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Efficiency Measurement using the Ideal Planning for Desirable and Undesirable Outputs Planning

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Abstract: Environmental assessment plays an important role in preventing various types of pollution. Data Envelopment Analysis (DEA) is one of the resources, which can be used as a benchmark measure of the operational efficiency. This paper uses DEA, in which the output is categorized into desirable (good) and undesirable (bad) outputs, in order to evaluate the environmental efficiency and at the same time, tries to measure Returns to Scale (RTS) for desirable outputs and Damages to Scale (DTS) for undesirable outputs.

Key words: Data Envelopment Analysis (DEA) . returns to scale . efficiency . single decision maker

INTRODUCTION

Environmental pollution is one of the important problems of the world created by the non-beneficial output of the industries. In order to solve this problem by applying DEA, most of the previous OR/MS studies (operation research, management science research) should be utilized for obtaining analytical evidence. The CCR model introduced by Cooper [1] is the origin of most of the researches in DEA. Return to scale is another issue, which has been given much attention in the development of DEA. In 1984, Banker [2] obtained returns to scale based on the CCR model. Furthermore, he managed to obtain the return to scale by using free variables of dual BBC. In addition, if the CCR model has multiple optimal solutions, Banker [3] proposed a model which could calculate returns to scale. Cooper [4] offered more than 100 research studies about using OR/MS for preventing the environmental problems. The first DEA model, which was about the application of measuring RAM, was also proposed by Cooper [5]. Similarly, Zhou [6] had over 100 projects related to energy and environmental policies in 2008. Moreover, Sueyoshi [7] cooperated with Cooper and stated the DEA history in 2009. According to the previous researches on the use of DEA in environment assessment and efficiency evaluation, we came to the conclusion that outputs should be classified as desirable and undesirable. For example, we have two electricity outputs; lighting generated from the electricity is the desirable output, while the generated and are considered as undesirable outputs, which are not useful for us.

RETURNS TO SCALE FOR DESIRABLE AND UNDESIRABLE OUTPUTS

In order to use RAM for assessing the efficiency, N Decision Maker Units (DMU) are taken into account. DMUj is the jth unit which is under assessment. In this unit, from the X jinputs, Gjare desirable outputs, while Bj are undesirable outputs.

Using the RAM model in output oriented, the efficiency of the specific kth organization is measured through the following model:

$$
\begin{aligned}
&\text{Max } \textstyle\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_i^x d_r^g \\
&\textstyle\sum_{i=1}^{m} x_{ij}\lambda_j + d_i^x = x_{ik} \quad j=1,\dots,n \qquad (1)\\
&\textstyle\sum_{j=1}^{n} g_{rj}\lambda_j + d_r^g = g_{rk} \quad r=1,\dots,m \\
&\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j > 0 d_i^x > 0 d_r^g > 0
\end{aligned}
$$

$\lambda = \lambda_1, \dots, \lambda_n$ λ_n is a vector of the unknown variables

These variables are in form of a convex combination of input and output variables. The auxiliary variables of and are related to input and output variables, respectively. The limitations of model 1 are determined by upper and lower bounds of desirable inputs and outputs.

Here, the upper bounds and the lower bounds are:

$$
\overline{X_j} = \mathrm{Max}\{x_{ij}\} \;.\; \overline{g_r} = \mathrm{Max}\{g_{rj}\}
$$
$$
\underline{X_j} = \mathrm{Min}\{x_{ij}\} \;.\; \underline{g_r} = \mathrm{Min}\{g_{rj}\}
$$

Therefore, the limitations of model 1 are converted as follows:

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$$R_i^x = \frac{1}{\left[(m+s)\left(\overline{x_i} - \underline{x_i}\right)\right]}$$

$$R_r^g = \frac{1}{\left[(m+s)\left(\overline{g_r} - \underline{g_r}\right)\right]}$$

$$R_r^g = \frac{1}{\left[(m+s)\left(\overline{g_r} - \underline{g_r}\right)\right]}$$

The θ efficiency of the Kth structure is measured by the following relation:

$$\theta = 1\text{-}\left(\sum_{i=1}^m R_i^x d_i^{x*} + \sum_{r=1}^s R_i^x d_r^{g*}\right)$$

All axillary variables are obtained in the optimal solution of the model (1). In this model, * mark indicates optimality. In addition, the equation in the parentheses represents the deficiency level of the efficiency of the optimality level of model1. Furthermore, effective efficiency is obtained by subtracting the deficiency level from 1.Moreover, the dual form of model (1) is as follows:

$$\text{Min} \quad \sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r g_{rk} + \sigma$$
$$\text{St}: \sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r g_{rk} + \sigma \geq 0 \quad (j=1,\ldots,n) \quad (2)$$
$$v_i \geq R_i^x (i=1,\ldots,m)$$
$$u_r \geq R_r^g \quad (r=1,\ldots,s)$$

Free in the sig σ v_r

where and v_r and dualvariables, whichare related to the first and the second group of constrains of the model (1). Variable s, however, is obtained from the third constrain of the model (1).

This is the Sekitani model [8] which was proposed in 2007.

Max\Min (s)

St: all constrains of models (1) and (2) are established. (3)

$$\sum_{i=1}^m v_i x_{ik} - \sum_{r=1}^s u_r g_{rk} + \sigma^* = \sum_{i=1}^m R_i^x d_i^x + \sum_{r=1}^s R_i^x d_r^g$$

and and represent upper and lower bounds, respectively, which can be obtained from Max and Min amounts.The optimal solutions of models (1) and (2) are feasible in the model (3) and vice versa. Therefore, ( ) and) are optimal solutions corresponding upper and lower bounds of (s) of model (2). Using upper and lower bounds, RTS classification of the desirable outputs is determined by the following relations:

If $\underline{\sigma^*} < \overline{\sigma^*} < 0$ RTS ascending

If $\overline{\sigma^*} \geq 0 \geq \underline{\sigma^*}$ RTS fixed

If $\overline{\sigma^*} \geq \underline{\sigma^*} > 0$ RTS descending

The DEA hybrid model for the kth unit under investigation is as follows:

$$\text{Max} \quad \sum_{i=1}^m R_i^x d_i^x + \sum_{r=1}^s R_i^x d_r^g + \sum_{f=1}^h R_f^b d_f^b$$
$$\text{St}: \sum_{j=1}^n x_{ij} \lambda_j^g + d_j^x = x_{ik} \qquad i=1,\ldots,m$$
$$\sum_{j=1}^n g_{rg} \lambda_j^g - d_r^g = g_{rk} \qquad r=1,\ldots,s \qquad (4)$$
$$\sum_{j=1}^n b_{fj} \lambda_j^b - d_f^b = b_{fk} \qquad f=1,\ldots,h$$
$$\sum_{j=1}^n \lambda_j = 1$$
$$\lambda_j^g \geq 0 \; \lambda_j^b \geq 0 \qquad d_j^x \geq 0 \; d_r^g \geq 0 \; d_f^b \geq 0$$

In the hybrid model, b and g signals are considered in order to identify desirable and undesirable outputs, respectively.

For example:

The Jth variable for the desirable outputs: (j=1,…, n)

The Jth variable for the undesirable outputs: (j=1,…, n)

In a similar case, and of the ith helping output variable are the interface between the desirable and the undesirable outputs, respectively.

The restrictions considered for model (4) are as follows:

$$R_i^x = \frac{1}{\left[(m+s)\left(\overline{g_r} - \underline{g_r}\right)\right]} \quad i = 1, \ldots, m,$$

$$R_r^g = \frac{1}{\left[(m+s)\left(\overline{g_r} - \underline{g_r}\right)\right]} \quad , \quad r = 1, \ldots,$$

$$R_f^b = \frac{1}{\left[(m+h+s)\left(\overline{b_f} - \underline{b_f}\right)\right]} \quad f=1,\ldots,h$$

One of the features of model (4) is that its mathematical structure includes 2 efficiency categories of operational and environmental efficiencies. Moreover, two output deviations $(d_i^{xb},$ and two unknown variables $(\lambda_j^b,$ are described for these two efficiency sets.

In addition, the amount of θ Efficiency in model (4) is obtained from the below relation:

$$\theta = 1\text{-}\left(\sum_{i=1}^m R_i^x (d_i^{xg*} + d_i^{xb*}) + \sum_{r=1}^s R_i^g d_r^{g*} + \sum_{f=1}^h R_f^b d_f^{b*}\right)$$

This equation shows that in order to obtain the efficiency level of a unit, we should subtract its inefficiency level from 1, because the efficiency level of a unit is always less than or equal to 1.

EVALUATION OF DECISION MAKER UNITS WITH DESIRABLE AND UNDESIRABLE OUTPUTS BY USING THE IDEAL PLANNING

In general, the present study aims to measure the efficiency level of the environment, obtained from the hybrid model (4), for both inputs and outputs (desirable and undesirable). However, since the calculations of the model (4) are complicated, the ideal programming model is utilized instead. At first, we just consider the desirable and undesirable outputs for the proposed without taking the inputs into account. Then, it is examined in case the inputs and outputs are desirable and undesirable. In one model, the inputs are considered as costs, while the outputs are considered as profits. Therefore, we aim to reduce the costs (inputs) and increase the profits (outputs). However, if the inputs and the outputs are not desirable, their interpretation will be vice versa. Undesirable inputs (without cost) could be used in the system, which is the ideal for any corporation or organization; in a way that they try to increase such inputs to the extent possible. In reality, however, there are no or very limited number of undesirable inputs. On the other hand, the process is reversed for undesirable outputs, which cost the system too much. Nevertheless, the desirable outputs are considered as profits; therefore, corporations and organizations try to decrease the undesirable outputs. In this paper, we attempt to increase the undesirable inputs and decrease the undesirable outputs by using the ideal programming model. In other words, we try to maximize the undesirable inputs and, at the same time, minimize the undesirable outputs. First, consider the following definitions:

$$\mathbf{b}_f = \mathrm{Max}\{b_{fj}\} \ , \ \underline{\mathbf{b}_f} = \mathrm{Min}\{b_{fj}\} \quad , \overline{X_L} \quad Max\{x_{ij}\} \ , \ X_L \quad Min\{x_{ij}\}$$

According to what was mentioned, the hybrid model (4) has two frontiers; one is considered for the desirable outputs and the other one for the undesirable outputs Fig. 1.

According to Fig. 1, the organization requires to extend the durability of the productivity by increasing a desirable output or decreasing an input, or improve its environmental efficiency by decreasing the undesirable output by point on Fig. 1. As it can be observed, these objectives are contradictory. In order to

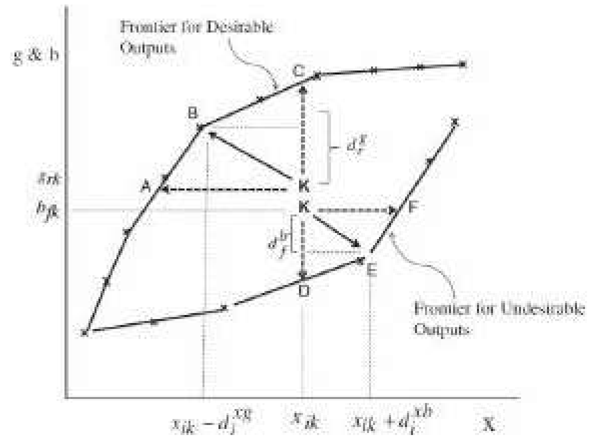

Fig. 1: An efficiency frontier for desirable outputs is located above that of undesirable outputs

solve this problem, we can try to minimize the level of undesirable outputs by using the ideal programming model.

As introduced earlier Min { } is the minimum of the undesirable outputs. Since the model (4) is a model with high computational complexity as well as unusual justification for the two sets of s, the usual DEA model can be proposed instead.

By using the ideal programming model, the proposed model can be written for both desirable and undesirable outputs as follows:

$$\begin{aligned}
&\mathrm{Min}\ \theta\text{-}e\left(\sum_{i=1}^{m} d_i^x + \sum_{r=1}^{s} d_r^g + \sum_{f=1}^{h} d_f^b + \sum_{f=1}^{h} Y_f\right)\\
&\sum_{j=1}^{n} x_{ij}\lambda_j + d_i^x = \Theta x_{ik} \quad i=1,\ldots,m\\
&\sum_{j=1}^{n} \lambda_j g_{rj} - d_r^g = g_{rk} \quad r=1,\ldots,s\\
&\sum_{j-1}^{n} \lambda_j b_{fj} + d_f^b = b_{fk} \quad f=1,\ldots,h \qquad (5)\\
&\sum_{j=1}^{n} \lambda_j = 1\\
&\lambda_j \geq 0 d_i^x \geq 0 \quad d_r^g \geq 0 \quad d_f^b \geq 0\\
&Y_f = b_{fk} - \underline{b_f} \ (f=1,\ldots,h)
\end{aligned}$$

In the proposed model, is the distance between and which should be zero. However, since it may not be practical to be zero, we use and try to minimize its value in the target funection as well as the other targets. It should be noted that (the deal case) is the minimum amount of , therefore, failure of the goal is impossible and, as a result, the variable related to the failure of the goal is not considered.

RESULTS AND RECOMMENDATIONS

As stated earlier, one of major problems in the world is environment pollution which indeed causes variety of other problems such as human and natural problems. In the other hand, many factories as well as companies have neglected this problem due to more efficiency and profits. In this research while attempting to present a model for measuring technical efficiency and profit, it is intended to take environmental elements into account as undesirable outputs. Therefore, regarding this idea that all generated outputs are not desirable, it was intended to calculate efficiency. In the model (3) proposed by Sueyoshi and Goto [9], this issue was explained by using two different frontiers for return to scale and damage to scale. However, the contradictory goals and defining two different boundaries are considered as the disadvantaged of this model. In this paper, considering the undesirable outputs in the lowest level possible, we tried to evaluate the efficiency by using the ideal programming model. In the proposed method to calculate the efficiency of such DMUs, unlike other methods, only one model should be solved. Hence, computational burden and complexity are less than the model proposed by [9]. Additionally, contradictories observed in Goto model do not exist here.

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