

Genetic and Lagrange Relaxation Algorithms for Solving Constrained Economic Dispatch Including Losses

¹Hussein Al-Masri and ²Hani Q. Al-Zoubi

¹Electrical Engineering Department,
Faculty of Engineering Yarmouk, University, Irbid, Jordan

²Department of Computer Engineering,
Faculty of Engineering Mutah University, Karak, Jordan

Abstract: Two techniques were presented to solve an economic dispatch problem. An iterative technique, represented by Lagrange relaxation algorithm, was implemented. The algorithm was based on selecting initial guess and continuing until finding the best iteration with the optimal solution. The iterative technique was compared to a stochastic one represented by genetic algorithm. This algorithm was based on principles inspired from the biological evolution using natural selection, genetic recombination and survival of the fittest. The objective was to minimize the total generation fuel cost and keep the power flows within the security limits. The two techniques were compared for solving an economic dispatch problem with two generators. Results showed that the genetic algorithm was efficient and reliable optimization technique. The stochastic nature of the genetic algorithm could provide higher optimal solution and less computation time when compared to those of the iterative method.

Key words: Genetic Algorithm • Iterative technique algorithm • Lagrangian multiplier • Economic Dispatch
• Electric power network

INTRODUCTION

The Economic Dispatch (ED) analysis schedule outputs of generating units by allocating their generation levels. This analysis aims at economizing the process when meeting the system's load. The ED problem is mirrored as an optimization algorithm that minimizes the core objective function represented by the cost of generation. Technically, the mathematical model representing the ED can be solved numerically using iterative techniques [1-3]. However, several optimization methods and algorithms can also be implemented with greater efficiency and higher accuracy than iterative methods [4-7]. Among these algorithms are genetic algorithms (GA) which their evolution was inspired the study of genetics [8, 9]. The GAs combine an artificial principal with genetic operation. The artificial principal is the survival of fittest principal and the genetic operation is abstracted from nature. Their dependence on natural biological evolution such as selection, crossover and mutation classified them as being stochastic search

methods [4, 9-11]. These algorithms work on finding populations of solutions rather than single solution as in other search techniques [12-15].

In this work, a GA is developed and implemented for the solution of the economic dispatch problem. The results of the GA economic dispatch were compared with those calculated using iterative techniques.

The Economic Dispatch Problem: The transmission of power, over long distances for low density areas, makes transmission losses appreciable to consider. These losses affect the optimal dispatch of generation. The power loss described by Kron's loss formula is written as [3],

$$P_{loss} = \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{Ng} B_{oi} P_{Gi} + B_{00} \quad (1)$$

where B_{ij} are called B -coefficients or loss coefficients. These coefficients are assumed constants as long as operation is near the value where these coefficients are

computed. The problem of ED focuses on minimizing the total cost C_t , which is a function of the real output power as follows [3]:

$$C_t = \sum_{i=1}^{Ng} C_i = \sum_{i=1}^{Ng} \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \quad (2)$$

This equation is subjected to two constraints; one is the equality constraint which indicate that the total real power generation should equal to the total system demand plus losses [3],

$$\sum_{i=1}^{Ng} P_{Gi} = P_D + P_{loss} \quad (3)$$

and the other is the inequality constraints

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \quad i = 1, 2, 3, \dots, N_g \quad (4)$$

Economic Dispatch Case Study: A small power plant is using two generating units operated on economic dispatch to serve a load. The variable operating (production) costs of the two units are given by:

$$C_1(P_{G1}) = 8P_{G1} + 0.020P_{G1}^2; \quad 100 \leq P_{G1} \leq 500 \quad (5)$$

$$C_2(P_{G2}) = 12P_{G2} + 0.010P_{G2}^2; \quad 50 \leq P_{G2} \leq 400 \quad (6)$$

The total system load is estimated as 550 MW and loss coefficients are given by:

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 0.400 \times 10^{-3} & 0 \\ 0 & 0.500 \times 10^{-3} \end{bmatrix},$$

$$B_0 = [B_{10} \quad B_{20}] = [0 \quad 0], B_{00} = 0$$

These units with their aforementioned data are implemented to test and compare the two algorithms discussed in this work.

Lagrange Relaxation Algorithm: Using the Lagrange multipliers and including the inequality constraints, the objective function is represented as [3]:

$$\zeta = C_t + \lambda(P_D + P_{loss} - \sum_{i=1}^{Ng} P_{Gi}) + \sum_{i=1}^{Ng} \mu_i^{(\max)} (P_{Gi} - P_{Gi}^{(\max)}) + \sum_{i=1}^{Ng} \mu_i^{(\min)} (P_{Gi} - P_{Gi}^{(\min)}) \quad (7)$$

To optimize the augmented cost function, we have to find the gradient of ζ with respect to λ , P_{Gi} , $\mu_i^{(\max)}$ and $\mu_i^{(\min)}$ and equate to zero; the following results are obtained after simplification [3]:

- Incremental loss calculated using:

$$\frac{\partial P_{loss}}{\partial P_{Gi}} = 2 \sum_{j=1}^{Ng} B_{ij} P_{Gj} + B_{0i} \quad (8)$$

The shadow price λ is a product of the incremental cost function times penalty factor:

$$L_i \frac{dC_i}{dP_{Gi}} = \lambda, \quad i = 1, \dots, N_g \quad (9)$$

where L_i is the *penalty factor* of plant i and given as:

$$L_i = \frac{1}{1 - \frac{\partial P_{loss}}{\partial P_{Gi}}} \quad (10)$$

Expanding into matrix form and obtain the following:

$$\begin{bmatrix} P_{G1}^K \\ P_{G2}^K \\ \vdots \\ P_{GNg}^K \end{bmatrix} = \begin{bmatrix} B_{11} + \frac{\gamma_1}{\lambda} & B_{12} & \dots & B_{1Ng} \\ B_{21} & B_{22} + \frac{\gamma_2}{\lambda} & \dots & B_{2Ng} \\ \vdots & \vdots & \ddots & \vdots \\ B_{Ng1} & B_{Ng2} & \dots & B_{NgNg} + \frac{\gamma_{Ng}}{\lambda} \end{bmatrix}^{-1} \times \frac{1}{2} \begin{bmatrix} 1 - B_{01} - \frac{\beta_1}{\lambda} \\ 1 - B_{02} - \frac{\beta_2}{\lambda} \\ \vdots \\ 1 - B_{0Ng} - \frac{\beta_{Ng}}{\lambda} \end{bmatrix} \quad (11)$$

- For each k -iteration, there is a new value of lambda λ and a real power error $\Delta P_G^{(K)}$ should be limited in equation (12) and (13):

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta \lambda^{(k)} \quad (12)$$

$$\Delta P_G^{(K)} = P_D + P_{loss}^{(k)} - \sum_{i=1}^{Ng} P_{Gi}^{(K)} \quad (13)$$

Genetic Algorithm: In genetic algorithm, the solution is transformed to chromosomes representation; each chromosome within the population represents a candidate solution. A flowchart for a GA is shown in Fig. 2. To formulate GA for ED problem, the k -th chromosome C_k can be defined as follows [12]:

$$C_k = [P_{k1}, P_{k2}, \dots, P_{kNg}] \quad (14)$$

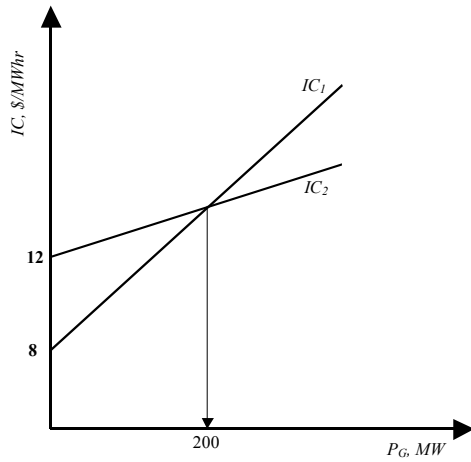


Fig. 1: Lagrange relaxation approach results.

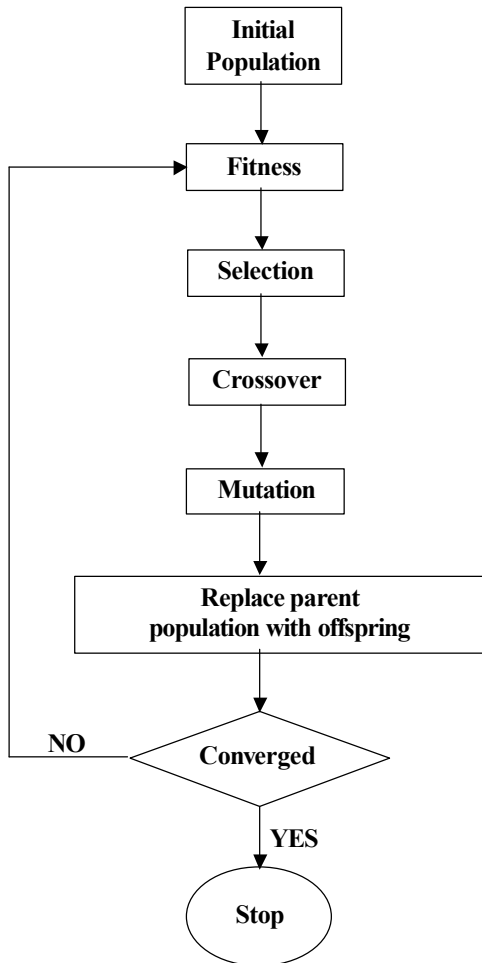


Fig. 2: Flow chart Genetic Algorithm

where $k = 1, 2, \dots, \text{population_size}$, Ng is the number of generating units and P_{kNg} is the real power production of the Ng -th, unit at k -th chromosome.

The objective function in ED problem is the cost function. The fitness function is used to transform the cost function value into a measure of relative fitness; the best fit individuals will have the lowest cost of the objective function. For the economic dispatch problem, the fitness function may be expressed as [12, 13]:

$$Fit(x) = g(f(x)) \quad (15)$$

Since $x = P_{Gi}$ and $f(x) = \sum_{i=1}^{Ng} C_i(P_{Gi})$, then

$$Fit(P_{Gi}) = g\left(\sum_{i=1}^{Ng} C_i(P_{Gi})\right) = g\left(\sum_{i=1}^{Ng} \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2\right) = \sum_{i=1}^{Ng} g(\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) \quad (16)$$

Crossover operator defines a linear combination of two chromosomes [12]. Two chromosomes are selected randomly for crossover, therefore, if the size range of the two parents, P_1 and P_2 , is $R = P_2 - P_1$, then the two children, C_1 and C_2 , will be given by the following linear equations [11]:

$$C_1 = (1 - a)P_1 + aP_2 \quad (17)$$

$$C_2 = (1 - a)P_2 + aP_1 \quad (18)$$

where a is a random number to be chosen in range of (0, 1). All constraints are to be satisfied in terms of load demand equation and the values of power outputs. If the solution has at least one constraint violated, a new random real value is used for finding a new value of the m -th element of the chromosome [13]. By elitist, the best solutions of each generations are saved and copies into the next generation. The genetic algorithm process repeats until the specified maximum number of generations is reached [13].

Simulation Results

Iterative Approach:

$$IC_1 = \frac{dC_1(P_{G1})}{d(P_{G1})} = 8 + 0.04P_{G1} \quad (19)$$

$$IC_2 = \frac{dC_2(P_{G2})}{d(P_{G2})} = 12 + 0.02P_{G2} \quad (20)$$

These equations are shown in Fig. 1. The system relaxation parameter is

$$\lambda = \frac{P_D + \sum_{i=1}^{Ng} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{Ng} \frac{1}{2\gamma_i}} = 18 \frac{\$}{MWhr} \quad (21)$$

The optimal dispatch is given by $P_{Gi} = \frac{\lambda - \beta_i}{2\gamma_i}$, then $P_{G1} = 300 \text{ MW}$ and $P_{G2} = 300 \text{ MW}$. The losses are considered for two units are calculated following using

$$\begin{bmatrix} P_{G1}^{(1)} \\ P_{G2}^{(1)} \end{bmatrix} = \begin{bmatrix} B_{11} + \frac{\gamma_1}{\lambda^{(1)}} & B_{12} \\ B_{21} & B_{22} + \frac{\gamma_2}{\lambda^{(1)}} \end{bmatrix}^{-1} \times \frac{1}{2} \times \begin{bmatrix} 1 - B_{01} - \frac{\beta_1}{\lambda^{(1)}} \\ 1 - B_{02} - \frac{\beta_2}{\lambda^{(1)}} \end{bmatrix} \quad (22)$$

By substituting the respective values of the parameters and taking $\lambda^{(1)} = 18\$$, then the calculated optimal dispatch including losses after the first trial for the two units are $P_{G1}^{(1)} = 183.84 \text{ MW}$ and $P_{G2}^{(1)} = 157.83 \text{ MW}$.

The transmission loss is estimated as

$$P_{loss}^{(1)} = \sum_{i=1}^{Ng} B_{ii} P_{Gi}^2 = B_{11} P_{G1}^2 + B_{22} P_{G2}^2 = 25.97 \text{ MW} \quad (23)$$

modifying the system lambda as

$$\Delta\lambda^{(1)} = \frac{\left[\lambda^{(1)} - \lambda^{(0)} \right] \Delta P_G^{(1)}}{\sum_{i=1}^{Ng} P_{Gi}^{(1)} - \sum_{i=1}^{Ng} P_{Gi}^{(0)}} = 12.344 > \epsilon \quad (24)$$

where for $P_D = 550 \text{ MW}$, $\Delta P_G^{(1)}$ is calculated as

$$\Delta P_G^{(1)} = P_D + P_{loss} - \sum_{i=1}^2 P_{Gi} = 234.304 \text{ MW} \cdot \text{Repeating the same}$$

procedure until $k=9$ then; $P_{G1}^{(9)} = 315.8095 \text{ MW}$ and

$P_{G2}^{(9)} = 327.8341 \text{ MW}$. These values results in $C_{total} = 9529.29 \$/hr$ for $P_{loss}^{(9)} = 94.6318 \text{ MW}$, $\Delta\lambda^{(9)} = |-0.0004| < \epsilon$ and $\lambda^{(9)} = 27.6073 \$/MWhr$.

Genetic Algorithm Approach: The genetic algorithm has been applied to the previous economic dispatch problem which is a two-unit power system. The GA approach is implemented using Matlab® [16] as shown in Fig. 2. Prior to starting the program, each blue point represented

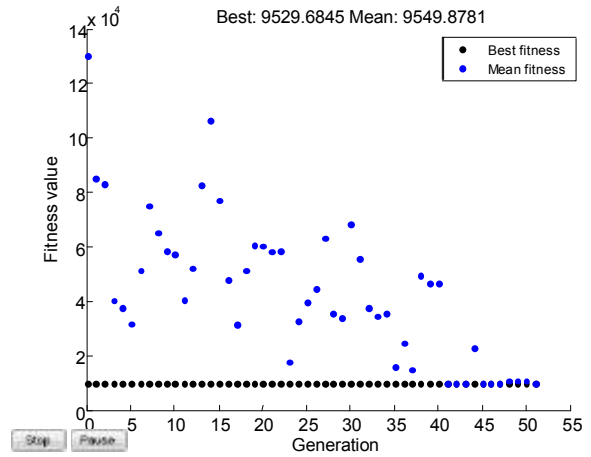


Fig. 3: GA for Economic Dispatch of 2-Units

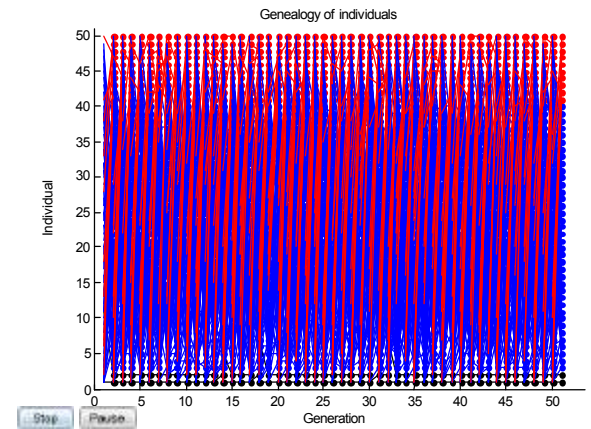


Fig. 4: Genealogy of individual

a candidate solution, after that it has clearly seen how the genetic algorithm search for the solution and choose the optimal one which is denoted as *Best* in Fig. 3.

To depict the offsprings of mutation, crossover and elitism Fig. 4, plots the genealogy of individuals. Lines from one generation to the next are color-coded as follows

- Red lines indicate mutation children.
- Blue lines indicate crossover children.
- Black lines indicate elite individuals.

After that a comparison is made to genetic algorithm with iterative techniques; the results of both techniques are compared in Table 1 for a demand of (550MW):

Approach	Unit#1 (MW)	Unit#2 (MW)	Total Cost (\$/hr)	Total Losses (MW)
Genetic Algorithm	316.3709	327.1930	9529.6	93.5638
Lagrange Relaxation	315.8095	327.8341	9529.95	94.6318

Table 2: Percent difference in computed values of total cost and losses by the two algorithms

Approach	Total Cost (\$/hr)	Total Losses (MW)
Genetic Algorithm	9529.6	93.5638
Lagrange Relaxation	9529.95	94.6318
% Difference	0.0037	1.1350

Regarding the time consumed, GA is faster than iterative techniques since the elapsed time to complete the operation of GA is 15.578 seconds. It can be seen that the genetic algorithm approach achieved lower operation cost and total loss. To calculate the percentage difference of total operation cost and total losses of both genetic algorithm and iterative techniques the equation (25) is used and the results are shown in Table 2:

$$\% \text{ Difference} = \frac{|x_1 - x_2|}{\left(\frac{x_1 + x_2}{2}\right)} \times 100\% \quad (25)$$

CONCLUSIONS

In this paper, an approach based on a genetic algorithm has been successfully presented and applied to the generation cost in electric power network to obtain the optimum solution of economic dispatch (ED). Operators are used in GA to generate a set of solutions for this problem. GA method is most useful for large power systems, it gave well results and it is much faster and more effective than iterative method. Methods are compared for solving an economic dispatch problem with two generators. Test results have shown GA algorithm can provide highly optimal solutions and reduces the computation time than those with the iterative method. An advantage of the GA solution is the flexibility it provides in modeling both time dependent and coupling constants [10]. Another advantage of the GA approach is the ease with which it can handle arbitrary kinds of constraints and objectives.

REFERENCES

- Happ, H.H., 1977. Optimal power dispatch – A comprehensive survey. IEEE Trans. Power App. Syst., 96: 841-854.
- Chowdhury, B.H. and S. Rahman, 1990. A review of recent advances in economic dispatch. IEEE Trans. Power Syst., 5: 1248-1259.
- Saadat, H., 1999. Power System Analysis. McGraw-Hill. New York.
- Prasanna, T.S. and P. Somasundaram, 2008. Fuzzy mutated Evolutionary Programming base algorithm for combined economic and emission dispatch. IEEE TENCON, pp: 1-5.
- Min, K., S.W. Lee and Y.H. Moon, 2008. An economic dispatch algorithm as combinatorial optimization problems. Int. J. Control Autom. Syst., 6(4): 468-476.
- Dekhici, L., K. Belkadi and P. Borne, 2012. Firefly algorithm for economic power dispatching with pollutants emission. Informatica Economica, 16(2): 54-57.
- Shaikh, F., P. Shaikh, M. Mirani and M. Aslam, 2012. Multi criteria optimization algorithm for economic dispatch complications for sustainable interconnected power system. Int. J. comput. Appl. 50(4): 22-25.
- Rennard, J.P., 2000. Introduction to Genetic Algorithms. <http://www.rennard.org>.
- Eiben, A.E., P.E. Raue and Z.S. Ruttkay, 1994. Genetic algorithms with multi-parent recombination. in Parallel problem solving from Nature-3, LNCS 8 Springer-Verlag, pp: 78-87.
- Garg, R. and A. Sharma, 2008. Economic generation and scheduling of power by genetic algorithm. J. Theor. and Appl. Inf. Technol., 4(12): 1142-1149.
- Lee, Y., A. Marvin and S. Porter, 1999. Genetic algorithm using real parameters for array antenna design optimization. Proc. High Frequency Postgraduate Student Colloquium, pp: 8-13.
- Yalcinoz, T., H. Altun and M. Uzam, 2001. Economic dispatch solution using a genetic algorithm based on arithmetic crossover. Proc. Of the IEEE Porto Power Tech Conference.
- Yalcinoz, T. and H. Altun, 2002. Environmentally constrained economic dispatch via a genetic algorithm with arithmetic crossover. IEEE africon, pp: 923-928.
- Koridak, L.A., M. Rahli and M. Younes, 2009. Hybrid optimization of the emission and economic dispatch by the genetic algorithm. Leonardo J. Sci., 8(14): 193-203.
- Nezamabadi-Pour, H., S. Yazdani, M. Farsangi and M. Neyestani, 2011. A solution to an economic dispatch problem by fuzzy adaptive genetic algorithm. Iran. J. Fuzzy Syst., 8(3): 1-21.
- MATLAB 7.0. Software package, MATHWORKS. <http://www.mathworks.com>.