

Modeling and Volatility Analysis of Share Prices Using ARCH and GARCH Models

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Abstract: We identify and estimate the mean and variance components of the daily closing share prices using ARIMA-GARCH type models by explaining the volatility structure of the residuals obtained under the best suited mean models for the said series. The parameters of ARIMA type simple specifications are routinely anticipated by applying the OLS methodology but it has two disadvantages when the volatility or ARCH effect is present. The first problem may be the autocorrelation in error terms. To handle this unwanted situation the lagged dependent variables can be incorporated as independent variables in the mean equation. The other problem may be the presence of ARCH effect. This problem can be resolved by employing the ARCH or GARCH specifications so we have taken advantage of such type of models in our study.

Key words: Closing Share Prices • Volatility • ARCH • GARCH

INTRODUCTION

Forecasting procedures are widely used in financial markets to evaluate companies and their stocks. Time series models play an important role in describing the underlying structure of the economic variables. Many time series especially occurring in the natural sciences and engineering cannot be modeled by linear processes. These kinds of time series can have trends which can be best modeled by nonlinear processes. The model building process for nonlinear time series is much more complicated than for linear time series. The important types of nonlinear time series includes bilinear, threshold autoregressive, exponential autoregressive, autoregressive conditional heteroscedastic (ARCH), generalized autoregressive heteroscedastic (GARCH) and stochastic and random coefficient models see e.g. [1].

In this paper, we study the financial assets relating to the daily closing share prices of Muslim Commercial Bank (MCB), a commercial bank in Pakistan. Our aim is to identify and estimate univariate time series models for the daily closing share prices of MCB. We have identified ARIMA models for the said series as mean models. The chosen estimated mean models gave the residuals which were white noise but having the ARCH effect. In literature, ARCH effect means that the time series variables or the residuals produced through the initial models show wide swings with respect to center line.

The ARCH effect or such influence is evidently persistent for long time periods. We have tried to capture this effect through different GARCH type models because high variability and high volatility has been seen in stock exchange rates, daily, weekly and monthly stock market returns, foreign exchange rates, CPI and many other variables. The existing literature describes that GARCH-type models are the better models in describing return series having the property of changing variance level. It has been tested statistically and empirically. This study is limited to the identification, estimation and diagnostic checking of the GARCH type models for MCB and to choose better models for them having maximum forecasting power.

The main idea underlying this study is to identify and estimate the mean and variance components of the daily closing share prices of the MCB through ARIMA-GARCH type models by explaining the volatility structure of the residuals obtained under the best suited mean models for the said series. The parameters of ARIMA type simple specifications are routinely anticipated by applying the OLS methodology but it has two disadvantages when the volatility or ARCH effect is present. The first problem may be the autocorrelation in error terms. To handle this unwanted situation the lagged dependent variables can be incorporated as independent variables in the mean equation. The other problem may be the presence of ARCH effect as discussed by [2].

The above stated problems can be resolved by employing the ARCH or GARCH specifications so we have taken advantage of such type of models in our study.

Literature Review: Volatility models may be of two types (1) symmetric and (2) asymmetric models. The main difference between these two classes is that symmetric models, including ARCH and GARCH do not capture leverage effects in the time-series, as opposed to the asymmetric models. Asymmetric models include exponential GARCH (EGARCH) proposed by [3], Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model proposed by [4] and threshold GARCH (TGARCH) proposed by [5] are the most popular. For details on asymmetric models see e.g. [6].

Hillmer and Tiao [7] used ARIMA technique for the seasonal adjustment as well as to introduce the decomposition the time series data into its mechanism like trend, seasonal and noise whereas such series will follow the assumption of the Gaussian ARIMA model.

Engle [8] proposed a model called ARCH model with the variation of conditional variance. In ARCH model the restricted variance depends on the previous squared error terms of different lags, even at higher lag, one can grasp the maximum of the restricted variance but a higher order indicates the model comprises of several parameters which makes the estimation work lengthy, difficult and different to intercept. Later, Bollerslev [9] proposed the GARCH model to conquer the higher order ARCH problem. The conditional variance depends on the previous squared errors and restricted variances of the GARCH model. The extension of ARCH through GARCH is like the extension of the AR to ARMA model.

Since the introduction of ARCH and GARCH these models have been extensively used in literature. Magnus and Fosu [10] modeled and forecasted volatility in GSE by taking an individual index and using the models or specifications like RW, GARCH(1,1), EGARCH(1,1) and TGARCH(1,1). Rafique and Kashif-ur-Rehman [11] studied the volatility clustering, excess kurtosis and heavy tails of the time series of KSE using ARCH, GARCH and Nelson's [12] EGARCH processes. It was found that GARCH (1,1) has done the best to fully capture the persistence in volatility. The "leverage effect" was successfully overcome by EGARCH (1,1) specification in KSE-100 index.

Rodriguez and Ruiz [15] studied the theoretical characteristics of a few and most trendy GARCH specifications having the component of leverage effect.

They compared their parameters to assure the conditions of positivity, stationarity and finite fourth order moment. Results showed that the EGARCH specification is the most flexible. The GJR specification may have important limitations if the restriction to have finite kurtosis is carried out.

Floros [14] used GARCH model and its subsequent variants for modeling and explaining volatility and financial market risk from daily observations from Egypt (CMA General Index) and Israel (TASE-100 index). Due to prices (and economy) uncertainty during the time period under considerations, Egyptian CMA index is the most volatile series.

MATERIALS AND METHODS

The stationarity of data is usually described by time plots and correlogram. The unit root test determines whether a time series is stable around its level or stable around the difference in its level. Two types of unit root tests are widely used (1) Dickey-Fuller (DF) test and (2) Augmented Dickey-Fuller (ADF) test. The most frequently used test for unit roots is the ADF-test. In this paper the ADF-test (1987) has been used.

Although in literature it is documented that for individual stocks leverage effect mostly remain insignificant as compared to significant leverage effect present in Stock market data and asymmetric models remains most popular models in estimating and forecasting volatility of Stock market index. In this study one objective is to test whether asymmetric effects are present in MCB daily closing share prices, therefore we used both of models.

ARCH Effect: ARCH effect can be tested in pre-estimation and as well as post-estimation analysis. In post-estimation, it tests remaining ARCH effect i.e. whether or not conditional heteroscedasticity has been removed. For this purpose, it is applied on standardized residuals of the fitted model. This is an LM-test for the ARCH effect in the residuals (Engle 1982). Normality tests are used to test the behavior of ARCH effect if the normality can be described by the conditional error distribution.

Another way is to inspect the autocorrelation structure of the residuals and squared residuals using portmanteau tests see e.g. [15, 16]. Portmanteau tests are used for diagnostic checking of fitted time series models. Results in literature show that Box-Pierce family of

portmanteau tests based on residual autocorrelation has poor power against non linearity see e.g. [17-19]. An indication of ARCH effect is that the residuals are uncorrelated but the squared residuals are correlated.

For GARCH models daily or intra-day returns are commonly used since the GARCH effects at lower frequencies are less apparent. To ensure that the likelihood function is well defined and that the models properly converge, a few years of data are needed, but not so many years that current market conditions are not reflected. If we take too short period data then parameter estimates may not be robust.

Estimation of GARCH models is done with the normal distribution. Pre-estimation analysis is performed on the returns and squared returns, which includes important tests applied to the two time series to ensure that conditional volatility modeling is appropriate. The main tests before actually estimating the conditional volatility are Engle's ARCH test and portmanteau tests.

Forecast Evaluation Methods: After making forecasts and choosing a proxy for actual volatility, next step is to choose statistical loss functions to see how close the forecast are to their target and compare forecast performance of models. Evaluation of performance of different volatility models is built on statistical loss functions present in literature. Statistical loss functions are based on moments of forecast errors such as root mean square error (RMSE), mean absolute error (MAE) and adjusted mean absolute percentage error (AMAPE). The best model would be the one that minimizes such a function of the forecast errors.

RESULTS AND DISCUSSION

In this section, first we study if the share prices data of MCB has any volatility structure in its variability. However to obtain a suitable model for short term forecasting of commercial bank, the Box –Jenkins methodology has been adopted to obtain ARIMA and GARCH models.

First we have tested the series for the presence of unit root. For this we have applied the ADF test to the original series X_t and differenced series ∇X_t .

The autocorrelation plots of the original series and of the first order differences were also supporting this observation. To look further into this issue, we apply the ADF test at level to the original series and the first order differenced series.

Table 2: ADF test for original and first order differenced series of MCB closing share prices.

	X_t		∇X_t	
	ADF	p-value	ADF	p-value
Intercept	-1.817	0.372	-27.8	0.000
Intercept + trend	-1.374	0.867	-27.8	0.000
No intercept+ trend	-0.359	0.555	-27.8	0.000

Table 3: Estimation and evaluation summary of tentative ARIMA models

No.	Models	Constant	AR(1)	MA(1)
1	ARIMA(1,1,0)	0.096	0.213	-
	with Drift	(0.6807)	(0.000)	(-)
2	ARIMA(0,1,1)	0.0952	-	0.197
	with Drift	(0.6645)	(-)	(0.000)
3	ARIMA(1,1,1)	0.095	0.292	-0.083
	with Drift	(0.6880)	(0.023)	(0.538)
4	ARIMA(1,1,0)	-	0.213	-
	without Drift	(-)	(0.000)	(-)
5	ARIMA(0,1,1)	-	-	0.197
	without Drift	(-)	(-)	(0.000)
6	ARIMA(1,1,1)	-	0.293	-0.083
	without Drift	(-)	(0.023)	(0.534)

The results Table 2 shows that the test fails to reject the null hypothesis of a unit root for series at level but this is rejected for the series at first difference. The results of ADF-test show that X_t series is non-stationary at level with intercept, with intercept and trend as well as with no intercept but stationary at first order differencing.

Linear Model Identification and Estimation: The tentative ARIMA models for MCB series based on the autocorrelations and partial autocorrelations would be ARIMA(1,1,0), ARIMA(0,1,1) and ARIMA(1,1,1). We estimated these models with and without drift. Results are shown in Table 3.

Model-4 and model- 5 are only two significant models. Hence the selected model for MCB closing rates series is ARIMA (1, 1, 0) with AR parameter $\phi = 0.213$.

Identification of ARCH and GARCH Effect: For the identification of ARCH effect in residuals, we use the correlogram for the squared errors obtained under the estimated ARIMA model and the structure of residuals and squared residuals is examined. If the ARCH effect exists in the error terms then they must be uncorrelated with each other meaning but the squared residuals show significant autocorrelations. The correlogram of squared

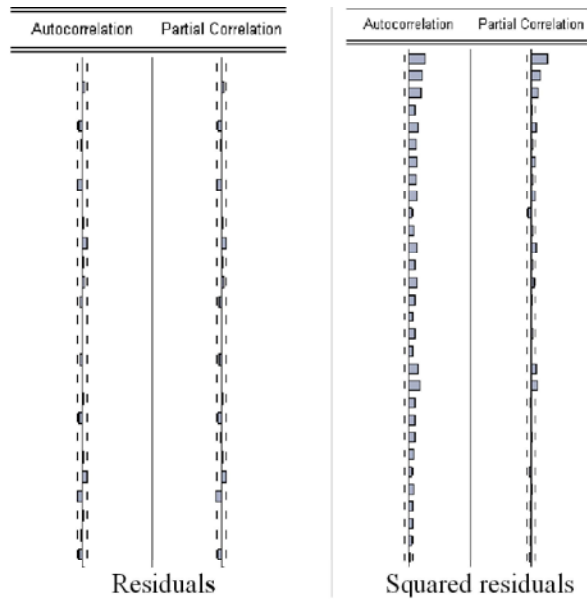


Fig. 3: Autocorrelation and partial autocorrelation function of residuals and squared residuals

Table 4: Model estimates for mean and variance components.

Models	Mean	Variance	
	AR(1)	ARCH	GARCH
AR(1) with	0.1098	0.4607	-
ARCH(1)	(0.000)	(0.000)	(-)
AR(1) with	0.1748	0.1790	0.822
GARCH(1,1)	(0.000)	(0.000)	(0.000)

Table 5: Model selection criterion

Models	AIC	SIC	LL
AR(1) with ARCH(1)	6.398	6.410	-3829.11
AR(1) with GARCH(1,1)	6.182	6.199	-3699.06

residuals shows a clear picture of absence or presence of ARCH effect. This model suggested that the variation of residuals the time depends on the squared terms of error for the past period.

Figure 3 shows that the residuals generated by model-4 are white noise. It is also very clear that various spikes of ACF and PACF of squared residuals are beyond the limits showing that the residuals under consideration have ARCH effect. Further results show that modeling using GARCH(1,1) results in residuals having no significant spike in the autocorrelation and partial autocorrelation plots. Thus, we can say that the GARCH(1,1) model is successful for capturing the ARCH effect.

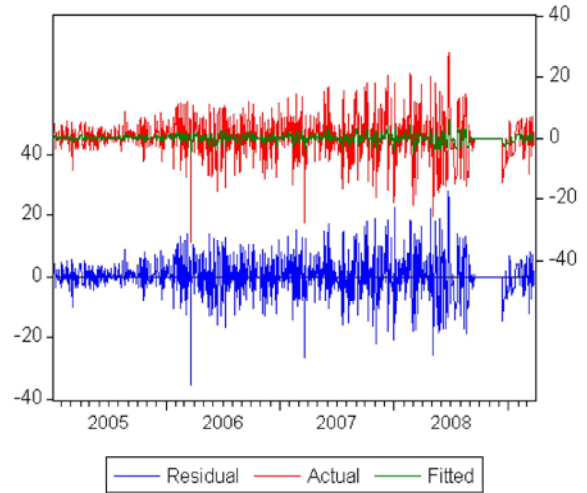


Fig. 4: Actual, fitted and residuals under ARIMA(1,1,0) with GARCH(1,1) model

Now we use model selection criteria to pick the final model among the models which have passed through the stage of diagnostic stage. We used Shawartz Information Criterion (SIC), Bayesian Information Criterion (BIC) and Log-likelihood (LL). The results are given in Table.

We have estimated ARCH (1) and GARCH (1, 1) models. Both AIC and SIC are minimum for GARCH (1, 1) model but ARCH (1) model has maximum value of LL. At the next stage we have to examine that which one of these two models has fully captured the ARCH effect.

All above diagnostic tests and helping graphs and correlograms give clear cut indication that GARCH (1,1) model with mean equation AR(1) is the most appropriate model among all the proposed models.

Now we examine forecasting ability out of sample from the fitted models. Different forecasting errors such as, RMSE, MAE, MAPE and Theil's U inequality and further collection criteria, have been presented in the following figures. The value of Theil's U statistic lie between 0 and 1. The value of Theil's U statistic if near to 1 points out the model is not fine fit and zero shows that model is fine fit and can be used for forecasting.

All the statistics, shown in above table, are in favor of GARCH (1,1) model except MAE and MAPE which is favor of EGARCH(1,1) model. In case of TGARCH model the threshold component is insignificant. From these statistics, it is very clear that GARCH(1,1) is the most suitable model among the models considered in this study.

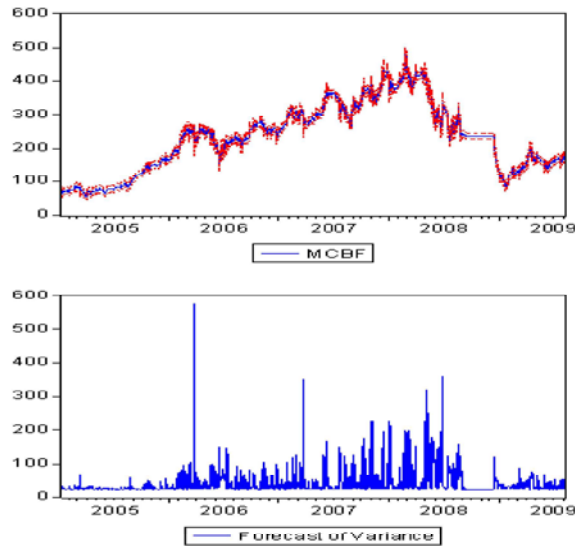


Fig. 5: Forecasts under for AR(1) under GARCH errors

Table 6: Forecast Evaluations

	GARCH	TGARCH	EGARCH
Adj.R ²	0.041462	0.040529	0.041074
SE	6.353686	6.356778	6.354973
LL	-3699.68	-3698.819	-3714.001
AIC	6.182084	6.183337	6.207014
SIC	6.199073	6.204574	6.224003
DW	1.929459	1.926058	1.919661
RMSE	6.343070	6.343499	6.344355
MAE	4.407178	4.406959	4.406552
MAPE	2.047237	2.047217	2.047183
TIC	0.012582	0.012583	0.012585

Forecast Summary and Evaluation: We have shown GARCH (1,1) model is superior fit for forecasting the daily MCB closing share prices as compared to ARCH(1) model. Theil's Inequality coefficient's value for GARCH(1,1) is less than that of ARCH(1) model i.e. $0.012582 < 0.012643$ and is very close up to zero which is an evidence of good fit of "GARCH (1, 1) model". Every new statistics for GARCH(1,1) model are clearly better than that of ARCH(1) model.

CONCLUSION

We estimated various ARIMA models for mean and ARCH type models for process variance through simultaneous estimation procedure of MLE for both of the time series under study. After fitting various ARCH type models each time the generated residuals have been examined by using the correlogram of squared residuals,

Q-statistics at different lags and the application of ARCH-LM test. All these techniques show that ARCH(1) model has failed to fully capture the ARCH effect from the residuals generated by the mean equation i.e. ARIMA(1,1,0). The model ARIMA(1,1,0)-GARCH(1,1) has fully captured the ARCH effect and left not any ARCH effect in the residuals and hence by using various goodness of fit tests as well as forecast evaluation criteria this model proves to be the best model among all competing models for MCB series.

GARCH (1,1) has better ability of capturing the volatility clustering among all estimated ARCH-type models in case of MCB data. For the daily closing share prices data set GARCH (1,1) model is the best among several considered models.

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REFERENCES

1. Brockwell, P.J. and R.A. Davis, 2002. Introduction to Time Series and Forecasting, 2nd edition, Springer texts in Statistics, pp: 469.
2. Berument, H., M.N. Coşkun and A. Şahin, 2006. Day of the week effect on foreign exchange market volatility: Evidence from Turkey. Research in International Business and Finance, 21: 87-97.
3. Nelson, D.B., 1991. Conditional Heteroskedasticity in assets returns: A New Approach. Econometrica, 59: 347-370.
4. Glosten, L.R., R. Jagannathan and D.E. Runkle, 1993. On the relation between expected value and the volatility of nominal excess return on stocks. The Journal of Finance, 48: 1779-1801.
5. Zakoian, J.M., 1994. Threshold Heteroskedastic Models, Journal of Economic Dynamics and Control, 18: 931-955.
6. Franses, P.H. and D. Van Dijk, 2000. Nonlinear Time Series Models in Empirical Finance. Cambridge University Press, pp: 289.
7. Hillmer, S.C. and G.C. Tiao, 1982. An ARIMA-Model based Approach to Seasonal Adjustment. Journal of the American Statistical Association, 77: 63-70.
8. Engle, R., 1982. Autoregressive Conditional Heteroskedasticity with Estimate of Variance of U.K Inflation. Econometrica, 50: 987-1008.

9. Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31: 307-328.
10. Magnus, F.J. and O. Fosu, 2006. Modeling and Forecasting Volatility of on Returns on the Ghana Stock Exchange Using GARCH Models *Journal of Applied Sci.*, 3: 2042-2048.
11. Rafique, A. and Kashif-ur-Rehman, 2011. Comparing the persistency of different frequencies of stock returns volatility in an emerging market: A case study of Pakistan. *African Journal of Business Management*, 5: 59-67.
12. Nelson, J.R., 1990. Evaluation report of a national mainstreaming project. Paper presented at the Montana Conference on Special Education, Billings, MT.
13. Rodriguez, M.J. and E. Ruiz, 2009. GARCH Models with leverage effect: Difference and Similarities, Working Paper, Statistics and Econometrics Series 02 January 2009, Departamento de Estadística Universidad, Spain.
14. Floros, C., 2008. Modeling Volatility using GARCH MODELS: Evidence from Egypt and Israel, *Middle Eastern Finance and Economics*, 2: 31-41.
15. Granger, C.W.J. and A.P. Andersen, 1978. Introduction to Bilinear Time Series Models, Vandenhoeck and Ruprecht, Göttingen.
16. Ljung, G.M. and G.E.P. Box, 1978. On a Measure of Lack of Fit in Time Series Models, *Biometrika*, 65: 297-303.
17. Escanciano, J.C., 2006. Goodness of Fit Tests for Linear and Nonlinear Time Series Models, *Journal of American Statistical Association*, 101: 531-541.
18. Chand, S., 2011. Goodness of Fit Tests and Lasso Variable Selection in Time Series Analysis, Unpublished PhD Thesis, University of Nottingham, United Kingdom.
19. Chand, S. and S. Kamal, 2006. A Comparative Study of Portmanteau Test for Univariate Time Series Models. *Pakistan Journal of Statistics and Operation Res.*, 2: 111-114.