

## A Study of the Possibility of Application of the Advanced Hypertrochoid Curve in the Inner Surface of the Stator of the Hydraulic Balance Vane Pump

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**Abstract:** In the latest our works, Performance of a fixed displacement-hydraulic balance vane pump, theoretically and practically was studied by application of the basic hypertrochoid curve in the inner surface of its stator. Also the effect of the inertial force of the vanes on the performance of the pump with this curve, a theoretical analysis of the internal pressure distribution in the pump and of the resulting forces and torques applied to its components were studied. This study presents the possibility of application of the advanced hypertrochoid curve in the inner surface of its stator. By using this curve in the inner surface of the stator, we are desirous of improving a sealing action between pressure and suction sides of the pump, in addition to improving of the inertial reaction of the vanes in each position where these vanes have a radial movement because, it causes a smooth sliding motion of the vanes and hence, a higher performance of the pump while attaining a longer life of it because of decreasing vane- tip and inner surface of stator wear and decreasing of vibration, noise and high stresses.

**Key words:** Hydraulic balance vane pump • Stator • Advanced hypertrochoid curve • Vane motion

### INTRODUCTION

Hydraulic balance vane pumps comprise a shaft, a tubular stator, a rotor having a plurality of vane grooves fixed on the shaft and vanes adapted to slide in said vane grooves. When the vane pump operates, the vanes reciprocate within the respective vane grooves while sliding on the inner surface of the stator in accordance with the rotation of the rotor. It is noted that the initial action of the vanes will be caused by centrifugal force and once the pump has developed pressure, the vanes will be exposed to pressure and will be held against the inner surface of the stator at all times. Rotation of the rotor, by virtue of the increasing area between the rotor and the stator surfaces, will cause inlet vacuum and entrance of oil into the pockets between the vanes. By increasing of rotation angle of rotor, the radius of

the stator and then the vane pocket area will gradually be decrease and a volume of oil will be delivered under pressure to the outlet port [2].

### The Hypertrochoid Curve Description:

The hypertrochoid curve is described by the following complex equations [3]:

$$Z = X + jY = \sum_{k=1}^{n+1} A_k \exp j(\alpha_k K + \beta_k) \text{ where } 0 < K < K^* \quad (1)$$

Wherein  $j$  is imaginary unit and  $\exp j$  is imaginary exponential function;  $A_k, \alpha_k, \beta_k$  are real number defining parameters of a particular form of the hypertrochoid;  $K$  is a real parameter varying between zero and a particular values  $K^*$ , where the affix once covers the hypertrochoid;  $n$  is an integer defining the order of the hypertrochoid (if  $n=1$ , Basic hypertrochoid and second order and if  $n=2$ , advanced hypertrochoid).

This curve has an order of symmetry,  $S_s$ , with respect to its centre point with centre point angle  $\theta_c$  ( $\theta_c \neq 0$ ). In a plane curve, an order of symmetry  $S_s$  with respect to a point represents the quality of the curve by which after one revolution with an amplitude  $2\pi/S_s$  radians about that point, the curve will be brought in coincidence with itself. By contrast, the closed advanced hypertrochoid curve has order of symmetry  $S_H$  with respect to a centre point with centre point angle  $\theta_H$  that differs from the order of symmetry  $S_s$  and is expressed by a rational number according to the formula:

$$S_H = \frac{\theta_c + \theta_H}{\theta_H} \cdot S_s = \frac{a}{b}$$

wherein  $\theta_H = \left( \frac{2\pi}{S_s} \right) - \theta_c$  (2)

The order of symmetry  $S_H$  of the advanced hypertrochoid is obtained by a suitable selection of the form parameter  $\alpha_k$ . One of the parameters  $\alpha_k$ ,  $\alpha_m$ , is chosen as the reciprocal value of  $S_H$  and The remaining form parameters  $\alpha_k$ , both positive and negative, differ from  $\alpha_m$  by an arbitrary integer.

For determining of  $A_k$ , assume that every hypertrochoid curve is contained in a circular ring whose outer circumference circumscribing the hypertrochoid is necessity real and has a radius  $R_e$  which is smaller than or equal to  $R_e^*$  and whose inner circumference has a radius  $R_i$  which is greater than or equal to  $R_i^*$ .

In this case the hypertrochoid surface of which such a hypertrochoid is the contour line, develops a cylindrical core whose contour line is the inner circumference inscribed in the hypertrochoid. This particular hypertrochoid type will be designated as a hypertrochoid with a core. Further conditions are set according to which the nominal radius  $R_i$  of the core of the hypertrochoid is identical with the nominal radius  $R_r$  of the rotor and the radius  $R_e$  of the circumscribed circumference of the annular stator equals:

$$R_e = R_r + H \quad (3)$$

Wherein H is the maximum stroke of the vanes.

Other necessary values are  $\beta_k$  that one of the parameters is determined so that the polar angle corresponding to the core is equal to  $\theta_c/2$  and the remaining parameters differ from it by amplitude  $2\pi/S_s$  radians.

**The Advanced Hypertrochoid Curve Characteristics:**  
By considering  $n = 2$  in the hypertrochoid curve equation, the advanced hypertrochoid curve will be achieved:

$$Z = X + jY = \sum_{k=1}^3 A_k \exp j(\alpha_k K + \beta_k) =$$

$$A_1 \exp j(\alpha_1 K + \beta_1) + A_2 \exp j(\alpha_2 K + \beta_2) +$$

$$A_3 \exp j(\alpha_3 K + \beta_3) \quad (4)$$

By considering the characteristics of the hypertrochoid curve, we can determine all of parameters of the advanced hypertrochoid curve, as below:  
Calculating of the Parameters  $\alpha_k$  and  $\beta_k$ .

$$\alpha_2 = \frac{b}{a} = \alpha_m, \alpha_1 = \alpha_m(-1) = \frac{la-b}{a},$$

$$\alpha_3 = \alpha_m(+1) = \frac{la+b}{a}, \beta_2 = \frac{\theta_c}{2} \text{ and}$$

$$\beta_1 = \beta_3 = \frac{2\pi}{S_s} + \frac{\theta_c}{2} \quad (5)$$

Calculating of the Parameter  $A_k$

$$n=2 \Rightarrow R_e^* = \sum_{k=1}^3 A_k = A_1 + A_2 + A_3$$

Because  $R_e \leq R_e^* \Rightarrow R_e = A_1 + A_2 + A_3$  And

If assume  $m=2$ :

$$R_i^* = A_m - \sum_{\substack{k=1 \\ k \neq m}}^3 A_k = A_2 - (A_1 + A_3)$$

Because  $R_i \geq R_i^* \Rightarrow R_i = A_2 - (A_1 + A_3)$

Because of assuming  $R_e = R_r + H$  and  $R_i = R_r$

$$\text{then: } A_2 = R_r + \frac{H}{2} \text{ and } A_1 = A_3 = \frac{H}{4} \quad (6)$$

By substituting the above parameters in the advanced hypertrochoid curve and using the definition of  $e^{j\theta}$  (or  $\exp j\theta$ ), the above equation will be as follow:

$$Z = X + jY = R \exp j\theta = R(\cos\theta + j\sin\theta) \Rightarrow$$

$$\begin{cases} X = R \cos\theta \\ Y = R \sin\theta \end{cases} \quad (7)$$

$$\Rightarrow \begin{cases} X = \frac{H}{4} \cos \left( \frac{la-b}{a} K + \frac{2\pi}{s_S} + \frac{\theta_C}{2} \right) + \\ \left( R_r + \frac{H}{2} \right) \cos \left( \frac{b}{a} K + \frac{\theta_C}{2} \right) + \\ \frac{H}{4} \cos \left( \frac{la+b}{a} K + \frac{2\pi}{s_S} + \frac{\theta_C}{2} \right) \\ Y = \frac{H}{4} \sin \left( \frac{la-b}{a} K + \frac{2\pi}{s_S} + \frac{\theta_C}{2} \right) + \\ \left( R_r + \frac{H}{2} \right) \sin \left( \frac{b}{a} K + \frac{\theta_C}{2} \right) + \\ \frac{H}{4} \sin \left( \frac{la+b}{a} K + \frac{2\pi}{s_S} + \frac{\theta_C}{2} \right) \end{cases} \quad (8)$$

**Description of the Preferred Embodiment:** For using the advanced hypertrochoid curve in the inner surface of the stator of a balance vane pump, we assume the recommended parameters as shown in Table 1:

The analysis of the advanced hypertrochoid curve characteristics with the above given parameters was carried out by FORTRAN Power station software and then the results was imported to TECHPLOT software for drawing the curve. The advanced hypertrochoid curve that drawn in accordance with the before described considerations is shown in Figure 1. The main disadvantage of this curve is the difficulty in providing a sufficient seal between the pressure side and the adjoining suction side between the stator and the rotor, caused by the fact that the hypertrochoid can not conform the profile of the rotor over a finite centre point angle. For using this curve at the inner surface of the stator of the hydraulic balance vane pump, only four adequate lobes of this curve can be selected. As shown in Figure 2 and as discussed earlier, four selected lobes of the advanced hypertrochoid curve have discontinuities at four points A, B, C and D; Then, if we can't use suitable transient curves at these discontinuous regions, the profile of the stator has difficulty in providing a sufficient seal between the pressure side and the adjoining suction side between the stator and the rotor (points C and D) and also due to existing of another discontinuity points at the inner surface of the stator (points A and B), the vanes sliding on the said area are made to move irregularly when they pass such discontinuous points and the irregular motion of the vanes causes chattering of the vanes resulting in local wear of the vanes, a poor seal between the vanes tip and the inner surface of the stator in discontinuous points, generation of noise and other troubles.

Table 1: Recommended parameters for the vane pump

Order of Symmetry, Ss	Radius of the Rotor, Rr	Max. Vane Stroke, H
2	31.75 mm	8.05 mm

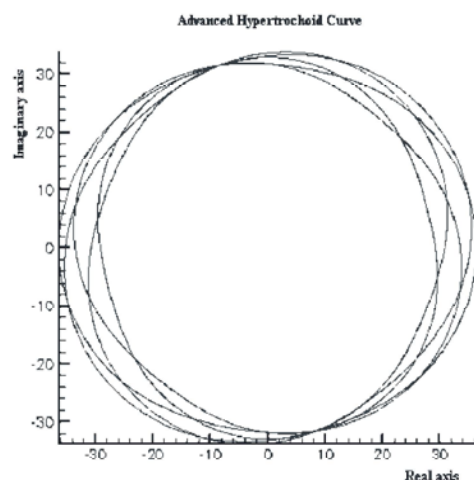


Fig. 1: Advanced hypertrochoid curve (n=2)

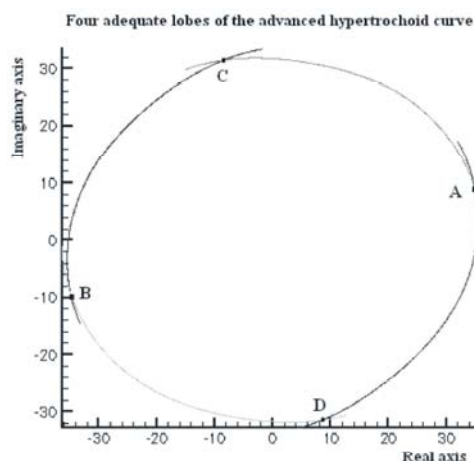
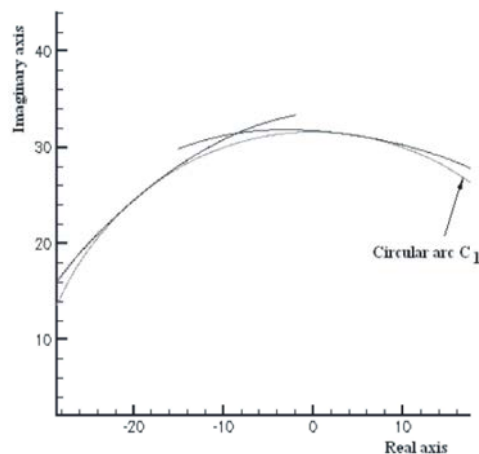


Fig. 2: Advanced hypertrochoid curve (n=2)

Fig. 3: Contacting of the advanced hypertrochoid curve by circle C<sub>1</sub>

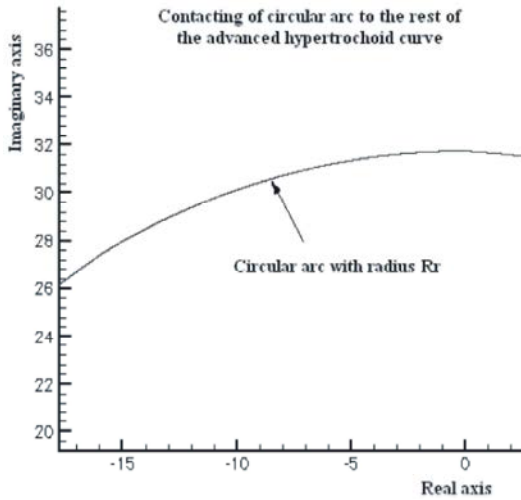


Fig. 4: Sealing zone of the contour

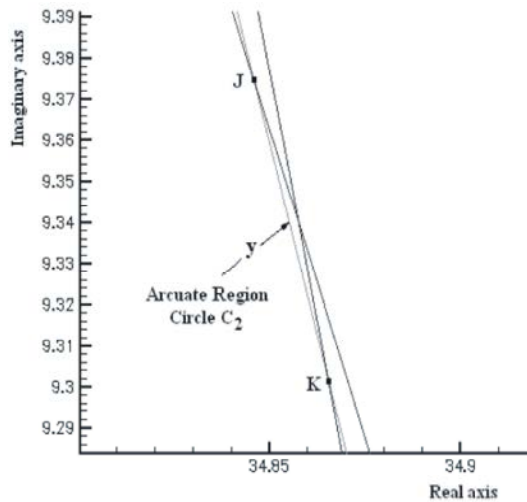


Fig. 5: Contacting of the advanced hypertrochoid curve by circle  $C_2$  at two points J and K

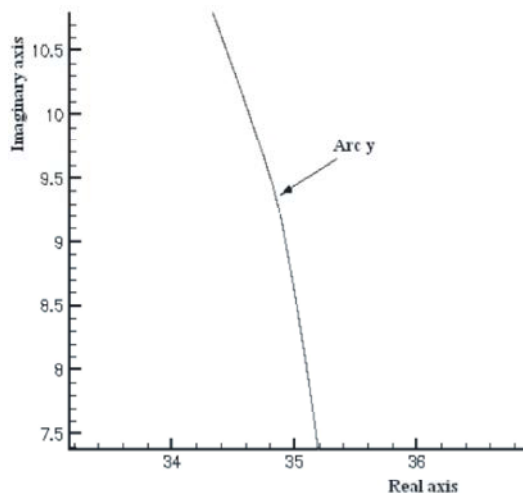


Fig. 6: Contacting arc y

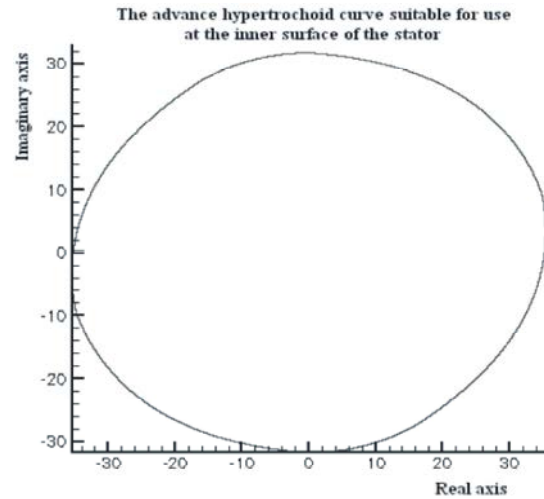


Fig. 7: Advanced hypertrochoid curve with circular arc in the sealing zone

**Discontinuities at Points C and D:** According to some techniques that mentioned in patent nos. 4,507,068, 4,432,711 and 4,556,372 [3-5], to avoid this disadvantage, as shown in Figure 3, at points C and D, the contour line of the stator surface is provided with a circular arc  $C_1$  having a smallest possible clearance relative to the rotor ( $R_r$ ) and conforming this profile over a small angle, ( $\theta_c$ ), sufficient for guaranteeing a sufficient sealing action and this circular arc is connected to the rest of the hypertrochoidal contour line of the stator (Figure 4). According to calculating, the centre point angle ( $\theta_c$ ) determined and its value is  $50^\circ$ .

**Discontinuities at Points a and B:** To avoid this disadvantage [3-5], as shown in Figure 5, at points A and B, the two lobes of the advanced hypertrochoid curve is contacted by a circle  $C_2$  at least at two points J and K and the discontinuity point at this location will be eliminated by using an arc y, the segment of the circle  $C_2$ , to obtain a contour constituting an inner peripheral surface of the tubular stator (Figure 6).

Then after using the above considerations that mentioned in 4.1 and 4.2, as shown in Figure 7, a special case of an advanced hypertrochoid suitable for using at the inner surface of the stator was obtained. Figure 8 illustrates a graphical picture of the ring of the stator by using the advanced hypertrochoid curve at the inner surface of it. Also the schematic of cartridge (the rotor, the shaft and the stator) of the vane pump with new profile is shown in Figure 9. By comparison between this cartridge and cartridge of

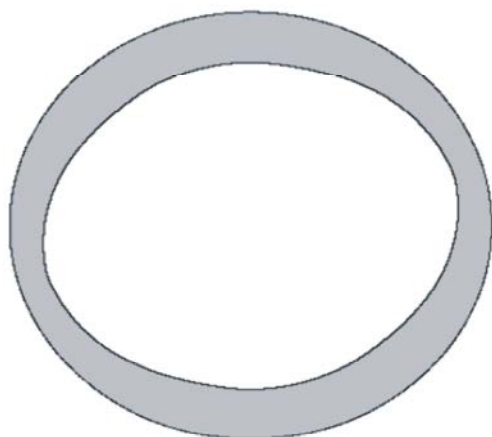


Fig. 8: Graphical picture of the stator with new profile

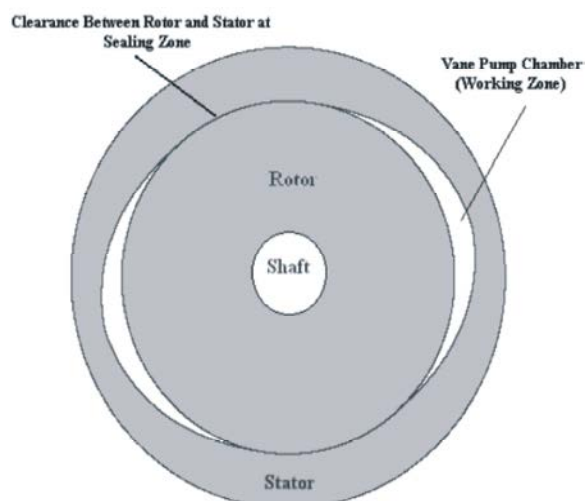


Fig. 9: Schematic of the cartridge with new profile

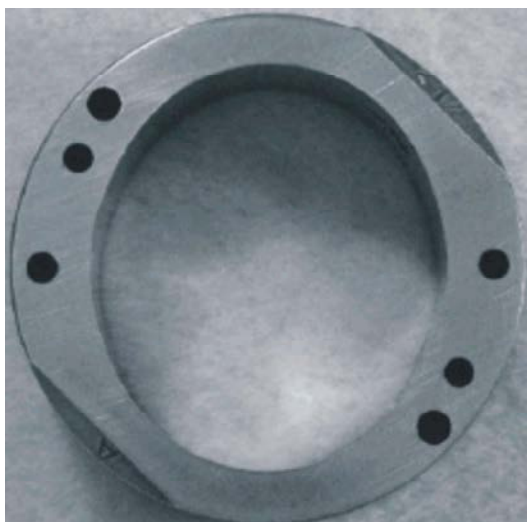


Fig. 10: the stator with basic hypertrochoid curve

another hydraulic balanced vane pump, we understand that the inner peripheral contour of an advanced hypertrochoid curve offers the same advantage as that provided by the inner peripheral contour of a basic hypertrochoid curve ( $n=1$ ). In the latest our works [6-8] that shown in Figure 10, the basic hypertrochoid curve was used at the inner surface of the stator of hydraulic vane pump- model 20V14 and after construction of the new stator with this profile and testing it, we obtained sufficient sealing action and good pump performance.

## CONCLUSIONS

The advanced hypertrochoid curve can be used in the inner surface of the stator of a fixed displacement vane pump, Hydraulic vane motor and vane type compressor. It will be caused improving a sufficient sealing action between pressure and suction sides of the pump (motor or compressor) and so decreasing the amount of leakage of working fluid from the clearance between the rotor and the stator and between the vanes tip and the inner surface of the stator in the sliding contact between the two and, accordingly, increasing the volumetric efficiency and its output flow because of the good continuity between circular arc and hypertrochoid curve at contact points in the end of sealing zone and smooth sliding motion of the vanes on the inner surface of the stator.

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