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The Determination of Nonlinear Equations to Predict Scour Hole Dimensions Downstream of Siphon Spillway

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Abstract: Siphon spillways are hydraulic structures which can pass surplus storm water automatically. Hence their stability has more importance. The reliable predictor derived physical modeling have significant role in this case. Different types of equations can be derived using statistical methods which each type has its applied limitations. This paper has been attempted to derive non-linear equations to predict scour hole dimensions downstream siphon spillway. Scour hole dimesions as: d_s(the maximum depth of scour hole), l_2 (upstream hill distance from the bucket), h_u (height of upstream hill) and h_d (height of downstream hill) were investigated through this research work. Using Bukingham pi theorem, three dimensionless parameters were derived as $\frac{d50}{h_t}, \frac{Q}{h_t^2\sqrt{g(G_S-1)h_t}}$. Each scour hole dimension was related to mentioned dimensionless parameters using multiple linear regression technique. The results of physical modeling and simulation values were compared using statics indices. The results of predictions had good agreement with measured valus.

Key words: Siphon spillway · Scour Hole Dimension · Non-Linear Equation · Bukingham Pi Theorem · Multiple Linear Regression

INTRODUCTION

Spillways are hydraulic structures which are designed to prevent overtopping of a dam at a place that is not designed for overtopping. These structures convey surplus store water to downstream of dam. Siphon spillways are one type of spillways which can pass fully design discharge with slight increase in water level at upstream of dam. Also, no need to mechanical equipments and other device driver duo to automatic performance is the benefit of siphon spillway. But, water flow over spillways and through the bottom outlets has a great potential in producing the scour on the bed material at the tail water. On the other hand, scour phenomena has great effect in view of structural stability of siphon spillway [1].

Decade 1930, was the beginning of scour hole study. As regards erosion is a two-phased (water-sediment) phenomenon, study on scour hole at downstream of siphon spillway is one of the most complicated hydrodynamic problems because scour phenomena is influenced by several variables such as: flow parameters, sedimentary materials, time, channel geometry, etc [1].

Generally, the study methods of scour phenomena are divided in two general groups: (a) numerical modeling (b) physical modeling. The first method attempts to predict the scour hole dimensions at downstream of spillway using numerical simulation of prototype properties. The outputs of these models aren't reliable alone for practical applications. The second method is based on physical model construction with dynamical and kinematical similarities and direct measurements. The results of this method are close to prototype. Thus, derived equations from physical modeling data will have more applications [2].

Many formulas have been developed to predict the scour hole dimensions at downstream of buckets which are based on laboratory as well as prototype observations. Schoklitsc was the first researcher who conducted a study to predict scour hole depth. He used sand particles with uniform and non-uniform sizes and developed the following equation [3]:

$$T=4.75H^{0.2}q^{0.57}d_{90}^{-0.32} \quad (1)$$

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Which T is scour hole depth under water surface (m), q is discharge per unit width of channel (m^2/s), H is height difference of upstream and downstream (m) and d_{90} is the particle size at which 90% by weight of the sample is finer (m).

Damel *et al*. used experimental and field data in India and presented the following equation [3]:

$$Y_s + Y_0 = 0.55(qH)^{0.5} \quad (2)$$

Which the left side of equation (y_s+y_o) is the flow depth from scour hole (m), q is discharge per unit width of channel (m^2/s) and H is height difference of upstream and downstream (m).

Hoffmans presented the following semi-analytical relation for scour hole of plunge pools [4]:

$$Y_m + h_t = c_{v2}\sqrt{\frac{qu_1 \sin\theta}{g}} \quad (3)$$

$$C_{v2} = \frac{20}{(D_{90}^*)^{\frac{1}{3}}} \qquad d_{90} < 0.125 \quad (4)$$

$$C_{v2} = 2.9 \qquad d_{90} \geq 0.125 \quad (5)$$

$$D_{90}^* = d_{90}(\frac{gG_s}{\theta^2})^{\frac{1}{3}} \quad (6)$$

Which G_s is specific gravity of sediment particles, θ is water kinematic viscosity, g is gravity acceleration, u_1 is mean velocity of jet, y_t is scour hole depth.

Mason presented the following equation [5]:

$$D=K\frac{q^A H^y h^w}{g^v d_z} \quad (6)$$

Which v=0.3, w=0.15, z=0.1, y=0.15-(H/200), x=0.6-(H/300), K= 6.42-3.1H^{01}. In this equation, D is scour hole depth (m), q is discharge per unit width of channel (m^2/s), H is height difference of upstream and downstream (m), h is water depth (m), g is gravity acceleration (m^2/s) and d is median size of sediment particle (m).

Azamathullah *et al*. developed the following equations using various experiment data [6]:

$$\frac{d_s}{d_w} = 6.914(\frac{q}{\sqrt{gd_w^3}})0.694(\frac{H}{d_w})0.0815(\frac{R}{d_w}) - 0.233(\frac{d_{50}}{d_w})0.196(\varphi)0.196 \quad (7)$$

$$\frac{I_s}{d_w} = 9.58(\frac{q}{\sqrt{gd_w^3}})0.42(\frac{H}{d_w})0.28(\frac{R}{d_w})0.043(\frac{d_{50}}{d_w})0.037(\varphi)0.34661 \quad (8)$$

$$\frac{w_s}{d_w} = 5.42(\frac{q}{\sqrt{gd_w^3}})0.15(\frac{H}{d_w})0.55107(\frac{R}{d_w})0.1396(\frac{d_{50}}{d_w})0.242(\varphi) - 0.16 \quad (9)$$

Which d_s is scour depth (m), q is flow discharge per unit width of channel (m^2/s), d_w is tail water depth (m), H is falling height (m), d_{50} is median size of sediment particle (m), R is curvature radius of bucket (m), l_s is scour hole length (m), w_s is scour hole width (m) and ö is lip angle of bucket (rad).

Sui *et al*. studied scour hole properties downstream square buckets and showed that ratio of channel width to jet width, tail water depth, Froud number as $\frac{U}{\sqrt{gG_s d_{50}}}$ and specific gravity of sediment particles have significance effects on scour hole [7].

Jafarinia *et al*. conducted a study to predict scour hole dimensions downstream of siphon spillway. They developed the following equations [2]:

$$\frac{d_s}{h_t} = 0.924(\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}) - 1.589(\frac{d_{50}}{h_t}) + 0.097\alpha - 0.0808 \quad (10)$$

$$\frac{I_0}{h_t} = 3.373\left(\frac{Q}{h_t^2\sqrt{g(G_s\text{-}1)h_t}}\right) - 0.421\left(\frac{d_{50}}{h_t}\right) + 4.25\alpha - 1.967 \tag{11}$$

$$\frac{I_s}{h_t} = 7.809\left(\frac{Q}{h_t^2\sqrt{g(G_s\text{-}1)h_t}}\right) - 20.815\left(\frac{d_{50}}{h_t}\right) + 3.627\alpha - 3.395 \tag{12}$$

Which d_s is scour hole depth (m), l_o is distance of start point of scouring from the bucket up (m), l_s is length of scouring (m), Q is flow discharge (m^3/s), d_{50} mean size of sediment particle (m), h_t is tail water depth (m), g is gravity acceleration (m/s^2), G_s is specific gravity of sediment particle and á is lip angle of bucket (rad). Correlation coefficient between measured and calculated values of d_s, l_o and l_s according to equations (10) to (12) were obtained 0.933, 0.975 and 0.927, respectively.

As seen, scour hole has several properties which their determination and prediction has high importance. But it has not been done more studies about it, recently. The existence relations have their limitations which make them restricted to determined condition of flow discharge, sediment size, etc. Thus, new developed equations will remove these restrictions. In this paper, it attempts to develop non-linear equations to predict scour hole dimensions using data.

MATERIALS AND METHODS

Experimental Set-up: Figure 1 shows an sketch of physical model of siphon spillway which was used to gather laboratory data. To make possible to photography and see scour pattern from side of flume, one side was constructed with Plexiglas. A rectangular gate was installed for flow measuring and control of tail water depth. The designed siphon spillway was made from Plexiglas and installed at the middle of the flume. The cross section of siphon spillway was rectangular with dimension of 7.5cm×30cm.in the experimental procedure, three bucket angle were investigated: 30°, 45° and 60°. Four different flow discharges as following were used: 0.039, 0.042, 0.045 and 0.05(m^3s^{-1}). The tests were carried out for four tail water depths: 15, 20, 25 and 30(cm). the canal bed profile was measured using a laser meter with accuracy of 1(mm).data were recorded over a net of wire which was generated over the model to fix the points of measurements. The dimensions of mesh were 10cm×10cm. On the other word, each test contained 279 points which were recorded. These data were used in this study. In this paper, as illustrated in the figure 1, the following properties of scour hole were considered d_s, l_2, h_u, h_d which are the maximum depth of scour hole, upstream hill distance from the bucket, height of upstream hill and height of downstream hill, respectively. Dimensional analysis and multiple linear regression were used to derive predictor equations of mentioned scour hole properties. Effective parameters on the scour hole dimensions can be written as:

$$d_s, l_2, h_u h_d = f(Q, \rho_w, \rho_s, h_v, \mu, d_{50}, g, \alpha) \tag{13}$$

Which Q is flow discharge, ρ_w is flow density, ρ_s is sediment density, h_t is tail water depth, μ is water dynamic viscosity, d_{50} is sediment mean size, g is gravity acceleration and α is lip angle of bucket. According to Bukingham pi theorem, three dimensionless parameters were derived which the parameters on the left hand side of equation (13) can be written as a function of them as following:

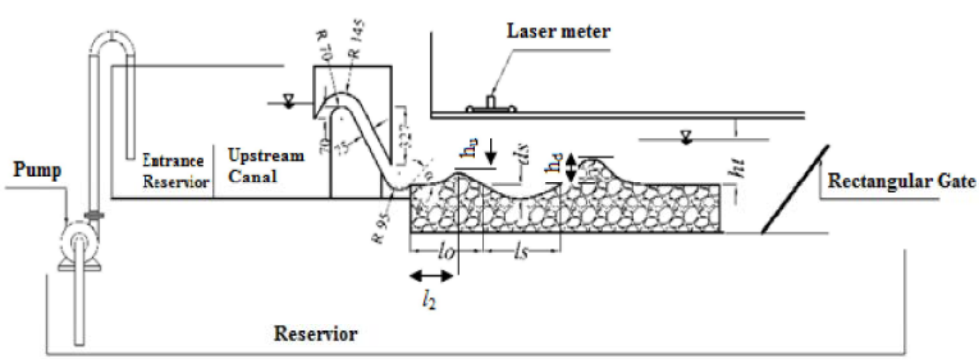

Fig. 1: Experiment installation

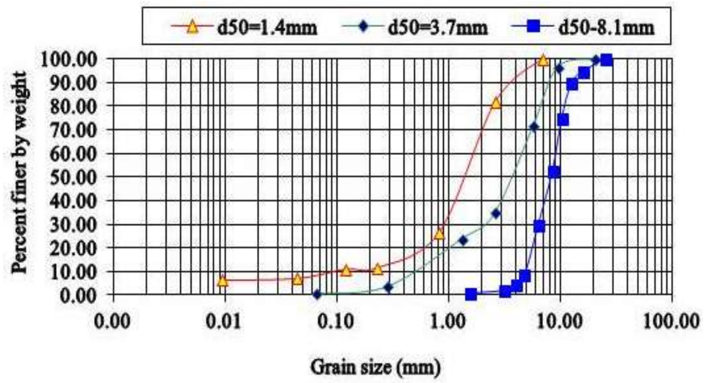

Fig. 2: Grain sizing curve of bed materials downstream the bucket

$$\frac{d_s}{h_t},\frac{l_2}{h_t},\frac{h_u}{h_t},\frac{h_d}{h_t}=f\left(\frac{d_{50}}{h_t},\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}\right) \quad (14)$$

Which $G_S=\frac{\rho_S}{\rho_W}$ is the specific gravity of sediment particle.

Figure 2 shows grain sizing curve of sediments which were used in this study. These particles were non-cohesive with size(d_{50}) of 1.4mm, 3.7mm and 8.1mm. Materials were full filled downstream of the bucket in length of 3m and thickness of 30cm. Three lip angle of bucket were used: 30°, 45° and 60°. Four different flow discharges were studied: 0.039, 0.042, 0.045 and 0.05m^3/sec. The canal bed was leveled carefully at the beginning of each experiment. All experiments were conducted for four tail waters: 15, 20, 25 and 30cm. a laser meter was used to measure canal bed profile with accuracy of 1mm. Data were recorded over a net with mesh size 10×10cm. On the word, 279 data were recorded during each test. At the end of canal, a rectangular spillway had been installed to measure passed discharge through the canal. Water level was measured by a liminimeter with accuracy of o.1mm. All tests were done in submerged jet and black-water condition.

The following equation describes the scour hole dimensions in non-linear format:

$$\frac{\varphi}{h_t}=x_1\left(\frac{d_{50}}{h_t}\right)^{x2}\left(\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}\right)^{x3}\alpha^{x4} \quad (15)$$

Which φ denotes scour hole properties, x_1, x_2, x_3 and x_4 are constants which must be determined. Multiple linear regression technique was used to calculate x_1, x_2, x_3 and x_4.

Extending multiple linear regression technique to dependent variable y_i and independent variables x_1, x_2, …, x_i yields the following equation [8]:

$$y_i=\beta_o+\beta_1x_{1k}+\ldots+\beta_kx_{ik} \qquad i=1,2,\ldots,n\ (n>k) \quad (16)$$

Which β_o is a constant value and (β_1, β_2,…, β_k) are the coefficients of independent variables. According to equation (16), equation (15) should be linear by taking logarithm from both side of equation (15):

$$\text{Log}\left(\frac{\varphi}{h_t}\right)=\text{Log}\,x_1+x_2\,\text{Log}\,\frac{d_{50}}{h_t}+x_3\,\text{Log}\,\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}+x_4\,\text{Log}\,\alpha \quad (17)$$

This equation was applied for properties of scour hole dimensions which are listed in table 1. Then, the final equations for prediction of scour hole properties are as following:

$$\frac{d_s}{h_t}=0.748\left(\frac{d_{50}}{h_t}\right)^{-0.068}\left(\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}\right)^{1.14}\alpha^{0.367} \quad (18)$$

$$\frac{l_2}{h_t}=1.784\left(\frac{d_{50}}{h_t}\right)^{-0.083}\left(\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}\right)^{-0.095}\alpha^{2.58} \quad (19)$$

$$\frac{h_u}{h_t}=0.74\left(\frac{d_{50}}{h_t}\right)^{-0.167}\left(\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}\right)^{-0.1}\alpha^{2.4} \quad (20)$$

$$\frac{h_d}{h_t}=0.614\left(\frac{d_{50}}{h_t}\right)^{0.018}\left(\frac{Q}{h_t^2\sqrt{g(G_s-1)h_t}}\right)^{0.618}\alpha^{-0.161} \quad (21)$$

Measured and calculated values of scour hole dimensions are plotted in figures 3 to 6. Table 2 shows the result of statistical analysis between measured and calculated values of scour hole dimensions.

Table 1: The final results of multiple linear regression technique for eq. (15)

		Scour hole properties dimensions			
Number	Exponent	d_s	l_2	h_u	h_d
1	x_1	0.748	1.748	0.074	0.612
2	x_2	-0.068	-0.083	-0.167	0.018
3	x_3	1.14	-0.095	-0.1	0.618
4	x_4	0.367	2.58	2.4	-0.161

Table 2: The result of statistical analysis between measured and calculted values of scour hole dimensions

	d_s/h_t		l_2/h_t		h_u/h_t		h_d/h_t	
	Calculated	Measured	Calculated	Measured	Calculated	Measured	Calculated	Measured
Number	230		180		150		220	
Mean	0.44	0.46	1.07	1.2	0.14	0.13	0.39	0.41
Standard diviation	0.31	0.29	0.48	0.53	0.053	0.051	0.14	0.15
Correlation Coefficient	0.923	0.912	0.84	0.921				

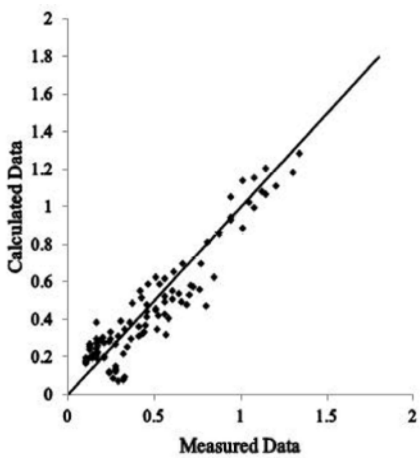

Fig. 3: Measured values versus Calculated values of d_s/h_t

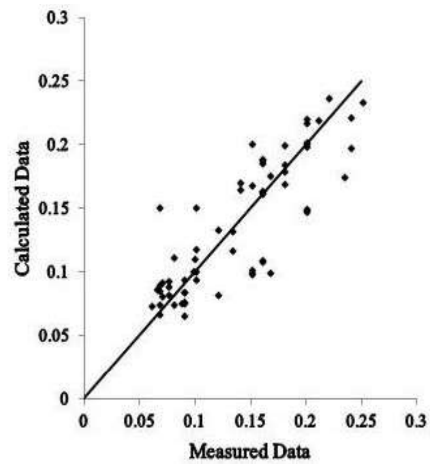

Fig. 5: Measured values versus Calculated values of h_u/h_t

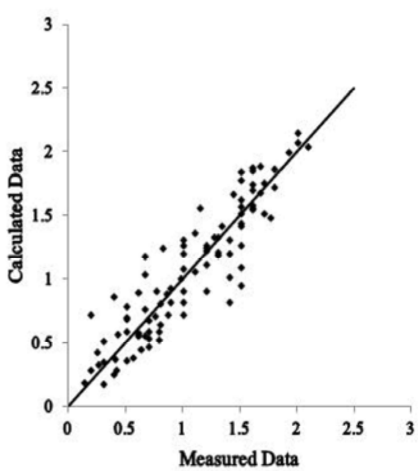

Fig. 4: Measured values versus Calculated values of l_2/h_t

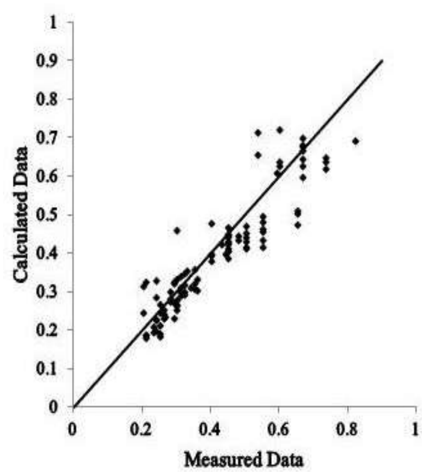

Fig. 6: Measured values versus Calculated values of h_u/h_t

According to figures 3 to 6 and table 2, there is good agreement between measured and calculated values of scour hole dimensions. It is worth noting that all scour hole dimensions were described using non-linear equations while derived linear equations by Jafarinia *et al*. described only three dimensions [2].

RESULTS AND DISCUSSION

In this paper, it was attempted to derive non-linear equations to describe scour hole dimensions at the downstream of siphon spillway using physical modeling and Bukingham pi theorem. A model of siphon spillway was made from plaxiglass and different conditions of flow properties (as: discharge, tail water), structural properties (as: lip angle of bucket) and sediment properties (as: size) were studied on the scour hole dimensions. The scour hole depth (d_s), upstream hill distance from the bucket (l_2), height of upstream hill (h_u) and height of downstream hill (h_d) are the dimensions which equations were derived to describe them. Three dimensionless parameters as: $\frac{d_{50}}{h_t}, \frac{Q}{h_t^2\sqrt{g(G_S-1)h_t}}, \alpha$ were used to predict of scour hole dimensions. Multiple linear regression technique was applied to determine the exponents of non-linear equations. Finally, it was showed, according to table 2 and figures 3 to 6, that these equations have good capability to describe mentioned properties of scour hole.

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