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A Note on Compactness in Intuitionistic Fuzzy Soft Topological Spaces

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Abstract: The aim of this paper is to construct a topology on an intuitionistic fuzzy soft set. Also the concepts of fuzzy soft compactness, fuzzy soft almost compactness and fuzzy soft near compactness of intuitionistic fuzzy soft topological spaces.

Key words: Soft sets · Intuitionistic fuzzy soft sets · Intuitionistic fuzzy soft topological spaces · Intuitionistic fuzzy soft compactness

INTRODUCTION

In [1] Atanassov introduced the fundamental concept of intuitionistic fuzzy set. Later, this concept was generalized to intuitionistic L-fuzzy sets by Atanassov and Stoeva [2]. Coker [3] introduced the notion of intuitionistic fuzzy topological spaces, fuzzy continuity, fuzzy compactness and some other related concepts. In 1999, D. Molodtsov [4] introduced the soft set theory to solve complicated problems in economics, engineering and environment. He has shown several applications of this theory in solving many practical problems. Later on authors like Maji *et al.* [5-9] have further studied the theory of fuzzy soft sets and introduced the concepts of fuzzy soft set and intuitionistic fuzzy soft set. Biswas Dinda and T.K. Samanta [10] introduced the concept of relations in intuitionistic fuzzy soft set and discussed its several properties.

In this paper we introduce the investigate intuitionistic fuzzy soft topological spaces, fuzzy soft continuity, fuzzy soft compactness, fuzzy soft almost compactness and fuzzy soft near compactness in intuitionistic fuzzy soft topological spaces.

Preliminaries

Definition 2.1: [4] Let U be an initial universe set and E be the set of parameters. Let P(U) denotes the power set of U. A pair (F, E) is called a soft set over U where F is a mapping given by F: E → P(U).

Definition 2.2: [5] Let U be an initial universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U where F is a mapping given by F: $A \to I^U$, where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3: [8] Let U be an initial universe set and E be the set of parameters. Let IF^U denotes the collection of all intuitionistic fuzzy subsets of U. Let $A \subseteq E$. A pair (F, E) is called an intuitionistic fuzzy soft set over U where F is a mapping given by F: $A \to IF^U$.

Example 2.4: [10] Consider the following example: Let (F, A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year. Let the set of students under consideration is U = {s1, s2, s3, s4}. Let $A \subseteq E$ and

A = {r=result, c=conduct, g= games and sports per formances}. Let

F(r) = {(s1,0.8,0.1), (s2,0.9,0.05), (s3,0.85,0.1), (s4,0.75,0.2)},

F(c) = {(s1,0.6,0.3), (s2,0.65,0.2), (s3,0.7,0.2), (s4,0.65,0.2)},

F(g) = {(s1,0.75,0.2), (s2,0.5,0.3), (s3,0. 5,0.4), (s4,0.7,0.2)}.

Then the family {F(r), F(c), F(g)} of IF^U is an intuitionistic fuzzy soft set.

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In general, for every e∈A, F(e) is an intuitionistic fuzzy set of U and it is called intuitionistic fuzzy value set of parameter e. Clearly, F(e) can be written as an intuitionistic fuzzy set such that

$F(e) = \{< x, \mu_{F(e)}x), \lambda_{F(e)}(x) >| x \in U \}$,

where

$\mu_{F(e)}$ and $\lambda_{F(e)}$ are the membership and non-membership functions, respectively. The set of all intuitionistic fuzzy soft sets over U with parameters from E is called an intuitionistic fuzzy soft class and it is denoted by IFS (U, E) [11].

Definition 2.5: [8] Let (F, A) and (G, B) be intuitionistic two fuzzy soft sets over U. We say that (F, A) is an intuitionistic fuzzy soft subset of (G, B) and write (F, A) $\subseteq E$ (G, B) if

- $A \subseteq B$,
- For any $e \in A$, $F(e) \subseteq G(e)$, that is, for all x∈U and e ∈ A, $\mu_{F(e)}(x) \le \mu_{G(e)}(x)$ and

$$\lambda_{F(e)}(x) \ge \lambda_{G(e)}(x).$$

(F, A) and (G, B) are said to be intuitionistic fuzzy soft equal and write (F, A) = (G, B) if (F, A) ⊆ (G, B) and (G, B) ⊆ (F, A).

Definition 2.6: [8] The union of two intuitionistic fuzzy soft sets (F, A) and (G, B) over U is an intuitionistic fuzzy soft set denoted by (H, C), where C= AuB and

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B, \\ G(e) & \text{if } e \in B - A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B, \end{cases}$$

For all e∈ C. This is denoted by (H, C) = (F, A)u(G, B).

Definition 2.7: [8] The intersection of two intuitionistic fuzzy soft sets (F, A) and (G, B) over U is an intuitionistic fuzzy soft set denoted by (H, C), where C=AuB and

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B, \\ G(e) & \text{if } e \in B - A, \\ F(e) \cap G(e) & \text{if } e \in A \cap B, \end{cases}$$

For all e∈C. This is denoted by (H,C) = (F, A) ∩ (G, B).

Definition 2.8: [11] The relative complement of an intuitionistic fuzzy soft set (F, A) over U is denoted by $(F, A)^r$ and is defined by (F^r, A), where for each e∈ A, $\mu_{F^r(e)} = \lambda_{F(e)}$ and $\mu_{F^r(e)} = \mu_{F(e)}$, that is,

$$F^r(e) = (\lambda_{F(e)}, \mu_{F(e)}).$$

Clearly, $((F, A)^r)^r = (F, A)$.

An intuitionistic fuzzy soft set (F, A) over U is said to be a relative null intuitionistic fuzzy soft set (with respect to the parameter A), denoted by $\tilde{\phi}_A$, if $F(e)=1_U$ for all e∈ A. An intuitionistic fuzzy soft set (F, A) over U is said to be a relative whole intuitionistic fuzzy soft set (with respect to the parameter set A), denoted by $\tilde{E}_A$, if $F(e)=1^U$ for all e∈ A.

Denote by 1_x and 1^x the intuitionistic fuzzy sets of X defined by $\mu_{1_x}(x) = 0$,
$\lambda_{1_x}(x) = 1$ and $\mu_{1^x}(x) = 1$, $\lambda_{1^x}(x) = 0$, respectively for all x∈ X [11].

Definition 2.9: [11] Let IFS(U, E) and IFS(U', E') be two intuitionistic fuzzy sets classes and let

$\varphi : U \to U'$ and $\phi : E \to E'$ be mappings. Then a mapping (φ, ϕ) :IFS(U, E)→IFS(U', E') is defined as:

for (F, A) ∈ IFS(U, E), the image of (F, A) under (φ, ϕ), denoted by

$(\varphi, \phi)(F, A)=((\varphi(F), \phi(A))$, is an intuitionistic fuzzy soft set in IFS(U', E') given by

$$\mu_{\varphi(F)(e')}(x') = \begin{cases} \vee\, \mu_{F(e)}(x) \\ e \in \phi^{-1}(e') \cap A,\ x \in \varphi^{-1}(x') & if\, \varphi^{-1}(x') \ne \phi \\ 0 & otherwise \end{cases}$$

and

$$\lambda_{\varphi(F)(e')}(x') = \begin{cases} \psi\, \lambda_{F(e)}(x) \\ e \in \phi^{-1}(e') \cap A,\ x \in \varphi^{-1}(x') & if\, \varphi^{-1}(x') \ne \phi \\ 0 & otherwise \end{cases}$$

for all e' ∈ ϕ(A) and x' ∈ U'. For (F', A') ∈ IFS(U', E'), the inverse image of (F', A') under (ϕ, ϕ), denoted by $(\phi, \phi)^{-1}(F', A')=(\phi^{-}(F'), \phi^{-}(A'))$ is an intuitionistic fuzzy soft set in IFS(U, E) given by

$$\mu_{\varphi^{-1}(F')(e)}(x) = \mu_{F'(\phi(e))}(\varphi(x))$$

and

$$\lambda_{\varphi^{-1}(F')(e)}(x) = \lambda_{F'(\phi(e))}(\varphi(x))\ (x)$$

for all e Є φ^{-1}(B) and x Є U.

Theorem 2.10: [11] Let (F, A), (G, B) Є IFS(U, E) and ϕ: U→U' and φ : E→E' be two mappings. Then

- (F, A) ⊆ $(\phi, \varphi)^{-1}$((ϕ, φ)(F, A)),
- ((ϕ, φ)(F, A))^r⊆(ϕ, φ)(F, A)^r if ϕ is surjective,
- (ϕ, φ)((F, A) u (G, B))=(ϕ, φ)(F, A) u (ϕ, φ)(G, B).

Theorem 2.11: [11] Let (F, A), (G, B)ЄIFS(U, E) and ϕ : U→U' and φ :E→E' be two mapping and an injective mapping, respectively.

- (F, A) = $(\phi, \varphi)^{-1}$((ϕ, φ)(F, A)), if ϕ is also injective.
- (ϕ, φ)((F, A)∩(G, B)) ⊆ (ϕ, φ)(F, A)∩(ϕ, φ)(G, B).

Theorem 2.12: [11] Let (F', A'), (G', B')ЄIFS(U', E') and ϕ : U→U' and φ :E→E' be two mappings. Then

- (ϕ, φ)($(\phi, \varphi)^{-1}$(F', A') ⊆ (F', A') and (ϕ, φ) ($(\phi, \varphi)^{-1}$(F', A')=) (F', A') if both ϕ and φ are surjective.
- ($(\phi, \varphi)^{-1}$(F', A'))^r=($(\phi, \varphi)^{-1}$((F', A')^r).
- $(\phi,\varphi)^{-1}$((F', A')u(G', B')) =$(\phi, \varphi)^{-1}$(F', A')u $(\phi, \varphi)^{-1}$(G', B').
- $(\phi, \varphi)^{-1}$((F', A')∩(G', B')) =$(\phi, \varphi)^{-1}$(F', A')∩ $(\phi, \varphi)^{-1}$(G', B').

Intuitionistic Fuzzy Soft Topological Spaces

Definition 3.1: An intuitionistic fuzzy soft topology τ on (U,E) is a family of intuitionistic fuzzy soft sets over (U,E) satisfying the following properties

- $\tilde{\phi}_E$, $\tilde{E}$ Є τ.
- If (F, A), (G, B) Є τ, then (F, A) ∩ (G, B)Є τ.
- If (F_α, A) Є τ for all α ϵ Λ, an index set, then $\underset{\alpha\in\wedge}{\mathrm{U}}$(Fα, A) ∈ ∧ .

In this case the (U, E, τ) is called an intuitionistic fuzzy soft topological space (IFSTS for short) and each IFSS in τ is known as an intuitionistic fuzzy soft open set (IFSOS for short) in (U, E). An intuitionistic fuzzy soft set is called τ-closed iff its complement is τ-open.

Definition 3.2: Let (U, E, τ) be an IFSTS topological space and (F, A) be an IFSS in (U, E). Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of (F, A) are defined by

int(F, A)= U {(G, A): (G, A) is an IFSOS in (U, E) and (G, A) ⊆ (F, A)} and

cl(F, A)= ∩{(K, A): (K, A) is an IFSCS in (U, E) and (F, A) ⊆ (K, A)}.

It can be also shown that cl(F, A) is an IFSCS and int (F, A) is an IFSOS in (U, E) and

- (F, A) is an IFSCS in (U, E) iff cl(F, A)=(F, A),
- (F, A) is an IFSOS in (U, E) iff int(F, A)=(F, A).

Definition 3.3: Let (U, E, τ) and (U', E', τ') be IFSTS s and let and ϕ : U→U' and φ :E→E' be two mappings. Then a mapping (ϕ, φ) : IFSTS(U, E, τ)>IFSTS(U', E', τ') is said to be intuitionistic fuzzy continuous iff the preimage of each IFSOS in (U', E', τ') is an IFSOS in (U, E, τ).

Proposition 3.4: Let (U, E, τ) be an IFSTS and (F, A), (G, B) ЄIFSS in (U, E). Then the following properties hold:

- Int(F, A)⊆(F, A) (a'). (F, A)⊆cl(F, A),
- (F, A) ⊆ then int (F, A) ⊆ int (G, B), (b'). (F, A) ⊆ then cl(F, A) ⊆ cl(G, B),
- Int(int(F, A))=int(F, A), (c'). cl(cl(F, A))=cl(F, A),
- İnt((F, A)∩(G, B))=int(F, A)∩ int(G, B),
- cl((F, A)u (G, B))=cl(F, A)ucl(G, B).

Proof: It is clear from the previous definitions.

Definition 3.5: Let (U, E, τ) and (U', E', τ') be IFSTS s and let and ϕ : U→U' and φ :E→E' be two mappings. Then a mapping (ϕ, φ) : IFSTS(U, E, τ)>IFSTS(U', E', τ') is said to be intuitionistic fuzzy open iff the image of each IFSOS in (U, E, τ) is an IFSOS in (U', E', τ').

Intuitionistic Fuzzy Soft Compactness: First we present the basic concepts:

Definition 4.1: Let (U, E, τ) be an IFSTS. If a family (Fα, A) of IFSOS s in (U, E, τ) satisfies the condition u(Fα, A)=1^U, α∈I, then it is called a fuzzy open cover of (U, E). A finite subfamily of a fuzzy open cover (Fα, A) of (U, E), which also an intuitionistic fuzzy open cover of (U, E), is called a finite subcover of (Fα, A).

Definition 4.2: Let (U, E, τ) be an IFSTS. If a family (G_α, B), α ∈ I, of IFSS s in (U, E, τ) satisfies the finite intersection property (FIP for short) iff every finite subfamily (G_{α_1}, B), i=1,2,…,n of the family satisfies the condition

$$\bigcap_{i=1}^{n}(G_{\alpha_i}, B) \neq \tilde{\phi}.$$

Definition 4.3: An IFSTS (U, E, τ) is called fuzzy compact iff every intuitionistic soft fuzzy open cover of (U, E) has a finite subcover.

Example 4.4: Let U = {s_1, s_2}, A ⊆ E and A ={r, c, g}. Define intuitionistic fuzzy soft sets (Fn, A) as follows

(n ∈N+):

Fn(r)= {(s_1, n/n+1,n+1/n+2), (s_2, 1/n+1,1/n+3)} (n=1,2,…),
Fn(c) ={(s_1, n+1/n+2,n+2/n+3), (s_2,1/n,1/n+2)},
Fn(g) ={(s_1, n+2/n+3, n+1/n+2), (s_2, 1/n,1/n+2)}.

In this case the family τ = $\{\tilde{\phi}, \tilde{E}\}$ U{(Fn, A): n=1,2,…} is an IFSTS on (U, E). But the intuitionistic fuzzy soft open cover {(Fn, A): n=1,2,…} has no finite subcover, i.e., (U, E, τ) is not intuitionistic fuzzy soft compact.

Theorem 4.5: Let (U, E, τ), (U', E', τ') be IFSTSs and (ϕ, φ) : IFSTS(U, E, τ)>IFSTS(U', E', τ') a fuzzy continuous function. If (F, A) is fuzzy compact in (U, E, τ), then so is (ϕ, φ)(F, A) i n (U', E', τ').

Proof: Let {(Gi, A): i∈I} be an intuitionistic fuzzy open cover of (ϕ, φ)(F, A). Since (ϕ, φ) is intuitionistic fuzzy soft continuous, {$(\phi, \varphi)^{-1}(G_i, A)$: i ∈ I} is an intuitionistic fuzzy soft open cover of (F, A), too. Since (F, A) is fuzzy soft compact, there exists a finite subcover of {$(\phi, \varphi)^{-1}(G_i, A)$: i∈I}, i.e., there exists {$(\phi, \varphi)^{-1}(G_i, A_i)$:i=1,2,…,n} such that

$$(F, A) \subseteq \bigcup_{i=1}^{n} (\varphi, \phi)^{-1}(G_i, A).$$

$$\text{Hence } (\phi, \varphi)(F, A) \subseteq (\varphi, \phi)\left(\bigcup_{i=1}^{n} (\varphi, \phi)^{-1}(G_i, A)\right).$$

$$= \bigcup_{i=1}^{n}(\varphi, \phi)^{-1}(G_i, A_i)) \subseteq \bigcup_{i=1}^{n}(G_i, A).$$

Therefore (ϕ, φ)(F, A) is also compact.

Theorem 4.6: An IFSTS(U, E, τ) is fuzzy compact iff every {(Gα, Aα):α∈ I} of IFSCSs in (U, E) having the finite intersection property (FIP for short) has a nonempty intersection.

Proof: Obvious.

Definition 4.7:

(a) An IFSTS (U, E, τ) is called fuzzy almost compact iff every fuzzy open cover of (U, E) has a finite subcollection whose closures cover (U, E), or equivalently, every fuzzy open cover contains a finite subcollection whose closures form a cover of (U, E).

(b)) An IFSTS (U, E, τ) is called fuzzy nearly compact iff every fuzzy open cover of (U, E) has a finite subcollection such that the interiors of closures of IFSSs in this subcollection covers (U, E).

It is clear that IFSTS s we have the following implications:

İntuitionistic fuzzy soft compactness ⇒ İntuitionistic fuzzy soft near compactness ⇒ İntuitionistic fuzzy soft almost compactness.

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