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A Review on Geometric Brownian Motion in Forecasting the Share Prices in Bursa Malaysia

Siti Nazifah Zainol Abidin and Maheran Mohd Jaffar

Department of Mathematics, Faculty of Computer and Mathematical Sciences,
Universiti Teknologi MARA, 40450 Shah Alam, Selangor Darul Ehsan, Malaysia

Abstract: The application of Geometric Brownian motion to forecast share prices is reviewed. Formula of Geometric Brownian motion is analyzed and examined to meet the fluctuation of share prices. Uncertainty and unpredictability share prices makes it difficult for investors to forecast future prices. Thus, this reviewed paper aims to state the importance of application of Geometric Brownian Motion into share prices and helps the investors to forecast future prices for the short-term investment. This paper will elaborate geometric Brownian motion involving the randomness, volatility and drift that can help investors in making their investment decision wisely.

Key words: Geometric brownian motion . forecasting . share prices . drift . volatility

INTRODUCTION

Stock market is a platform for investors to own some shares of a company. Investors will become a part of the company members and share in both profits and losses of that company. This is the opportunity for the investors to generate extra income apart from their daily jobs.

Changes of share prices on daily basis make them more volatile and difficult to predict. When purchasing a stock, it does not guarantee anything in return. Thus, it makes stocks risky in investment, but investors can gain high return. Wrong decision in choosing the counters may end up in capital loss.

Therefore, this paper is available as a basic guide for investors to predict future share prices using geometric Brownian motion. This model can predict share prices in a short period of time [1] by taking into account the important elements of the share prices. Investment in short period of time is the time awaited by every investor to earn profit immediately. This model is very efficient for investors who want immediate share prices outlook.

There are many mathematical models introduced by researchers in predicting share prices. Among the models are Hidden Markov Model (HMM), high-order fuzzy time-series model, moving average autoregressive exogenous (ARX) with combination of Grey System (GS) theory and Rough Set (RS), Markov-Fourier Grey Model (MFGM), Clustering-Genetic Fuzzy System (CGFS) which were introduced by [2-6] respectively. Unfortunately, these models are not suitable for short-term investments as desired by most of the investors. It is suitable for long-term investment and forecasting the next day's closing price.

For example [5] used the combination of the grey model, Fourier series and Markov and come out with the new method called as MFGM. The researchers stated that the MFGM is a powerful model and can predict accuracy but it is only suitable for long-term operation.

The other examples are CGFS model. The CGFS is outperformed since it gives a lowest Mean Absolute Percentage Error (MAPE) value by comparing with HMM, Hybrid of HMM, Artificial Neural Network (ANN) and Genetic Algorithms (GA), Hybrid of HMM and Fuzzy logic, ARIMA and ANN. This CGFS model gives the same result as HMM model used by [2]. It can only be applied to predict the next day's closing price [6].

Meanwhile, the method such as ANN is problematic because it requires the use of fuzzy systems and architectures in predicting share prices [2]. In addition, it also requires some background knowledge of experts.

Thus, a mathematical model as simple as Geometric Brownian Motion (GBM) is required to assist investors in forecasting share prices for a short period of investment time. Our result shows that GBM is highly accurate model proven by the MAPE value and it can be used to predict the future share prices for the next two weeks of investment in

Corresponding Author: Siti Nazifah Zainol Abidin, Department of Mathematics, Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor Darul Ehsan, Malaysia

Bursa Malaysia. Therefore it gives some room for investors to evaluate the decision to be taken now and gain profit in a maximum of two weeks of investments undertaken.

This paper is organized as follows. In section II, we will briefly describe about derivation of GBM involving the formula, properties and its application in forecasting share prices. Section III will be the elaboration, implementation and result of applying GBM in share prices. Section IV will be the conclusion.

DERIVATIVE OF GEOMETRIC BROWNIAN MOTION

The mathematical model used is GBM. GBM grows in the stochastic calculus. Stochastic calculus is a branch of the mathematic that deals with uncertainty such as in stock market and foreign exchange.

According to [7], investor main concern will be the return on investment which is referred to the percentage growth in the value of an asset. The quantity S_i is the asset value on the ith day and the return from day ito day i+1 is given by

$$R_i = \frac{S_{i+1} - S_i}{S_i}$$

Rate of return can be explained as the rate of profit or loss in investment. For instance, if yesterday the price in counter A is RM0.50 and today it was RM0.55 then the rate of return was 0.1. Meaning that, if investors invest in counter A, the rate of return will be 10% increase in capital investment.

The positive value of rate of return indicates increase of profit, while a negative value, means that the investor will face the loss. Higher rate of return value gives higher profit gaining. By knowing the rate of return, the mean of returns distribution of drift, μ can be calculated as follows, where M is the number of returns in the sample.

$$\mu = \bar{R} = \frac{1}{M}\sum_{i=1}^{M} R_i$$

And the sample standard deviation (or volatility, σ) is

$$\sigma = r = \sqrt{\frac{1}{(M-1)\delta t}\sum_{i=1}^{M}(R_i - \bar{R})^2}$$

Volatility refers to the fluctuation of the share prices, which the price of a security moves up and down [8]. Volatility is found by calculating the annualized standard deviation of daily change in price where standard deviation is a statistical measure of dispersion around a central tendency.

High volatility refers to share prices rapidly moves up and down over the short periods of time. In simple words, it refers to the risk level, since the fluctuation of the prices is unpredictable and uncertain. Investing in stock market is risky. Investor will face either loss or profit after investment. Therefore, volatility of the rate of return (or standard deviation) can be used as the measurement of risk level [9]. Higher volatility refers to the higher level of risk.

According to [7], he believes that the returns can be written as random variables, drawn from a normal distribution with a known, constant, non-zero mean and a known, constant and non-zero deviation since the return is closed enough to normal distribution. The usage of normal distribution because the return value changes in one unit of time by an amount that is normally distributed with mean and standard deviation. The normal distribution is a good choice because the return variable is being affected additively by many independent random variables. Reference [7] standardizes the normal distribution of asset return by entering the standard normal variable, ϕ into the asset return model as bellow:

$$R_i = \frac{S_{i+1} - S_i}{S_i} = \text{mean} + \text{standard deviation} \times \phi \qquad (1)$$

Time step for one day denotes as ∂t. Mean of the scales follows the size of the time step. By assuming μ to be constant, it can be written as

$$\text{mean} = \mu\partial t$$

And let the standard deviation of the asset return over time steps, ∂t will be as below by letting σ to be some parameter in measuring the amount of randomness.

$$r = \sigma\partial t^{1/2}$$

The mean and the standard deviation over the time steps by assuming μ and σ are constant will be as follows:

$$R_i = \frac{S_{i+1} - S_i}{S_i} = \mu\partial t + \sigma\phi\,\partial t^{1/2} \qquad (2)$$

Equation (2) can be simplified as

$$S_{i+1} - S_i = S_i\mu\partial t + S_i\sigma\phi\partial t^{1/2} \qquad (3)$$

Left hand side shows the changes of the asset price, while in the right hand side shows the random walk model in discrete time step.

According to [7], stock markets are varying continually over very small intervals of time which follows the Brownian motion (BM). BM refers to the limiting process for a random walk as the time steps go to zero [7]. This change on the asset pricing is being altered by random amounts. BM is the fundamental tools to describe the mathematical model on all the financial asset pricing. This was strongly supported by [10], who stated that the behavior of the stock market's price are unpredictable and follow the random walk in GBM. It is out performing compares with the other model.

A GBM model is a continuous-time stochastic process explained by [1], in which the logarithm of the randomly varying quantity follows a BM also known as Wiener process.

Wiener process or a BM process can be defined as the stochastic process $\{X(t), t\geq 0\}$ is called a Wiener process (or a Wiener Einstein process or a BM process) [11].

By using Wiener process notation, asset price model in continuous-time limit, can be written as in (4), where dS refers to the change in the asset price. The limit will be $\partial t\to 0$. The first term in ∂t on the right-hand side of (3) will be changed to dt, but it is wrong to change the second term since it can't be written as $dt^{1/2}$ instead of $\partial t^{1/2}$. Thus, dX will be a random variable, from normal distributions with mean zero and variance dt

$$E[dX]=0 \qquad E[dX^2]=dt$$

$$dS=\mu S dt+\sigma S dX \tag{4}$$

Learning of the asset return model has bought us to the BM theory, where BM refers to the limiting process for a random walk as the time steps go to zero indicates as X(t). The properties of BM are very important for financial model and it is explained by [7], are as below:

- Finiteness: Either random walk is going to infinity in a finite time or a limit in which there is no motion at all, is the result for any increment of scaling over time step. It denotes in term of square root of time step ($\partial t^{1/2}$) as in equation (3).
- Continuity: X(t) is the continuous-time limit of discrete time random walk.
- Markov property: the conditional distribution of X(t) given information up until $\tau<t$ depends only on X(t) It only depends on the previous value.
- Martingale property: the conditional expectation of X(T) given information up until $\tau<t$ is $X(\tau)$. It is only the amount that is already hold.
- Quadratic variation: the time between 0 to t in the partition is divided with n+1 partition points where $t_i=i(t/n)$. Here

$$\sum_{i=1}^{n}(X(t_i)-X(t_{i-1}))^2\to t$$

- Normality: the increment of X(t) over finite time of t_{i-1} to t_i, $X(t_i)$-$X(t_{i-1})$ is Normally distributed with mean zero and variance t_i-t_{i-1}.

The BM's properties will be used in GBM.

Back to the stochastic differential model as in (4), we will discuss the stochastic integration. Let assume that a(t) and b(t) are some functions, then

$$S(t)=a(t)+b(t)$$

The ordinary differential equation will be as

$$dS=a(t)dt+b(t)dX$$

The equation above is as in (4). The integration of it will be

$$S(t)=\int_0^t a(\tau)d\tau+\int_0^t b(\tau)dX(\tau)+S_0$$

Before discussing the Ito's lemma, the most important rule of the stochastic calculus, the mean square limit, that is defined first as

$$E[(\sum_{i=1}^{n}(X(t_i)-X(t_{i-1}))^2-t)^2]$$

Which follow the quadratic variation as $t_i = i(t/n)$. And as $n\to\infty$ this tends to zero, therefore

$$\sum_{i=1}^{n}(X(t_i)-X(t_{i-1}))^2=t$$

From the definition of mean square limit, this is often written as

$$\int_0^t (dX)^2=t$$

The mean square limit is useful in definition of the stochastic integration. Now, Ito's lemma will be introduced. In learning of Ito's lemma, the knowledge of Taylor series expansion is needed to derive the Ito's lemma.

Let a function of F(X(t)), with the smallest timescales as $\partial t = nh$. Thus, the function of F(X(t+h)) can be approximated by a Taylor series as below

$$F(X(t+h)) - F(X(t)) = (X(t+h) - X(t)\frac{dF}{dX}(X(t)) + \frac{1}{2}(X(t+h) - X(t))^2\frac{d^2F}{dX^2}(X(t)) + \ldots$$

Now, substitute the smallest timescales and it forms

$$\begin{aligned}&(F(X(t+h)) - F(X(t))) + (F(X(t+2h)) \\ &-X(t)\frac{dF}{dX}(X(t)) + \frac{1}{2}(X(t+h) - X(t))^2\frac{d^2F}{dX^2}(X(t)) + \ldots \\ &+(F(X(t+nh)) - F(X(t+(n-1)h)))\end{aligned} \quad (5)$$

By applying the Taylor series in (5) it will be as below

$$\begin{aligned}&\sum_{i=1}^{n}(X(t+ih) - X(t+(i-1)h))\frac{dF}{dX}(X(t+(i-1)h)) \\ &+\frac{1}{2}\frac{d^2F}{dX^2}(X(t))\sum_{i=1}^{n}(X(t+ih) - X(t+(i-1)h))^2 + \ldots\end{aligned} \quad (6)$$

Approximation used is

$$\frac{d^2F}{dX^2}(X(t+(i-1)h)) = \frac{d^2F}{dX^2}(X(t))$$

By Substitute $\partial t = nh$ into the first line of $F(X(t+nh) - F(X(t))$ and will get $F(X(t+\partial t)) - F(X(t))$, thus

$$F(X(t+nh) - F(X(t)) = F(X(t+nh) - F(X(t)) \quad (7)$$

The second line is

$$\int_{t}^{t+\partial t}\frac{dF}{dX}dX \quad (8)$$

The last definition is

$$\frac{1}{2}\int_{t}^{t+\partial t}\frac{d^2F}{dX^2}(X(t))\partial t \quad (9)$$

Substitute into (8) and (9) into (7) we get

$$\begin{aligned}&F(X(t+nh) - F(X(t)) = \int_{t}^{t+\partial t}\frac{dF}{dX}(X(\tau))dX(\tau) \\ &+\frac{1}{2}\int_{0}^{t}\frac{d^2F}{dX^2}(X(\tau))d\tau\end{aligned} \quad (10)$$

If we extend the equation (10) to the longest timescales, from zero to t, thus, the integral of Ito's lemma, F(X), will be

$$F(X) = F(X(0)) + \int_{0}^{t}\frac{dF}{dX}(X(\tau))dX(\tau) + \frac{1}{2}\int_{0}^{t}\frac{d^2F}{dX^2}(X(\tau))d\tau \quad (11)$$

The differential version of the Ito's lemma in (11) is written as

$$dF = \frac{dF}{dX}dX + \frac{1}{2}\frac{d^2F}{dX^2}dt \quad (12)$$

Suppose, the stochastic differential equation as in equation (4) and the function of F(S) = log S, will describe the asset price as the lognormal random walk. By using Ito's lemma as follows:

$$dF = \frac{dF}{dS}dS + \frac{1}{2}\frac{d^2F}{dS^2}dS^2 + \ldots$$

$$dF = \frac{dF}{dS}[\mu Sdt + \sigma SdX] + \frac{1}{2}\frac{d^2F}{dS^2}[\mu Sdt + \sigma SdX]^2$$

$$\begin{aligned}&dF = \mu Sdt\frac{dF}{dS} + \sigma SdX\frac{dF}{dS} + \frac{1}{2}\mu^2S^2dt^2\frac{d^2F}{dS^2} + \\ &\frac{1}{2}(2\mu\sigma S^2dtdX)\frac{d^2F}{dS^2} + \frac{1}{2}\sigma^2S^2dX^2\frac{d^2F}{dS^2}\end{aligned}$$

$$\begin{aligned}&dF = \mu Sdt\frac{dF}{dS} + \sigma SdX\frac{dF}{dS} + \frac{1}{2}\mu^2S^2dt^2\frac{d^2F}{dS^2} + \\ &\mu\sigma S^2dtdX\frac{d^2F}{dS^2} + \frac{1}{2}\sigma^2S^2dX^2\frac{d^2F}{dS^2}\end{aligned} \quad (13)$$

Cancel all the insignificant term by using the rule of thumb as below

$$\begin{aligned}&dX \cdot dX = dt \\ &dt \cdot dt = dX \cdot dt = dt \cdot dX = 0\end{aligned}$$

And (13) will be as below

$$dF = \mu Sdt\frac{dF}{dS} + \sigma SdX\frac{dF}{dS} + \frac{1}{2}\sigma^2S^2\frac{d^2F}{dS^2}dt$$

It can be simplified as

$$dF = \frac{dF}{dS}dS + \frac{1}{2}\sigma^2S^2\frac{d^2F}{dS^2}dt \quad (14)$$

Then, substitute dS, $\frac{dF}{dS} = \frac{1}{S}$ and if $\frac{d^2F}{dS^2} = -\frac{1}{S^2}$ into (14), then it will form

$$dF = \frac{1}{S}dS + \frac{1}{2}\sigma^2 S^2(-\frac{1}{S^2})dt$$
$$dF = \frac{1}{S}(\mu S dt + \sigma S dX) - \frac{1}{2}\sigma^2 dt$$
$$dF = \mu dt + \sigma dX - \frac{1}{2}\sigma^2 dt$$
$$dF = \mu dt - \frac{1}{2}\sigma^2 dt + \sigma dX$$
$$dF = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dX$$

Integrate both sides

$$\int dF = \int(\mu - \frac{1}{2}\sigma^2)dt + \int \sigma dX$$
$$\ln S = (\mu + \frac{1}{2}\sigma^2)t + \sigma(X(t) - X(0)$$
$$e^{\ln S} = e^{(\mu + \frac{1}{2}\sigma^2)t + \sigma(X(t) - X(0) + c}$$
$$e^{\ln S} = e^{(\mu + \frac{1}{2}\sigma^2)t + \sigma(X(t) - X(0)} \cdot e^c$$

Here e^c = S(0), μ is drift, σ is volatility, X(t) is random value and S(t) is the price of stock at time, t. The stochastic differential equation for log S is

$$S(t) = S(0)e^{(\mu + \frac{1}{2}\sigma^2)t + \sigma(X(t) - X(0))} \quad (15)$$

This stochastic differential equation is particularly important in modeling of many asset classes. Equation (15) is the asset price model that is able to predict an asset price at specific time t.

According to [12], there are three measurement of forecasting model which involve time period, t. The measurements are number of period forecast, n, actual value in time period at time, t, Y_t and forecast value at time period t, F_t. The widely used to evaluate the forecasting method that considers the effect of the magnitude of the actual values, is the mean absolute percentage error (MAPE). It can be calculated as follows:

$$MAPE = \frac{\sum \left| \frac{Y_t - F_t}{Y_t} \right|}{n}$$

Table 1 shows a scale of judgment of forecast accuracy using MAPE equation.

ELABORATION ON GEOMETRIC BROWNIAN MOTION

GBM model is also known as exponential of BM or model of stock prices. It also refers to a process often used to model the price of a security as it evolves over time [13]. This model is widely used model of stock price behavior. With the combination of volatility, randomness and expected rate of return make the model equivalent with the stock price behavior.

According to [13], when used to model the price of a security over time, the GBM process possesses neither the flaws of the BM process. Because it is the logarithm of the share prices that is assumed to be normal random variable, the model does not allow for negative share prices. Furthermore, because the values are in ratios, rather than differences of prices separated by a fixed amount of time that have the same distribution, the GBM makes what many feel is the more reasonable assumption. It is the percentage, rather than the absolute, change in price whose probabilities do not depend on the current prices.

Table 1: A scale of judgment of forecast accuracy

MAPE	Judgment of forecast accuracy
<10%	Highly accurate
11% to 20%	Good accurate
21% to 50%	Reasonable forecast
>51%	Inaccurate forecast

Source [12]

Table 2: A sample of forecast prices vs. actual prices for hingyap counter

Date	Actual (RM)	Forecast (RM)
31-Mar-10	1.18	1.18
1-Apr-10	1.20	1.14
2-Apr-10	1.14	1.18
5-Apr-10	1.19	1.19
6-Apr-10	1.19	1.16
7-Apr-10	1.18	1.16
8-Apr-10	1.17	1.20
9-Apr-10	1.18	1.17
12-Apr-10	1.19	1.20
13-Apr-10	1.18	1.17
14-Apr-10	1.18	1.18
15-Apr-10	1.17	1.18
16-Apr-10	1.18	1.15
19-Apr-10	1.15	1.18
20-Apr-10	1.15	1.22
21-Apr-10	1.16	1.18
22-Apr-10	1.16	1.21
23-Apr-10	1.16	1.17
26-Apr-10	1.16	1.20
27-Apr-10	1.16	1.19
28-Apr-10	1.15	1.19
29-Apr-10	1.13	1.19
30-Apr-10	1.12	1.18

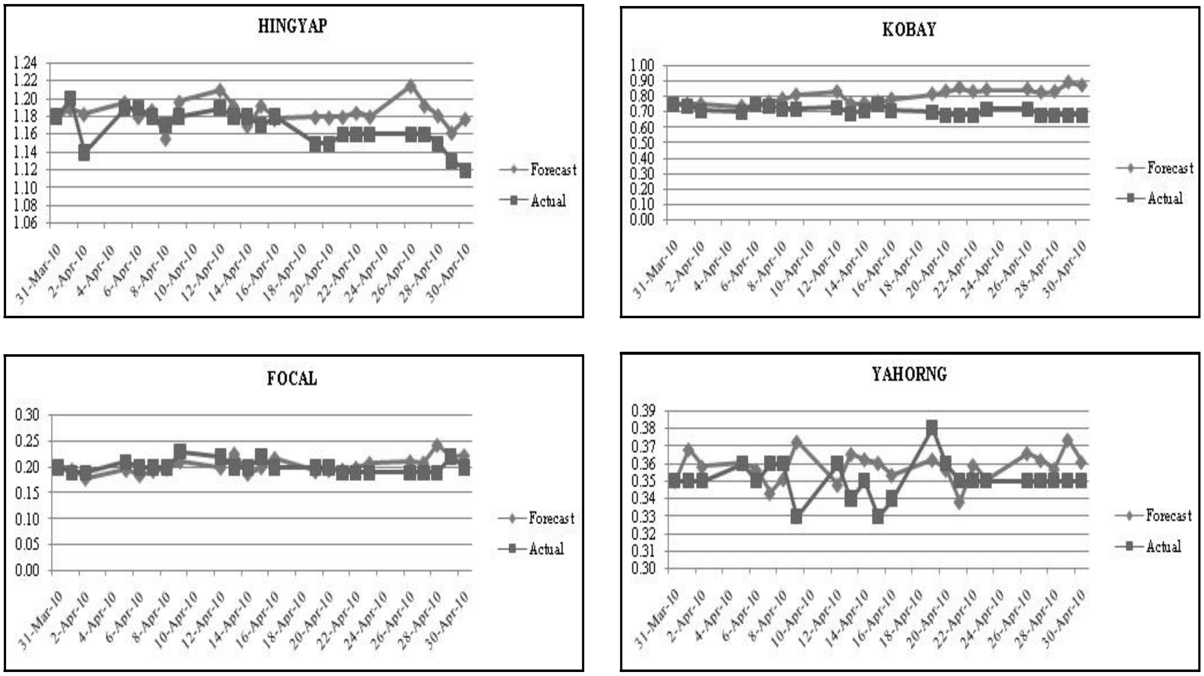

Fig. 1: A sample of forecast prices vs. actual prices

The expected returns of GBM are independent of the value of the process which agrees with what we would expect in reality. This model shows the same kind of movement in its paths as we see in real stock prices as shown in the Fig. 1 and Table 1 as a sample.

Figure 1 and Table 2 are obtained by analyzing the small size companies under the range of RM10 to RM50 million market capitalizations. For this case, there are 77 counters that are normally distributed asset return and listed in Main Board of Bursa Malaysia that are analyzed.

Figure 1 shows that 4 graphs of forecasting price using GBM compared with the actual price in Bursa Malaysia for 1 month data. Meanwhile Table 2 shows a sample of forecasting value for HINGYAP counter. Its 1 month forecasting data is compared with the actual prices.

Let say, investors invested on 31st March 2010 and gain profit on 14th April 2010. Table 2 illustrates the prices on 1st April 2010 until 14th April 2010. The forecast prices are closed to the actual prices. And after 14th April 2010, the forecast prices become more volatile and far from the actual prices. Thus, this is the good opportunity for the investors to decide on 31st March 2010 and gain profits on 14th April 2010, which is a two week working day of investment. Figure 1 gives a clear view of it.

To give more confident to the researchers, let us measure the accuracy of the forecast model by looking at the MAPE value. Table 3 shows the forecast price and actual price on 14th April 2010 and MAPE value for two week investment of 24 counters as samples. It shows that the MAPE values are lower than 10% and most of the forecast prices are closest to actual prices for the two week investment.

Table 3: A sample of actual prices, forecast prices and MAPE Values

Counter	Actual (RM)	Forecast (RM)	MAPE (%)
HINGYAP	1.18	1.18	1.78
XIANLING	0.46	0.45	3.50
DUFU	0.56	0.56	3.53
GEFUNG	0.25	0.24	5.39
KOBAY	0.71	0.75	7.72
BGYEAR	0.75	0.71	9.61
SPK	0.36	0.34	9.84
FOCAL	0.20	0.24	9.31
FARLIM	0.38	0.32	5.87
SBCCORP	0.55	0.52	8.05
PATIMAS	0.08	0.09	3.75
EMICO	0.36	0.34	4.58
FARMBES	0.53	0.57	4.40
HWATAI	0.49	0.49	8.68
SINARIA	0.33	0.33	2.09
FBO	0.95	0.95	2.09
KBES	0.40	0.40	1.74
LFECORP	0.23	0.20	8.51
VASTALX	0.14	0.12	13.51
TOCEAN	0.95	0.96	1.65
ABRIC	0.18	0.17	8.77
CNASIA	0.51	0.49	2.74
KOMARK	0.28	0.26	6.43
YAHORNG	0.35	0.34	3.44

The advantages of GBM are relatively easy involvement of calculation and do not need a lot of data to forecast the future closing price. It has been stated in the Martingale and Markov properties of GBM.

CONCLUSION

This model is very suitable for the short-term investment at least a maximum of two week investment. This have been proven in Table 2 and illustrated in Fig. I. GBM gives a good opportunity to decide new and gain profit after two weeks of investment. Moreover, GBM is the highly accurate model, proven in Table 3, the most of MAPE value are below than 10%. The calculation of GBM is much easier and less data is needed to forecast the future closing prices are compared with the other forecasting models. Although, GBM model has some weaknesses, but it proven that it gives the accurate value to the actual prices. Therefore GBM is the best model used to forecast the future closing for at least a maximum of two week investment.

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