

Analytical Study of Temperature Distribution of the Electroosmotic Flow in Slit Microchannels

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Abstract: Presence of externally applied electric field can cause the flow field in the micron and submicron channels; this generated flow field is called electroosmotic. In this type of flow field, the presence of the external electric field causes the extra generated heat in the domain that may affect the performance of the system (Joule heating effect). In the present study, an analytical approach has been proposed to find the temperature distribution of the electroosmotic flow field in the slit microchannels. Using the obtained method, the effects of microchannel height and external electric field on the temperature distribution of the flow field have been examined. It is revealed that, for the larger microchannels, the effect of the external electric field can be intensified on the temperature of the flow field. However, in the small microchannels, its effect may be negligible.

Key words: Microchannels • Electrokinetic • Electroosmotic • Joule heating

INTRODUCTION

By recent advancements in micro-fabrication, miniaturization takes into consideration for many applications spanning from biological to cooling of microelectronics applications [1-8]. Operational state of many micro-instruments deals with fluid flow in microscale channels (microchannel is the channel that its smallest dimension is between 10 μ m to 200 μ m). Microfluidic devices offer many advantages such as a significant reduction in the consumption of required materials, ability to perform in-vitro experiments on the continuous flow in a manner similar to the real situation in a living biological system, being portable and vibration free.

One of the most important characteristics of micro-scale phenomena is large surface-to-volume ratio. It means that factors related to surface effects, which may be neglected in macro scales, have more influence on micro/nano- scale fluid flow and heat transfer. Electroosmosis is one of these surface effects that becomes prominent and generate flow field in the microchannels. Figure 1 illustrates the nature of the electroosmotic flow. By definition, electroosmosis is the

motion of ionized liquid relative to the stationary charged surfaces by an applied external electric field [8]. Electroosmotic phenomenon is present due to the electric double layer effect (EDL), which is formed as a result of the interaction of ionized solution with static charges on dielectric surfaces. When the ionic liquid comes into contact with most non-conductive materials such as glass and silicon capillary walls, the small immobile charges appear on the wall surfaces (Figure 1). Ionization, absorption and dissociation of the ions in the capillary material lead to the charge build up which is counteracted by charges being drawn out of the solution toward the interface. The charges on the solution side of the interface, unlike that on the capillary surface, are mobile and move parallel to the applied electric field. The thin layer of mobile charges drags the neutral bulk of the solution along the channel in a plug-like manner due to viscous forces.

Electroosmotic flow has many applications in microscale devices such as microfluidic liquid chromatography systems, microreactors, microenergy systems, microelectronic cooling systems and biomedical microdevices [1]. Although the electroosmotic flow has

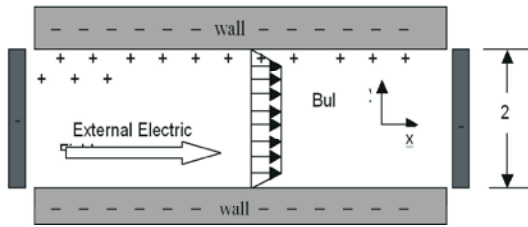


Fig. 1: Schematic of the electroosmotic flow in a slit channel with the finite EDL.

many advantages in comparison with pressure driven flow such as being vibration free and not requiring any external mechanical pumps or moving parts, there are some drawbacks associate with this type of flow field that must be carefully taken into account. Joule heating is one of these downsides: external electric field generates heat in the system due to the electrical resistance. If σ is the electrical conductivity and E is the applied external electric field, the produced heat of Joule heating effects can be found as $q = \sigma E^2$ [6]. Joule heating may affect the performance of the microfluidic devices. For example, increasing the local temperature of fluid flow can cause decrease in the fluid viscosity that leads to the increase of slip velocity on the boundaries. Thus, having a comprehensive understanding on the temperature distribution of the electroosmotic flow in microchannels seems to be essential. Although many researchers conducted numerical methods to model heat transfer and find temperature distribution for electroosmotic flow in microchannels [6], lack of any analytical studies is completely evident. Analytical approaches are completely essential for analysis and control of any industrial and laboratory micro-devices. Unlike numerical methods that associated with errors, analytical studies are usually exact. Using analytical methods, parametric study can be performed easier. Many control approaches are also based on the closed-form governing equations of the system. In the present study, an analytical approach is used to find the temperature distribution of electroosmotic flow in a slit microchannel; consequently, the effects of different parameters such as external electric field and dimension on the temperature distribution will be investigated. The rest of this paper is organized as follow: following section is the mathematical modeling of electroosmotic flow field in microchannels; the results and discussions will be presented in section 3; and finally concluding remarks will be presented in section 4.

Mathematical Modeling: Figure 1 depicts the schematic diagram of the assumed model of the current study. A unidirectional, steady state, incompressible, fully developed flow of the Newtonian fluid is assumed in the slit microchannel. The flow field is generated because of the electroosmotic effect. No-slip velocity and constant temperature is assumed on the walls of the microchannel.

In this study, h is the half-height of the slit-microchannel. u and p represent velocity and pressure, respectively. T is temperature; E_x stands for external electric field; ϕ denotes the electric potential of the electric double layer (EDL). μ and ρ are the viscosity and density of the fluid; c and k are the specific heat capacity and thermal conductivity of the assumed fluid. \hat{a} and \hat{a}_0 are the relative and absolute permittivity. z is the valence of the ion, e is the elementary charge, k_b is Boltzmann constant and n_0 is the ionic bulk number concentration.

By considering the above mentioned assumptions of the current study, the simplified form of the governing equations become:

$$\frac{\partial u}{\partial x} = 0 \quad (1)$$

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \epsilon \epsilon_0 E_z \frac{\partial^2 \phi}{\partial y^2} \quad (2)$$

$$\frac{\partial^2 \phi}{\partial y^2} = \frac{2ze n_0}{\epsilon \epsilon_0} \sinh\left(\frac{ze\phi}{k_b T}\right) \quad (3)$$

$$\rho c_p \left[u \frac{\partial T}{\partial x} \right] = k \frac{\partial^2 T}{\partial y^2} + \sigma E_x^2 + \phi_{inc}. \quad (4)$$

$$\phi_{inc.} = 2\mu(D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) \quad (5)$$

$$D_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (6)$$

Eq. (1) is the continuity equation. Eq. (2) is the modified Navier-Stokes equation. $\epsilon \epsilon_0 E_z \frac{\partial^2 \phi}{\partial y^2}$ is the effect

of body force caused by the electroosmotic effect on the fluid flow. Eq. (3) is the Poisson-Boltzmann equation. Eq. (4) is the energy equation. In this equation, term $\sigma_{eff} E_x^2$ represents Joule heating effect, ϕ_{inc} is the dispersion energy that for the incompressible fluid flow can be calculated by solving Eqns. (5-6).

s-potential is the considered potential of the electric double layer on the walls of the microchannel (Eq. (7)). No slip velocity (Eq. (8)) and constant temperature (Eq. (10)) are assumed on the walls of the microchannel. The mathematical representations of these equations are:

$$\varphi(x, \pm h) = \zeta_0 \quad (7)$$

$$u(x, \pm h) = 0 \quad (8)$$

$$\frac{\partial u}{\partial y}(x, 0) = 0 \quad (9)$$

$$T(x, \pm h) = T_s \quad (10)$$

$$\frac{\partial T}{\partial y}(x, 0) = 0 \quad (11)$$

Velocity Field: In the current study, no external pressure gradient is induced to the microchannel. No turn and cross-sectional variation are also considered in the geometry. Thus, there is not any induced pressure gradient in the system and therefore, there is a pure electroosmotic flow in the system. The pure electroosmotic velocity for the assumed flow field of the current study and in the slit microchannel can be evaluated as [8]:

$$u = L_0 + L_1 \tanh^{-1}[L_2 \exp(L_3(h - |y|))] \quad (13)$$

In this equation, L_0 , L_1 , L_2 and L_3 are defined as:

$$L_0 = \frac{-2\zeta\epsilon\epsilon_0 E}{\mu} \quad (14)$$

$$L_1 = \frac{\epsilon\epsilon_0 E}{\mu} \frac{4}{k_2} \quad (15)$$

$$L_2 = \tanh\left(\frac{\zeta k_2}{4}\right) \quad (16)$$

$$L_3 = -\sqrt{k_1 k_2} \quad (17)$$

Temperature Distribution: Because of the uni-directional flow field of the current study, the dispersion energy term of the energy equation can be simplified as follow:

$$D_{12} = \frac{1}{2} \left(\frac{\partial u}{\partial y}\right)^2, D_{11} = D_{22} = D_{33} = D_{13} = D_{23} = 0 \quad (18)$$

$$\phi_{inc.} = \mu \left(\frac{\partial u}{\partial y}\right)^2$$

By substituting Eq. (18) in the Eq. (4), the energy equation becomes:

$$\rho c \left[u \frac{\partial T}{\partial x} \right] = \mu \left(\frac{\partial u}{\partial y} \right)^2 + k \frac{\partial^2 T}{\partial y^2} + \sigma E_x^2 \quad (19)$$

The following equation is valid for the fully developed flow:

$$\frac{\partial}{\partial x} \left[\frac{T_s(x) - T(y, x)}{T_s(x) - T_m(x)} \right] = 0 \quad (20)$$

By expanding the above equation, $\partial T / \partial x$ can be expressed as:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} - \frac{(T_s - T)}{(T_s - T_m)} \frac{dT_s}{dx} + \frac{(T_s - T)}{(T_s - T_m)} \frac{dT_m}{dx} \quad (21)$$

The temperature is assumed to be constant on the walls of the channel ($dT_s/dx = 0$); thus, Eq. (21) can be simplified as follow:

$$\frac{\partial T}{\partial x} = \frac{(T_s - T)}{(T_s - T_m)} \frac{dT_m}{dx} \quad (22)$$

For more simplicity, following constants and parameters are defined:

$$A = \frac{1}{\rho_f c_p} \quad (23)$$

$$B = \frac{k_f}{\rho_f c_p} \quad (24)$$

$$C = \frac{\sigma_{eff} E^2}{\rho_f c_p} \quad (25)$$

$$D = \frac{\partial T}{\partial x} = \frac{(T_s - T)}{(T_s - T_m)} \frac{dT_m}{dx} \quad (26)$$

$$L_5 = \frac{D}{B} \quad (27)$$

$$L_6 = -\frac{\mu A}{B} \quad (28)$$

$$L_7 = -\frac{C}{B} \quad (29)$$

By considering the above parameters, Eq. (19) can be re-write as:

$$\frac{\partial^2 T}{\partial y^2} = L_5 u + L_6 \left(\frac{\partial u}{\partial y} \right)^2 + L_7 \quad (30)$$

Using the software *Mathematica*, we have solved the above equation. The obtained solution can be expressed by the following set of equation. In these equations,

subscripts u and l represent the upper and lower half-channels:

$$T_u(y) = H_1(y) + C_1y + C_2 \quad 0 \leq y \leq h \quad (31)$$

$$T_l(y) = H_2(y) + C_3y + C_4 \quad -h \leq y \leq 0 \quad (32)$$

Where:

$$H_1(y) = g_1(y) + g_2(y) + g_3(y) + g_4(y) + g_5(y) \quad (33)$$

$$H_2(y) = g_6(y) + g_7(y) + g_8(y) + g_9(y) \quad (34)$$

$$g_1(y) = -\frac{1}{4}l_1^2l_6(Ln(1 - f_1(y)) - 2yl_3) \quad (35)$$

$$g_2(y) = l_8y^2 + l_9y^4 \quad (36)$$

$$g_3(y) = l_{10}(y^2(f_3(y) + Ln(\frac{1-f_2(y)}{1+f_2(y)}))) \quad (37)$$

$$g_4(y) = l_{11}(y \times pl(2, f_1(y)) - 4y \times pl(2, f_2(y))) \quad (38)$$

$$g_5(y) = l_{12}(pl(3, f_1(y)) - 8pl(3, f_2(y))) \quad (39)$$

$$g_6(y) = -\frac{1}{4}l_1^2l_6(Ln(-1 + f_4(y)) - 2yl_3) \quad (40)$$

$$g_7(y) = g_2(y) \quad (41)$$

$$g_8(y) = l_{11}(y \times pl(2, f_4(y)) - 4y \times pl(2, f_5(y))) \quad (42)$$

$$g_9(y) = l_{12}(pl(3, f_4(y)) - 8pl(3, f_5(y))) \quad (43)$$

$$f_1(y) = \frac{\exp(2(-h+y)l_3)}{l_2^2} \quad (44)$$

$$f_2(y) = \frac{\exp((-h+y)l_3)}{l_2} \quad (45)$$

$$f_3(y) = 2 \tanh^{-1}(\exp(-(-h+y)l_3)l_2) \quad (46)$$

$$f_4(y) = l_2^2 \exp(2(h+y)l_3) \quad (47)$$

$$f_5(y) = l_2 \exp((h+y)l_3) \quad (48)$$

In these equations, constants l_8 to l_{12} are defined as:

$$l_8 = \frac{(l_7 + l_5 - l_4l_5)}{2} \quad (49)$$

$$l_9 = \frac{1}{12}(4l_6l_4^2 + l_4l_5) \quad (50)$$

$$l_{10} = -\frac{l_1l_5}{4} \quad (51)$$

$$l_{11} = \frac{l_1l_4l_6}{l_3} \quad (52)$$

$$l_{12} = \frac{l_1(l_5 - 8l_4l_6)}{8l_3^2} \quad (53)$$

Boundary conditions (10) and (11) should be utilized to find the constants C_1 to C_4 of the equations (31) and (32):

$$c_1 = -\frac{\partial H_1(y)}{\partial y} \Big|_{y=0} \quad (54)$$

$$c_2 = T_s - H_1(h) - c_1h \quad (55)$$

$$c_3 = -\frac{\partial H_2(y)}{\partial y} \Big|_{y=0} \quad (56)$$

$$c_4 = T_s - H_2(-h) + c_3h \quad (57)$$

RESULTS AND DISCUSSION

In this section, using the obtained analytical formulas, the temperature distribution is simulated. Furthermore, the effects of external electric field (E) and microchannel half height (h) are studied on the temperature. The assumed constants and parameters of the current study have been presented in Table 1.

Figure 2 illustrates the temperature distribution at the cross section of the microchannel. By comparing Figures (2.a) and (2.b), one can conclude that the effects of E on the temperature distribution increases by size. As an example, if $h = 10^{-6}m$, increase of E has less influence on the temperature (Fig. 2.a); however, by increasing the microchannel height to 30 μm , the effect of E on the temperature distribution is significantly intensified. Here, ten times increase in E boosts temperature variation to $300^\circ C$.

For the different values of external electric field, Fig. 3 depicts the effect of microchannel height (h) on the averaged temperature at the cross section of the microchannels. As it is clear from this figure, for the higher values of external electric field (E), increase of the height has great influence on the temperature. However, this effect becomes lesser for the smaller channels. It can explain by considering the effect of Joule heating in the microchannels. Joule heating is defined as $Q = IV$, where I and V are electric current and electric potential, respectively. For the smaller microchannels, It is true that the increase of external electric field amplifies the electric potential ($\vec{E} = -\vec{\nabla}V$), but the electric current becomes lower in these smaller channels. This is because the channel's cross-section area becomes smaller. Electric current, I , can be calculated as [9]:

$$I = \iint Fz_i c_i U^{bulk} dA + \iint F^2 z_i^2 v_i c_i E dA$$

Here, F is Faraday's constant, z_i , v_i and c_i are the valence, mobility and concentration of the ion type i , U^{bulk} is the bulk velocity and E is the external electric field.

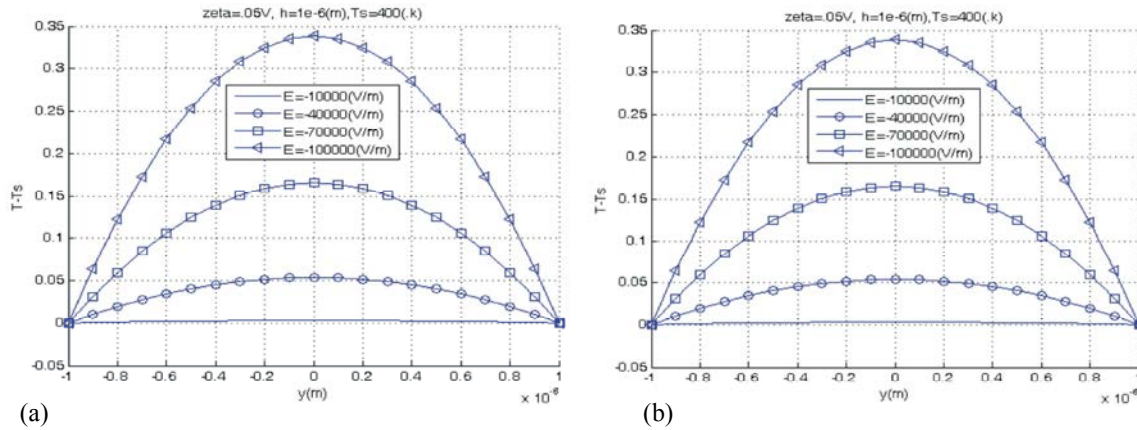


Fig. 2: This figure shows the influence of external electric field (E) on the temperature distribution of the electroosmotic flow in the slit microchannel ($T_s = 400.K$, $h = 10^{-6}m$ and $\zeta = 0.05Volt$). (a) $h = 1 \mu m$, (b) $h = 30 \mu m$

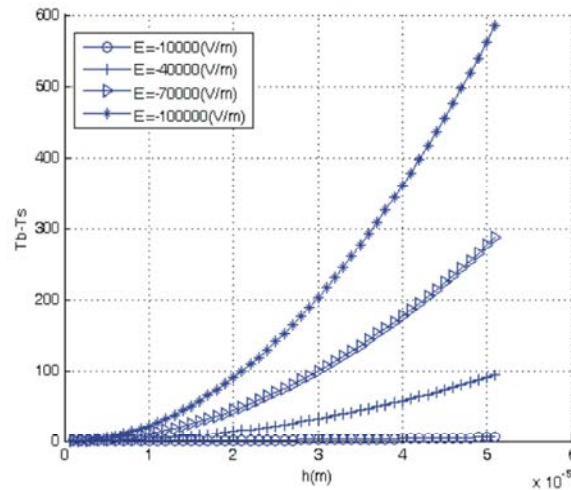


Fig. 3: For the different values of external electric field (E), this figure shows the effect of increasing microchannel height on the temperature.

Table 1: The values for constants and parameters used in the simulations

Parameter	Value/range	Unit
ϵ_r	80	-
ϵ_0	8.85×10^{-12}	F/m
ρ (liquid density)	1000	Kg/m ³
μ	1×10^{-3}	Pa.s
F	96 485.3415	A.s/mol
k_B	1.381×10^{-23}	J/.K
h (microchannel height)	1~50	μm
E_z (external electric field)	-10 ~ -100	kV/m
s -potential	0.05	V
e (elementary charge)	1.602×10^{-19}	C
z (valence)	1	-
c_p (specific heat capacity)	4.1813	kJ/kg.K
k (Thermal conductivity)	0.6	W/m.K
T_s (Temperature at the walls)	300	.K

From this equation, it is clear that for the same applied electric field, the smaller microchannels have lower electric current. Thus, the effect of Joule heating decreases by reduction in size.

CONCLUSION

Electroosmotic flow is one of the widespread forms of flow fields in microscale systems. These microscale devices can have different applications ranging from biological studies to micro-electronic cooling and micro-energy systems. Although this type of flow field has many advantages such as being compact and vibration free, few drawbacks are also associated with it. Joule heating is one of these weak points. Having comprehensive study on the thermal characteristics of the

electroosmotic flow helps the researcher to reduce the negative aspects of Joule heating. In this study, the analytical formula was obtained to find the temperature distribution of the electroosmotic flow in the slit microchannels. By using the obtained formula, the effects of microchannel dimension (height) and externally applied electric field on the temperature distribution of the flow field were investigated. It was shown that, for the larger microchannels, the influence of the external electric field can be intensified on the temperature of the flow field. However, in the small microchannels, its effect may be negligible.

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Nomenclature:

- u : Velocity
 P : Pressure
 μ : Dynamic viscosity
 ρ : Liquid density
 ε : Relative permeability
 ε_0 : Absolute permeability
 E_z : External electric field
 φ : Potential of electric double layer
 ζ_0 : Potential of electric double layer at the walls
 z : Valance
 e : Elementary charge
 n_0 : Ionic bulk number concentration
 k_b : Boltzmann constant
 T : Temperature
 T_s : Temperature at the walls
 k : Thermal conductivity
 c_p : Specific heat capacity
 σ : Electric conductivity
 h : Half channel height