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A Solution Method for Solving Systems of Nonlinear PDEs

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Abstract: In this study, Reduced Differential Transform Method (RDTM) is applied to some systems of nonlinear PDEs. The concept of RDTM is introduced birefly and RDTM is examined for system of nonlinear PDEs. The main advantage of the RDTM is the fact that it provides to its user an analytical approximation, in many cases an exact solution, in a rapidly convergent sequence with elegantly computed terms. Results obtained by using RDTM are compared with the exact solution of given systems and the results clearly reveal that reliability and efficiency of the RDTM. Also, results indicate that introduced method is promising for solving other type systems of nonlinear PDEs.

Key words: Reduced differential transform method . system of nonlinear PDEs . brusselator system . two-dimensional burger's equation . homotopy perturbation method

INTRODUCTION

Differential equations theory is an important mathematical branch which is used to describe practical problems in physics, chemistry, biology and so on [1]. It is well known that many phenomena in scientific fields such as reaction-diffusion process, population growth, solid physics, fluid dynamics, mathametical biology and chemical kinetics, can be modelled by systems of linear or nonlinear PDEs.

In order to understand and analyze these phenomenas well, it is need to know solution of systems of these linear or nonlinear PDEs. So, it is a crucial work to obtain solutions of systems of linear or nonlinear PDEs in the science. With this idea; scientists and mathematicians have developed and searched some methods such as Hirota bilinear method [2], Exp-function method [3], tanh method [4], sine-cosine method [5], Galerkin method [6] and Differential transform method (DTM) [7, 8].

It is more difficult to obtain solutions of nonlinear PDEs than those of linear differential equations [9]. Therefore, it may not always be possible to obtain analytical solutions of these equations. In this case, it is used semi-analytical methods giving series solutions. In these kinds of methods, the solutions are sought in the form of series [9]. Semi-analytical methods are based on finding the other terms of the series from given initial conditions for the problem being considered [9]. At this point, it is encountered the concept of convergence of the series. So, it is necessary to perform convergence analysis of these methods. As this convergence analysis can be carried out theoretically, one can gain information about the convergence of the series solution by looking at the absolute error between the numerical solution and the analytical solution. In some semi-analytic methods, a very good convergence can be achieved with only a few terms of the series, but more terms can be needed in some problems. That is, if the terms of the series increase, this provides better convergence to the analytical solution [9].

The concept of Differential transform was first introduced by Zhou [7, 8], who solved linear and non-linear initial value problems in electric circuit analysis. Later, DTM was applied to many problems in the literature by several authors [10-13].

Recently, Keskin et al. introduced a reduced form of DTM as reduced DTM (RDTM) and applied to approximate some PDEs [14] and fractional PDEs [15]. More recently, Abazari and Malek [16] extended RDTM to generalized Hirota-Satsuma coupled KdV equation and P.K. [17] Gupta applied RTDM to Fractional Benney-Lin equation. The reduced differential transform recursive equations produce exactly all the Poisson series coefficients of solutions, whereas the differential transform recursive equations produce exactly all the Taylor series coefficients of solutions [9].

The solution procedure of the RDTM is simpler than that of traditional DTM and the amount of computation required in RDTM is much less than that in traditional DTM. The solution obtained by the

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reduced differential transform method is an infinite power series for initial value problems, which can be, in turn, expressed in a closed form, the exact solution

In this study, we introduced main properties of RDTM and applied the RDTM to some systems of nonlinear PDEs such as Brusselator system and two-dimensional Burger's equation. Later, we compared the obtained results with the exact solution and Homotopy Perturbation Method solutions (HPM) [18, 19].

REDUCED DIFFERENTIAL TRANSFORM METHOD

Consider a function of two variables u (x,t) and suppose that it can be represented as a product of two single-variable functions, i.e., u(x,t) = f(x)g(t). Based on the properties of one-dimensional differential transform, the function u (x,t) can be represented as

$$u(x,t)=\left(\sum_{i=0}^{\infty}F(i)x^{i}\right)\left(\sum_{j=0}^{\infty}G(j)t^{j}\right)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}U(i,j)x^{i}t^{j} \quad (1)$$

where U(i,j) = F(i)g(j) is called spectrum of u (x,t). The basic definitions of reduced differential transform method [20, 21] are introduced as follows:

Definition 2.1: If a function u (x,t) is analytic and differentiated continuously with respect to time t and space x in the domain of interest then

$$U_k(x)=\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0} \quad k\geq 0, k\in N \quad (2)$$

where the t-dimensional spectrum function U_k (x) is the transformed function and u (x,t) is the original function.

Definition 2.2: The differential inverse transform of U_k (x) is defined as follows:

$$u(x,t)=\sum_{k=0}^{\infty}U_k(x)\cdot t^k \quad (3)$$

From Eq. (2) and Eq. (3), we get

$$u(x,t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{\partial^k}{\partial t^k}u(x)\right]_{t=0} \quad (4)$$

The following rules in Table 1 can be deduced from Eq. (2) and Eq. (3).

APPLICATIONS

Example 1: Consider the following system of two non-linear equations [19]:

Table 1: Fundamental operations of reduced differential transform method

Original function	Transformed function
$f(x,y,t)=\alpha g(x,y,t)\pm\beta h(x,y,t)$	$F(x,y)=\alpha G(x,y)\pm\beta H(x,y)$
$f(x,y,t)=x^a y^b t^n$	$F_k(x,y)=x^a y^b\delta(k-n)$, $\delta(k)=\begin{cases}1, & k=0\\ 0, & k\neq 0\end{cases}$
$f(x,y,t)=g(x,y,t)\cdot h(x,y,t)$	$F_k(x,y)=\sum_{r=0}^{k}G_r(x,y)\cdot H_{k-r}(x,y)$
$f(x,y,t)=\frac{\partial^n}{\partial t^n}g(x,y,t)$	$F_k(x,y)=(k+1)(k+2)\cdots(k+n)G_{k+n}(x,y)$
$f(x,y,t)=\frac{\partial^{n+m}}{\partial x^n\partial y^m}g(x,y,t)$	$F_k(x,y)=\frac{\partial^{n+m}}{\partial x^n\partial y^m}G_k(x,y)$
$f(x,y,t)=x^a y^b t^n g(x,y,t)$	$F_k(x,y)=x^a y^b G_{k-n}(x,y)$
$f(x,y,t)=\sin(\alpha x+\gamma y+\beta t)$	$F_k(x,y)=\left(\frac{\beta^k}{k!}\right)\cdot\sin\left(\frac{k\pi}{2}+\alpha x+\gamma y\right)$
$f(x,y,t)=\cos(\alpha x+\gamma y+\beta t)$	$F_k(x,y)=\left(\frac{\beta^k}{k!}\right)\cdot\cos\left(\frac{k\pi}{2}+\alpha x+\gamma y\right)$

$$\begin{aligned}&\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial t}+u\frac{\partial v}{\partial t}=-1+e^{x}\sin t\\&\frac{\partial v}{\partial x}+\frac{\partial u}{\partial t}\frac{\partial v}{\partial x}+\frac{\partial v}{\partial t}\frac{\partial u}{\partial x}=-1-e^{-x}\cos t\end{aligned} \quad (5)$$

with boundary conditions

$$\begin{aligned}&u(x,0)=0\\&v(x,0)=e^{-x}\end{aligned} \quad (6)$$

The exact solutions of this system are

$$u(x,t)=e^{x}\sin t \quad (7)$$

$$v(x,t)=e^{-x}\cos t \quad (8)$$

Applying the related rules in Table 1, the recurrence relation will be as follows:

$$\begin{aligned}&\frac{\partial}{\partial x}U_k(x)-\sum_{r=0}^{k}V_r(x)(k+1)U_{k+1-r}(x)\\&+\sum_{r=0}^{k}U_r(x)(k+1)V_{k+1-r}(x)=-\delta(k)+\frac{e^{x}}{k!}\sin(\frac{k\pi}{2})\\&\frac{\partial}{\partial x}V_k(x)+\sum_{r=0}^{k}(k+1)U_{k+1-r}(x)\frac{\partial}{\partial x}V_r(x)\\&+\sum_{r=0}^{k}\frac{\partial}{\partial x}U_r(x)(k+1)V_{k+1-r}(x)=-\delta(k)-\frac{e^{-x}}{k!}\cos(\frac{k\pi}{2})\end{aligned} \quad (9)$$

From Eq. (2), the initial conditions given in Eq. (6) can be transformed at t = 0 as

$$U_0(x) = 0,\ V_0(x) = e^{-x} \tag{10}$$

Using Eqs. (9-10) and inverse transformation Eq. (3), for k = 0,1,2,...,n,..., we get approximate solutions of (5) as:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k = e^{x}\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - ...\right)$$

which is the taylor series of Eq. (7)

$$v(x,t) = \sum_{k=0}^{\infty} V_k(x)t^k = e^{-x}\left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!} - ...\right)$$

which is the taylor series of Eq. (8).

Example 2: Consider the following system of three-dimensional PDEs [19]:

$$\begin{aligned} &\frac{\partial u}{\partial t} - v\frac{\partial u}{\partial x} - \frac{\partial v}{\partial t}\frac{\partial u}{\partial y} = -1 - x + y + t \\ &\frac{\partial v}{\partial t} - u\frac{\partial v}{\partial x} - \frac{\partial u}{\partial t}\frac{\partial v}{\partial y} = -1 - x - y - t \end{aligned} \tag{11}$$

with initial conditions

$$\begin{aligned} u(x,y,0) &= x + y - 1 \\ v(x,y,0) &= x - y + 1 \end{aligned} \tag{12}$$

The exact solutions are

$$u(x,y,t) = x + y + t - 1 \tag{13}$$

$$v(x,y,t) = x - y - t + 1 \tag{14}$$

Applying the related rules in Table 1, the recurrence relation will be as follows:

$$\begin{aligned} &(k+1)U_{k+1}(x,y) - \sum_{r=0}^{k} V_r(x,y)\frac{\partial}{\partial x}U_{k-r}(x,y) \\ &-\sum_{r=0}^{k}(k+1)V_{k+1-r}(x,y)\frac{\partial}{\partial y}U_r(x,y) = \delta(k) - x + y + \delta(k-1) \\ &(k+1)V_{k+1}(x,y) - \sum_{r=0}^{k} U_r(x,y)\frac{\partial}{\partial x}V_{k-r}(x,y) \\ &-\sum_{r=0}^{k}(k+1)U_{k+1-r}(x,y)\frac{\partial}{\partial y}V_r(x,y) = \delta(k) - x - y - \delta(k-1) \end{aligned} \tag{15}$$

From Eq. (2), the initial conditions given in Eq. (12) can be transformed at t = 0 as:

$$\begin{aligned} U_0(x,y) &= x + y - 1 \\ V_0(x,y) &= x - y + 1 \end{aligned} \tag{16}$$

Using Eqs. (15-16) and inverse transformation Eq. (3), for k = 0,1,2,...,n,..., we get approximate solutions of (11) as:

$$u(x,y,t) = \sum_{k=0}^{\infty} U_k(x,y)t^k = x + y + t - 1$$

$$v(x,y,t) = \sum_{k=0}^{\infty} V_k(x,y)t^k = x - y - t + 1$$

which are exact solutions of Eq. (11).

Example 3: Consider the two-dimensional Burger's equation [18]:

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \\ &\frac{\partial v}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \end{aligned} \tag{17}$$

subject to the initial contions

$$u(x,y,0) = x + y,\ \ v(x,y,0) = x - y$$

$$(x,y,t) \in R^2 \times [0, \frac{1}{\sqrt{2}}] \tag{18}$$

The exact solutions are

$$u(x,y,t) = \frac{x + y - 2xt}{1 - 2t^2} \tag{19}$$

$$v(x,y,t) = \frac{x - y - 2xt}{1 - 2t^2} \tag{20}$$

Applying the related rules in Table 1, the recurrence relation will be as follows:

$$\begin{aligned} &(k+1)U_{k+1}(x,y) + \sum_{r=0}^{k} U_{k-r}(x,y)\frac{\partial}{\partial x}U_r(x,y) \\ &+\sum_{r=0}^{k} V_{k-r}(x,y)\frac{\partial}{\partial y}U_r(x,y) = \frac{\partial^2}{\partial x^2}U_k(x,y) + \frac{\partial^2}{\partial y^2}U_k(x,y) \\ &(k+1)V_{k+1}(x,y) + \sum_{r=0}^{k} U_{k-r}(x,y)\frac{\partial}{\partial x}V_r(x,y) \\ &+\sum_{r=0}^{k} V_{k-r}(x,y)\frac{\partial}{\partial y}V_r(x,y) = \frac{\partial^2}{\partial x^2}V_k(x,y) + \frac{\partial^2}{\partial y^2}V_k(x,y) \end{aligned} \tag{21}$$

From Eq. (2), the initial conditions given in Eq. (18) can be transformed at t = 0 as:

$$\begin{aligned} U_0(x,y) &= x + y \\ V_0(x,y) &= x - y \end{aligned} \quad (22)$$

Using Eqs. (21-22) and inverse transformation Eq. (3), for k = 0,1,2,...,n,..., we get approximate solutions of (17) as:

$$\begin{aligned} u(x,y,t) &= \sum_{k=0}^{\infty} U_k(x,y)t^k \\ &= x + y - 2xt + (2x+2y)t^2 - 4xt^3 + (2x+2y)t^4 - 8xt^5 + \ldots \\ v(x,y,t) &= \sum_{k=0}^{\infty} V_k(x,y)t^k \\ &= x - y - 2yt + (2x-2y)t^2 - 4yt^3 + (2x-2y)t^4 - 8yt^5 + \ldots \end{aligned} \quad (23)$$

And with aid of some algebraic-sembolic compution tool, the solutions in closed forms are given by

$$u(x,y,t) = \frac{x + y - 2xt}{1 - 2t^2}$$

and

$$v(x,y,t) = \frac{x - y - 2yt}{1 - 2t^2}$$

which are the exact solution of the Eq. (17).

Example 4: Consider the following non-linear system of inhomogeneous PDEs [19]:

$$\frac{\partial u}{\partial t} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial t} - \frac{1}{2}\frac{\partial w}{\partial t}\frac{\partial^2 u}{\partial x^2} = -4xt$$

$$\frac{\partial v}{\partial t} - \frac{\partial w}{\partial t}\frac{\partial^2 u}{\partial x^2} = 6t \quad (24)$$

$$\frac{\partial w}{\partial t} - \frac{\partial^2 u}{\partial x^2} - \frac{\partial v}{\partial x}\frac{\partial w}{\partial t} = 4xt - 2t - 2$$

subject to the initial conditions

$$u(x,0) = x^2 + 1,\ v(x,0) = x^2 - 1, w(x,0) = x^2 - 1 \quad (25)$$

The exact solutions are

$$u(x,t) = x^2 - t^2 + 1 \quad (26)$$

$$v(x,t) = x^2 + t^2 - 1 \quad (27)$$

$$w(x,t) = x^2 - t^2 - 1 \quad (28)$$

Applying the related rules in Table 1, the recurrence relation will be as follows:

$$\begin{aligned} &(k+1)U_{k+1}(x) - \sum_{r=0}^{k}(k+1)V_{k+1-r}(x)\frac{\partial}{\partial x}W_r(x) \\ &\quad - \frac{1}{2}\sum_{r=0}^{k}(k+1)W_{k+1-r}(x)\frac{\partial^2}{\partial x^2}U_r(x) = -4x\delta(k-1) \\ &(k+1)V_{k+1}(x) - \sum_{r=0}^{k}(k+1)W_{k+1-r}(x)\frac{\partial^2}{\partial x^2}U_r(x) = 6\delta(k-1) \\ &(k+1)W_{k+1}(x) - \frac{\partial^2}{\partial x^2}U_k(x) - \sum_{r=0}^{k}(k+1)W_{k+1-r}(x)V_r(x) \\ &= (4x-2)\delta(k-1) - 2\delta(k) \end{aligned} \quad (29)$$

From Eq. (2), the initial conditions given in Eq. (25) can be transformed at t = 0 as:

$$U_0(x) = x^2 + 1,\ V_0(x) = x^2 - 1, W_0(x) = x^2 - 1 \quad (30)$$

Using Eqs. (29-30) and inverse transformation Eq. (3), for k = 0,1,2,...,n,..., we get approximate solutions of (24) as,

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k = x^2 - t^2 + 1$$

$$v(x,t) = \sum_{k=0}^{\infty} V_k(x)t^k = x^2 + t^2 - 1 \quad (31)$$

$$w(x,t) = \sum_{k=0}^{\infty} W_k(x)t^k = x^2 - t^2 - 1$$

which are the exact solutions of Eq. (24).

Example 5: Consider the two-dimensional Brusselator system [18]:

$$\begin{aligned} \frac{\partial u}{\partial t} &= u^2 v - 2u + \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ \frac{\partial v}{\partial t} &= u - u^2 v + \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \end{aligned} \quad (32)$$

subject to the initial conditions

$$u(x,y,0) = \exp[-x-y],\ v(x,y,0) = \exp[x+y]$$

and

$$(x,y,t) \in R^2 \times [0,2] \quad (33)$$

the exact solution of system is

$$u(x,y,t) = \exp[-x - y - \frac{t}{2}] \quad (34)$$

$$v(x,y,t) = \exp[x + y + \frac{t}{2}]$$

Now, let's assume nonlinear term as $u^2v = r(x,y,t)$ in Eq. (32). We know from main properties of RDTM that

$$u(x,y,t) = U_0(x,y) + U_1(x,y)\cdot t + U_2(x,y)\cdot t^2 + \cdots + U_n(x,y)\cdot t^n + \cdots$$

and

$$v(x,y,t) = V_0(x,y) + V_1(x,y)\cdot t + V_2(x,y)\cdot t^2 + \cdots + V_n(x,y)\cdot t^n + \cdots$$

By means of these equations, r (x,y,t) is re-written as follows:

$$r(x,y,t) = u^2v = U_0^2V_0 + \left(U_0^2V_1 + 2\,U_0U_1V_0\right)\cdot t + \left(U_0^2V_2 + 2U_0U_1V_1 + U_1^2V_0 + 2U_0U_2\right)\cdot t^2 + \cdots .$$

Then applying the related rules in Table 1, we obtained RDT of r (x,y,t) as follows :

$$R(x,y) = \delta(k)\,U_0^2V_0 + \delta(k-1)\left(U_0^2V_1 + 2U_0U_1V_0\right) + \delta(k-2)\left(U_0^2V_2 + 2\,U_0U_1V_1 + U_1^2V_0 + 2\,U_0U_2\right) + \cdots.$$

Applying the related rules in Table1 and puting into R (x,y) in the recurrence relation, the last form of the recurrence relation will be as follows:

$$(k+1)U_{k+1}(x,y) = \delta(k)U_0^2V_0 + \delta(k-1)(U_0^2V_1 + 2U_0U_1V_0) + \delta(k-2)\left(U_0^2V_2 + \ldots\right) - 2U_k(x,y) + \frac{1}{4}\left(\frac{\partial^2}{\partial x^2}U_k(x,y) + \frac{\partial^2}{\partial y^2}U_k(x,y)\right) \quad (35)$$

$$(k+1)V_{k+1}(x,y) = U_k(x,y) - (\delta(k)(U_0^2V_0) + \delta(k-1)(U_0^2V_1 + 2\,U_0U_1V_0) + \delta(k-2)(U_0^2V_2 + \ldots) + \frac{1}{4}\left(\frac{\partial^2}{\partial x^2}V_k(x,y) + \frac{\partial^2}{\partial y^2}V_k(x,y)\right) \quad (36)$$

From Eq. (2), the initial conditions given in Eq. (33) can be transformed at t = 0 as:

$$U_0(x,y) = \exp[-x-y]$$

and

$$V_0(x,y) = \exp[x+y] \quad (37)$$

Using Eqs. (35-37) for k = 0,1,2,...,n,..., we get $U_1(x,y)$, $U_2(x,y)$,... and $V_1(x,y)$, $V_2(x,y)$,... as follows:

$$k=0 \Rightarrow U_1(x,y) = -\frac{e^{-x-y}}{2}$$

and

$$V_1(x,y) = \frac{e^{x+y}}{2}$$

$$k=1 \Rightarrow U_2(x,y) = \frac{e^{-x-y}}{8} = \frac{1}{2^2 2}e^{-x-y}$$

and

$$V_2(x,y) = \frac{e^{x+y}}{8} = \frac{1}{2^2 2}e^{x+y}$$

$$k=2 \Rightarrow U_3(x,y) = \frac{-e^{-x-y}}{48} = \frac{-1}{2^3 3!}e^{-x-y}$$

and

$$V_3(x,y) = \frac{e^{x+y}}{48} = \frac{1}{2^3 3!}e^{x+y}$$

$$k=3 \Rightarrow U_4(x,y) = \frac{e^{-x-y}}{384} = \frac{1}{2^4 4!}e^{-x-y}$$

and

$$V_4(x,y) = \frac{e^{x+y}}{384} = \frac{1}{2^4 4!}e^{x+y}$$

$$\vdots$$

Using the inverse transformation Eq. (3), for k = 0,1,2,...,n,... we get the approximate solution of (32) as

$$\begin{aligned} u(x,y,t) &= \sum_{k=0}^{\infty} U_k(x,y)t^k \\ &= e^{-x-y} - \frac{e^{-x-y}}{2}t + \frac{e^{-x-y}}{2^2 2!}t^2 - \frac{e^{-x-y}}{2^3 3!}t^3 + \frac{e^{-x-y}}{2^4 4!}t^4 - \ldots \\ v(x,y,t) &= \sum_{k=0}^{\infty} V_k(x,y)t^k \\ &= e^{x+y} + \frac{e^{x+y}}{2}t + \frac{e^{x+y}}{2^2 2!}t^2 + \frac{e^{x+y}}{2^3 3!}t^3 + \frac{e^{x+y}}{2^4 4!}t^4 + \ldots \end{aligned} \quad (38)$$

The solutions in closed forms are given by

$$u(x,y,t) = \exp[-x - y - \frac{t}{2}]$$

and

$$v(x,y,t) = \exp[x + y + \frac{t}{2}]$$

which are the exact solution of the Eq. (32).

CONCLUSION

In this study, we tested the Reduced Differential Transform method (RDTM) for solving systems of nonlinear PDEs. Obtained results point out that RDTM is appropriate method for finding solution of system of nonlinear PDEs. Comparison with RDTM and HPM for solving systems of nonlinear PDEs showed that RDTM is more effective, powerful and simple than HPM. Also, RDTM has less of computational work than HPM.

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