

Optimal Synthesis of Planar Mechanisms with PSO Algorithms

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Abstract: This paper studies the solution methods of optimal synthesis of planar mechanisms. The method is defined by using of pso techniques and a common kind of goal function which is used to find the appropriate dimension and to minimize the error and find the best mechanism with accurate Solution. The possibility of extending is the advantage of this method. Unlike others we do not consider input angle as design variable, because in those cases when many precision points are available, computation will increase without having exact solution. So we divided the path to some section and find minimum error between desired points and design points. Using this method, we can easily decrease the path error and processing time.

Key words: Planar mechanism • PSO algorithm • Path generation • Optimal Synthesis

INTRODUCTION

Mechanisms which compose some connected rigid members are exclusively used in the area of mechanical engineering to transfer energy from one member to another. Hrones and Nelsone [1] improve the atlas of mechanisms with a lot of curves and use these curves to solve the mechanism problems. These methods are easy and fast to use but offer a low precision rate. Dimensional synthesis can be classified as motion generation, function generation and path or trajectory generation. Both graphical and analytical methods have been used for dimensional synthesis [2-5]. Using precision points which are traced by a mechanism is also classified but such methods are relatively restrictive because of their low precision rates and cannot be used when we have variety of precision points mechanism [6,7].

By increasing the power of computers, numerical methods are used commonly to minimize the goal function. Han [8] studied this method first and then Kramer and Sandor [9] and Sohoni and Hung [10] went on his approach. They used some optimization method to optimize the goal function, the error between the points traced by the coupler and its desired trajectory. Multi objective techniques are studied by Roa and Kaplan [11]

and Krishnamurty and Turcic [12]. Moreover, some of the researchers are used analytical and numerical methods for solving nonlinear functions [13-17]. But all the solutions have a disadvantage of failing if the solutions appear in a local minimum.

In this paper the approach presented to the synthesis of mechanisms deals with PSO algorithm and we can compare it with other solutions like the method Fang and Kunjur and Krishnamurty have presented by evolutionary techniques [18-20].

Solution

Particle Swarm Optimization: Particle swarm optimization (PSO), is based on the behavior of a colony or swarm of insects. The PSO algorithm mimics the behavior of these social organisms. The word particle denotes a bee in a colony or a bird in a flock. Each individual or particle in a swarm behaves in a distributed way using its own intelligence and the collective or group intelligence of the swarm. As such, if one particle discovers a good path to food, the rest of the swarm will also be able to follow the good path instantly even if their location is far away in the swarm. The PSO algorithm was originally proposed by Kennedy and Eberhart in 1995 [21]. Algorithm composed of two important parts which described as below.

New Velocity: In canonical version of PSO, each particle is moved by two elastic forces. One attracting it with random magnitude to the fittest location so far encountered by the particle and one attracting it with random magnitude to the best location so far encountered by the best location [22]. If the problem is n-dimensional, each particle as position and velocity can be represented as a vector with n components. The following alternative velocity-update equation was developed [23] in the following equation:

$$V_{i+1} = wV_i + c_1 \text{rand}(n_1)(P_{pbi} - x_i) + c_2 \text{rand}(n_2)(P_{gb} - x_i) \quad (1)$$

Where V_i is the velocity of particle, P_{pbi} is the best position of particle, P_{gb} is the best position in all particle, x_i is the position of particle, w is constant hardness, c_1 and c_2 are constant, n_1 and n_2 are two random value.

New Position: The same formula is used independently for each dimension of the problem and synchronously for all particles. The position of a particle is updated every time step using the following equation.

$$X_{ik} = X_{ik-1} + V_{ik} \quad (2)$$

Classification: The most important part of solution is classification [24]. Sometimes we cannot design a real mechanism because the trajectory is traced by an imaginary mechanism which has two branches. Thus, we should disassemble and assemble the mechanism for tracing the path. By classifying we can defeat the problem. Consider the planar mechanism shown in Fig. 1. The relationship between the input angle (θ) of driving link to the output angle of the output link is:

$$\psi(\theta) = \arctan\left(\frac{B}{A}\right) \pm \arccos\left(\frac{C}{\sqrt{A^2 + B^2}}\right) + \pi \quad (3)$$

$$\begin{aligned} A(\theta) &= 2ab \cos \theta - 2gb, B(\theta) = 2absin \theta \\ c(\theta) &= g^2 + b^2 + a^2 + h^2 - 2ga \cos \theta, B(\theta) = ab \sin \theta \end{aligned} \quad (4)$$

The argument of the arccosine term in Eq. (3) must be in the range of -1 to +1 for a solution to exist. Therefore, $A(\theta)^2 + B(\theta)^2 - C(\theta)^2 \geq 0$ and this relationship defines the range of the angular movement of the input link. Expanding the inequality yields a quadratic equation in $\cos(\theta)$ that has two roots.

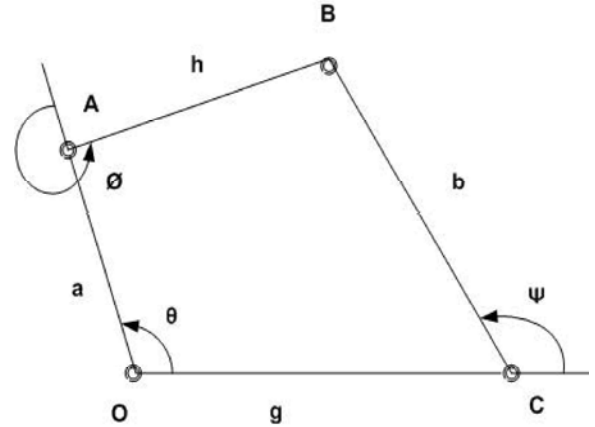


Fig. 1: A schematic of the planar mechanism

$$c_1 = \frac{(g^2 + a^2) - (h - b)^2}{2ag} \quad (5)$$

$$c_2 = \frac{(g^2 + a^2) - (h + b)^2}{2ag} \quad (6)$$

The Input Link: Introducing the parameters $T_1 = g - a + h - b$ and $T_2 = g - a - h + b$ and $T_3 = h + b - g - a$ then $T_1 T_2 = (g - b)^2 - (h - b)^2$ so the input link can pass $\theta = 0$ if $T_1 T_2 \geq 0$. Angle θ_1 generated by the root C_1 is the smallest positive angle of input link and angle θ_2 generated by the root C_2 defines the biggest positive angle of input link.

Parameters $\{T_i\}_{i=1}^3$ classify the rotation of input link:

- Input link fully rotates if $T_1 T_2 \geq 0$ and $T_3 \geq 0$.
- Input link rocks through $\theta = 0$ if $T_1 T_2 \geq 0$ and $T_3 \geq 0$.
- Input link rocks through $\theta = 0$ if $T_1 T_2 \geq 0$ and $T_3 \geq 0$.

Algorithms

First Algorithm: In this method like other algorithms, design variable depends on the numbers of desired points which we want to be traced. There are 9 independent geometrical variables to define a four bar mechanism:

These are: $\{a_1, a_2, a_3, a_4, x_0, y_0, \theta, \phi, \gamma\}$ which are shown in Fig. 2.

So estimation increases by increasing the number of points. Objective function [25] for optimizing is also defined as:

$$Fobj = \left(\sum_{i=1}^n (X_{pi} - X_{di})^2 \right)^{1/2} \quad (7)$$

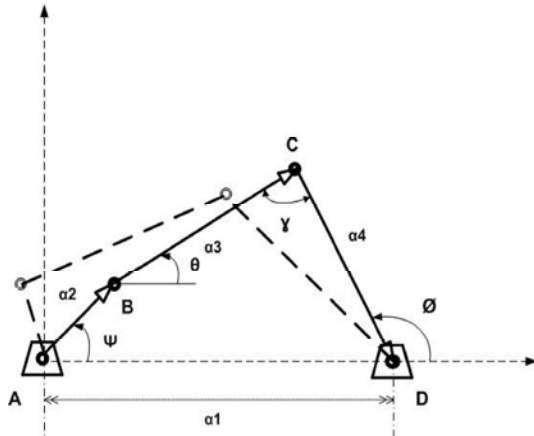


Fig. 2: Schematic of the mechanism

X_{pi} denotes precision points or points that we like to be traced by the mechanism and X_{di} denotes desired points or real points that coupler trace them.

Second Algorithm: Accuracy of optimization algorithms depends on design space. Decreasing of design variables results higher accuracy. At the beginning we need a big population and to convergence more iteration are done. Disadvantage of first algorithm is amount of design variable which is increase when have many points (more than 10 points). In this case we cannot design a mechanism with minimum error. To defeat this problem we do not consider target points as design variable and by imposing some constraints the optimization algorithm guess a mechanism randomly. This mechanism creates a path in space. After that we divided the path into smaller parts and find the minimum error between each point and parts. By repeating this action for all iterations the error decreased and an optimum result found.

RESULTS AND DISCUSSIONS

This section analyzes a set of results found when applying the algorithm described in the previous sections. All examples were programmed on a Pentium 6 and implemented in Matlab. As described, a set of target points is first input by the designer to define the problem. Here we compare the behavior of mechanisms by using PSO algorithm and we will compare the differences between each method which has described before. For all cases, the final error is computed by a definite goal function of each method.

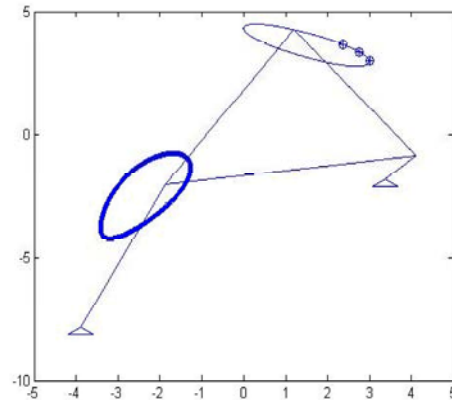
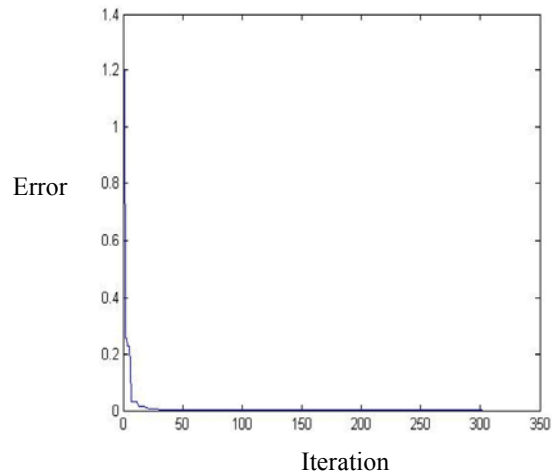


Fig. 3: Schematic of the optimized mechanism with three points First branch (-), second branch (-)



Example 1: Target points: $P_1 = (3,3)$, $P_2 = (2.75,3.36)$, $P_3 = (3.37,3.66)$,

Parameters of Algorithm: $C_1 = 1$, $C_2 = 2$, $W=0.4$, particle=100, iteration=300

By using second algorithm, optimized design variables are:

$$\begin{aligned} r_1 &= 9.425, r_2 = 1.228, r_3 = 6.105, r_4 = 6.124 \\ \alpha x &= 3.378, \alpha y = -1.838 \\ c x &= 1.848, c y = -5.584, \theta_o = -2.452 \end{aligned}$$

We can observe in Fig. 3 that input link has a full rotation to trace the path and there are two separate branches and we need disassembling to transfer from one into another. Below (Fig. 4) show the error between the desired points and actual points which is 4.396×10^{-6} in the last iteration.

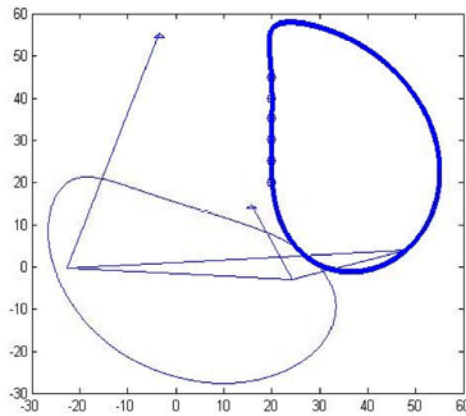


Fig. 5: Schematic of the optimized mechanism with six points First branch (-), second branch (-).

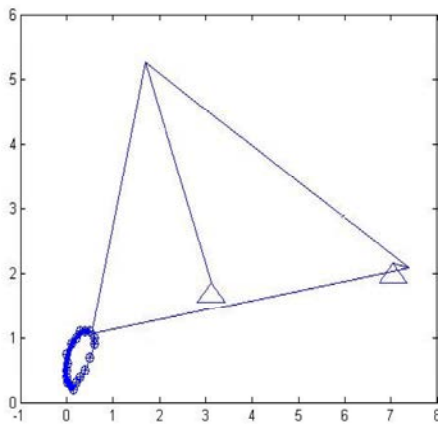


Fig. 6: Schematic of the optimized mechanism with 18 points first branch (-), second branch (-)

Example 2: Target Points:

$$\begin{aligned} P_1 &= (20, 20) & P_2 &= (20, 25) & P_3 &= (20, 30) \\ P_4 &= (20, 35) & P_5 &= (20, 40) & P_6 &= (20, 45) \end{aligned}$$

Parameters of Algorithm: $C_1 = 1$, $C_2 = 2$, $W = 0.4$, particle number=100, iteration=300, $n=10000$

By using second algorithm, optimized design variables are:

$$\begin{aligned} r_1 &= 48.145 & r_2 &= 37.106 & r_3 &= 55.351, & r_4 &= 20.645 \\ o_x &= -46.176 & o_y &= 37.152 \\ c_x &= -14.664 & c_y &= 28.08 \end{aligned}$$

Six points are straight and coupler should trace them. As Fig. 5 shows there are two separate branches again and all desired points are located in one branch so we do

not need disassemble the mechanism. Cabera and Simson [26] did the same problem with 0.0261 error, but using of this method decreases the error down to 0.0035.

Example 3: This example indicates the efficiency of the second algorithm compared with first algorithm and all other methods. This efficiency increases when target points increase as described before. Here are 18 points to be traced. These points are:

$$\begin{aligned} P_1 &= (0.5, 1.1), & P_2 &= (0.4, 1.1), & P_3 &= (0.3, 1.1), \\ P_4 &= (0, 2.1), & P_5 &= (0.1, 0.9), & P_6 &= (0.005, 0.75) \\ P_7 &= (0.02, 0.6), & P_8 &= (0, 0.5), & P_9 &= (0, 0.4), \\ P_{10} &= (0.03, 0.3), & P_{11} &= (0.1, 0.25), & P_{12} &= (0.15, 0.2) \\ P_{13} &= (0.2, 0.3), & P_{14} &= (0.4, 0.5), & P_{15} &= (0.4, 0.5), \\ P_{16} &= (0.5, 0.7), & P_{17} &= (0.6, 0.9), & P_{18} &= (0.6, 1) \end{aligned}$$

Parameters of algorithm: $C_1 = 1$, $C_2 = 2$, $W = 0.4$, particle number=100, iteration=300, $n=50$.

By using second algorithm, optimized design variables are:

$$\begin{aligned} r_1 &= 3.93, & r_2 &= 3.69, & r_3 &= 6.51, & r_4 &= 0.352 \\ o_x &= 3.126 & o_y &= 1.846 \\ c_x &= 1.035 & c_y &= -4.224 & \theta_o &= 0.0785 \end{aligned}$$

We can see that the design mechanism (Figure 6) has two branches and the mechanism can reach each branch without disassembling. Thus, some points are traced in the first branch and the rest are traced in the second one. These characteristics make the method different from others. It should be noted that the mechanism transfer from one branch into another by passing through death point. Cabera and Simon [26] did the same problem with error 4.3×10^{-2} and error decrease down to 2.6×10^{-2} by using first algorithm. By using second algorithm we have the least error which is 8.2×10^{-3} .

CONCLUSIONS

The paper presents a new method based on PSO algorithms for path synthesis of mechanisms. In this method, which we can find out the simplicity, an algorithm is used to solve the optimization problems. The creative idea of dividing the path into smaller parts and finding the minimum error of each part from the points made the problem easier and help us to decrease the error. specially

when the number of precision points increases, convergence occurs earlier. The superiority of the obtained mechanisms was shown in comparison with those recently reported. Three cases are solved and compared with the same problems which are solved by other algorithms. The result has more accurate solution and faster convergence. The advantage of this method appears when we have lots of points and variety of mechanism for tracing the path as mentioned in case 3 with 18 points. Another advantage of this method beside the previous one is the ability of this procedure to discriminate the branches in each iteration.

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