd)^{0.5}$	P	W	R^2
1	0.009716	2.50E-13	---	---	---	---	4.24E-23	4.10E-65	0.986
2	1.33E-14	---	---	---	---	---	2.49E-15	2.05E-54	0.970
3	---	4.61E-25	---	---	---	---	2.69E-22	1.64E-64	0.985
4	---	---	1.53E-20	---	---	---	4.29E-19	3.77E-60	0.979
5	---	---	---	0.005114	---	---	1.20E-09	1.09E-43	0.938
6	---	---	---	---	0.003795	---	1.07E-09	8.48E-44	0.938
7	---	---	---	---	---	3.02E-20	6.81E-19	7.32E-60	0.979

Table 8: Section width, overall unloaded diameter, inflation pressure, vertical load and deflection for radial-ply tire No. 4 used in evaluating model No. 1

				Deflection δ (mm)			
Section width b (mm)	Overall unloaded diameter d (mm)	Inflation pressure P (kPa)	Vertical load W (kN)	Measured by test apparatus	Predicted by model No. 1	Average of measured and predicted deflection (mm)	Difference of measured and predicted deflection (mm)
216	650	30	5.8690	26.0	26.5	26.2	-0.5
			7.8250	33.5	33.4	33.4	0.1
			9.7810	40.0	40.3	40.1	-0.3
			11.738	46.0	47.2	46.6	-1.2
			13.694	52.0	54.0	53.0	-2.0
		32	5.8690	25.0	25.0	25.0	0.0
			7.8250	32.5	31.9	32.2	0.6
			9.7810	38.0	38.7	38.4	-0.7
			11.738	44.0	45.6	44.8	-1.6
			13.694	50.5	52.5	51.5	-2.0
		34	5.8690	24.0	23.5	23.7	0.5
			7.8250	31.5	30.3	30.9	1.2
			9.7810	37.5	37.2	37.4	0.3
			11.738	42.5	44.1	43.3	-1.6
			13.694	50.0	51.0	50.5	-1.0
		36	5.8690	23.0	21.9	22.5	1.1
			7.8250	30.5	28.8	29.7	1.7
			9.7810	35.0	35.7	35.4	-0.7
			11.738	42.0	42.6	42.3	-0.6
			13.694	48.5	49.5	49.0	-1.0
		38	5.8690	23.0	20.4	21.7	2.6
			7.8250	29.0	27.3	28.2	1.7
			9.7810	34.5	34.2	34.3	0.3
			11.738	40.5	41.1	40.8	-0.6
			13.694	46.0	48.0	47.8	-2.0

Table 9: Paired samples t-test analyses on comparing deflection determination methods

Determination methods	Average difference (mm)	Standard deviation of difference (mm)	p-value	95% confidence intervals for the difference in means (mm)
Test apparatus vs. model No. 1	0.23	1.25	0.3695	-0.28, 0.74

No. 1. The deflection values predicted by model No. 1 were compared with the deflection values measured by test apparatus and are shown in Table 8. A plot of the deflection values predicted by model No. 1 and the deflection values measured by test apparatus with the line of equality (1.0: 1.0) is shown in Fig. 4. Also, a paired samples t-test and the mean difference interval approach were used to compare the deflection values predicted by model No. 1 with the deflection values measured by test apparatus. The Bland-Altman approach [12] was also used to plot the agreement between the deflection values measured by test apparatus with deflection values

predicted by model No. 1. The average deflection difference between two methods was 0.23 mm (95% confidence interval: -0.28 mm and 0.74 mm; $P = 0.3695$). The standard deviation of the deflection difference was 1.25 mm (Table 9). The paired samples t-test results showed that the deflection values predicted by model No. 1 were not significantly different than the deflection values measured by test apparatus. The deflection difference values between two methods were normally distributed and 95% of these differences were expected to lie between μ-1.96σ and μ+1.96σ, known as 95% limits of agreement [13-17]. The 95% limits of agreement for comparison of the deflection values determined by test apparatus and model No. 1 was calculated at -2.22 mm and 2.68 mm (Fig. 5). Thus, the deflection values predicted by model No. 1 for radial-ply tire No. 4 may be 2.22 mm lower or 2.68 mm higher than the deflection values measured by test apparatus for this radial-ply tire. The average percentage difference for the deflection values predicted by model No. 1 and measured by test apparatus was 2.92%.

CONCLUSIONS

It can be concluded that the multiple variables regression model $\delta = 75.67 + 0.104\ b - 0.107\ d - 0.758\ P + 3.519\ W$ with $R^2 = 0.986$ can be strongly suggested to predict deflection of radial-ply tire based on section width, overall unloaded diameter, inflation pressure and vertical load.

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