World Applied Sciences Journal 15 (6): 853-859, 2011
ISSN 1818-4952

Effect of Axial Force on Natural Frequency of Lateral Vibration of Flexible Rotating Shafts

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Abstract: In this paper, effect of an axial force and shaft characteristics on the lateral natural frequencies of a flexible rotating shaft with a cubic nonlinearity is investigated. Also this research performed for viscoelastic Voigt-Kelvin rotating shaft. The shaft is assumed to be uniform, and the Euler-Bernoulli theory is used to model the rotating shaft. Method of multiple scales is used to solve the dimensionless partial differential equation of the motion. Linear and nonlinear lateral natural frequencies are plotted for various shaft parameters and effects of these parameters and cubic nonlinearity is also discussed.

Key words: Flexible rotating shaft · Lateral vibration · Viscoelastic shaft · Multiple scales method · Linear and nonlinear frequency

INTRODUCTION

Rotating shafts and rotors are important issue in rotating machinery. Accurate consideration of dynamics of them is necessary for successful design step. Bishop [1] analyzed dynamic stability of rotating shafts, with omission of the compressive force. Melanson and Zu [2] studied the free vibrations and stability of internally damped rotating shafts with general boundary conditions. Bokian [3] presented changes in the lateral natural frequency of Euler-Bernoulli beams under axial load with various boundary conditions. Chen and Ku [4] examined the dynamic stability of a cantilever shaft-disk system subjected to a periodic axial force by the finite element method and gave boundaries of the regions of dynamic instability. Shaw and Shaw [5] considered instabilities and bifurcations in non-linear rotating shaft made of viscoelastic Voigt-Kelvin material without compressive force. Hosseini and Khadem [6] investigated free vibrations analysis of a rotating shaft with nonlinearities in curvature and inertia.

In current research, lateral vibration of a flexible uniform rotating shaft subjected to axial force is studied. The Euler-Bernoulli theory is used to model the rotating shaft and the multiple scales method is applied to the partial differential equation and solved nonlinear equation of motion.

Several methods are applied to solve nonlinear equation in literatures [7-13]. Multiple scales method is very powerful and efficient method for solving partial differential equations especially vibration equations [14]. So we obtain linear and nonlinear frequencies and effect of system characteristics and shaft parameters in frequencies by means of multiple scales method.

Equation of Motion: Fig. 1 shows an element of a flexible rotating shaft. With considering nonlinear Euler-Bernoulli beam theory and equilibrium equations of force and momentum, we can extract the equation of motion as mention below.

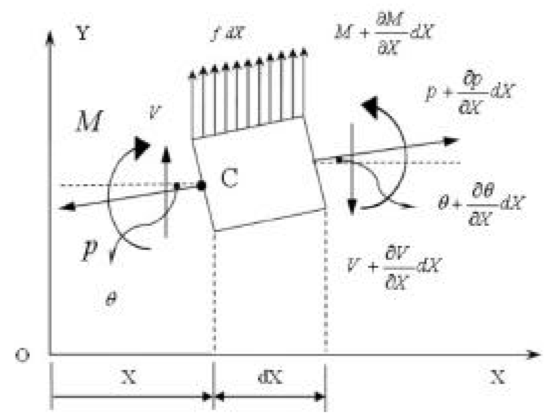

Fig. 1: An element of flexible rotating shaft

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By writing equilibrium force relation in Y direction, it is obtained:

$$fdX + V + (p+dp)\sin(\theta + d\theta) - p\sin(\theta) - (V + dV) = \rho A \frac{\partial^2 W}{\partial t^2} dX \quad (1)$$

Writing equilibrium momentum relation around of c:

$$fdX\frac{dX}{2} + (M + dM) - M - (V + dV)dX = 0 \quad (2)$$

And writing suitable approximation obtained from Taylor expansion for *sin* function and note that the power terms of $d\theta$ are negligible, as follows:

$$\sin(\theta) = \theta - \frac{\theta^3}{6} \quad (3)$$

$$\sin(\theta + d\theta) = \theta + d\theta - \frac{\theta^3}{6} - \frac{\theta^2}{2}d\theta \quad (4)$$

By assuming uniform properties, internal damping (Voigt-Kelvin model) and the plane section remains plain, the bending moments due to the internal forces in x-y plane, respectively, is written by following usual Euler-Bernoulli beam theory as

$$M(X,t) = EI(X)\frac{\partial^2 W}{\partial X^2}(X,t) \quad (5)$$

By using Eqs. (1-5) and with considering Voigt-Kelvin internal damping in the shaft and constant section area along the shaft and simplifying the obtained equation, we have:

$$EI\frac{\partial^4 W}{\partial X^4}(X,t) + EI\eta\frac{\partial^5 W}{\partial X^4 \partial t}(X,t) - p\frac{\partial^2 W}{\partial X^2}(X,t) + \frac{p}{2}\frac{\partial^2 W}{\partial X^2}(X,t)(\frac{\partial W}{\partial X}(X,t))^2 + \rho A\frac{\partial^2 W}{\partial X^2}(X,t) = f(X,t) \quad (6)$$

To analyze free vibration of the system, we consider $f(X,t)=0$.

For simplifying the analysis, we define the following dimensionless system variables:

$$w = \frac{W}{\sqrt{\frac{I}{A}}},\ x = \frac{X}{L},\ \mathrm{P} = \frac{p}{EA},\ \tau = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}},\ k = \frac{AL^2}{I},\ \zeta = \frac{\eta}{L^2}\sqrt{\frac{EI}{\rho A}} \quad (7)$$

One can obtain nonlinear equation of motion as follows:

$$\frac{\partial^4 w}{\partial x^4} + \zeta\frac{\partial^5 w}{\partial x^4 \partial \tau} + \frac{\partial^2 w}{\partial \tau^2} - \mathrm{P}k\frac{\partial^2 w}{\partial x^2} + \frac{\mathrm{P}}{2}\frac{\partial^2 w}{\partial x^2}(\frac{\partial w}{\partial x})^2 = 0 \quad (8)$$

To obtain an analytical solution for this case, we use the method of multiple scales in the following section.

Method of Multiple Scales [15]: In this section, we use the multiple scales method to analyze the first and second resonances of the system. Here, we apply multiple scales method directly to the partial differential equation of motion derived in past section, i.e. Eq. (8).

We expand w in the form:

$$w(x,T_0,T_1,\varepsilon) = w_0(x,T_0,T_1) + \varepsilon w_1(x,T_0,T_1) + \ldots \quad (9)$$

Where $T_0=\tau$ and $T_1=\varepsilon\tau$ are fast and slow time scales, respectively and ε is a small dimensionless parameter.

Also we have

$$\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial T_0^2} + 2\varepsilon\frac{\partial^2}{\partial T_0 \partial T_1} + \ldots \quad (10)$$

Substituting Eqs. (9, 10) into Eq. (8) and then equating the coefficients of ε^0 and ε^1, we arrive at

$$\varepsilon^0 : \frac{\partial^4 w_0}{\partial x^4} + \frac{\partial^2 w_0}{\partial T_0^2} - \mathrm{P}k\frac{\partial^2 w_0}{\partial x^2} = 0 \quad (11)$$

$$\varepsilon^1 : \frac{\partial^4 w_1}{\partial x^4} + \frac{\partial^2 w_1}{\partial T_0^2} - \mathrm{P}k\frac{\partial^2 w_1}{\partial x^2} = -2\frac{\partial^2 w_0}{\partial T_0 \partial T_1} - \frac{1}{2}\mathrm{P}k\frac{\partial^2 w_0}{\partial x^2}(\frac{\partial w_0}{\partial x}) \quad (12)$$

The unperturbed Eq. (11) is the linear governing equation corresponding to the free vibration of flexible rotating shaft. The perturbed Eq. (12) can solved based on the unperturbed solution.

Unperturbed Solution: The solution to Eq. (11) can be written in a complex form as

$$w_0(x,\ \tau) = Ae^{is\tau + imx} = Ae^{isTo + imx} \quad (13)$$

By substituting the solution into Eq. (11), zero order characteristic equation can be written as:

$$m^4 + i\zeta sm^4 + \mathrm{P}km^2 - s^2 = 0 \quad (14)$$

This algebraic equation has solution of the form

$$m = \pm m_f,\ m = \pm im_e \quad (15)$$

Where

$$m_f = \sqrt{\frac{1}{2}\left(\frac{\sqrt{P^2k^2 + 4s^2 + 4i\zeta s^3} - Pk}{1 + i\zeta s}\right)} \tag{16}$$

$$m_e = \sqrt{\frac{1}{2}\left(\frac{\sqrt{P^2k^2 + 4s^2 + 4i\zeta s^3} + Pk}{1 + i\zeta s}\right)} \tag{17}$$

Then the solution to Eq. (11) can be expressed as

$$w_0(x, \tau) = \phi(x)e^{is\tau} \tag{18}$$

Where the mode function is in the form

$$\phi(x) = c_1 e^{-im_f x} + c_2 e^{+im_f x} + c_3 e^{-m_e x} + c_4 e^{+m_e x} \tag{19}$$

The boundary conditions corresponding to the assumed pined-pined supports are specified at $x=0$ and 1 as follows:

$$W(0, \tau) = W(1, \tau) = 0$$
$$M(0, \tau) = M(1, \tau) = 0 \tag{20}$$

Substituting Eq. (19) into Eq. (20) yields

$$\sin(m_f) = 0 \tag{21}$$

The corresponding lateral natural frequencies are determined from Eqs. (16, 21) and are

$$is_n = -\frac{n^4\pi^4}{2}\zeta \pm i\frac{n\pi}{2}\sqrt{4n^2\pi^2 + 4Pk - \zeta^2 n^6\pi^6} \tag{22}$$

$$\omega_{d_n} = \frac{n\pi}{2}\sqrt{4n^2\pi^2 + 4Pk - \zeta^2 n^6\pi^6} \tag{23}$$

Perturbed Solution: To solve Eq. (12) we may use the unperturbed solution.

$$w_0(x, T_0, T_1) = \sin(m_f x)[A_n(T_1)e^{is_nT_0} + \bar{A}_n(T_1)e^{-is_nT_0}] \tag{24}$$

Substituting Eq. (24) into Eq. (12) yields

$$\frac{\partial^4 w_1}{\partial x^4} + \zeta\frac{\partial^5 w}{\partial x^4 \partial\tau} + \frac{\partial^2 w_1}{\partial T_0^2} - Pk\frac{\partial^2 w_1}{\partial x^2} = -2is_n \sin(m_f x)\frac{dA_n(T_1)}{dT_1}e^{is_nT_0}$$
$$+\frac{3}{2}Pkm_f^2 \sin(m_f x)\cos^2(m_f x)A_n^2(T_1)\bar{A}_n(T_1)e^{is_nT_0} + CC + NST \tag{25}$$

Where CC stands for complex conjugates and NST denotes not secular terms.

$$\int_0^1 \begin{bmatrix} -2is_n \sin(m_f x)\dfrac{dA_n(T_1)}{dT_1}e^{is_nT_0} + \dfrac{3}{2}Pkm_f^4 \sin(m_f x)\cos^2(m_f x) \\ \times A_n^2(T_1)\bar{A}_n(T_1)e^{is_nT_0} \end{bmatrix} \times dx = 0 \tag{26}$$

Integrating over the length of the shaft, gives a complex ordinary differential equation with respect to $A_n(T_1)$as follows

$$-2is_n\frac{dA_n(T_1)}{dT_1} + \frac{1}{2}Pkm_f^2 A_n^2(T_1)\bar{A}_n(T_1) = 0 \tag{27}$$

Express the solution to Eq. (27) in polar form, i.e.

$$A_n(T_1) = a_n(T_1)e^{i\beta_n(T_1)} \tag{28}$$

Where $A_n(T_1)$ and $\beta_n(T_1)$ are respectively the amplitude and the phase angle of the nonlinear free vibration of the shaft.

Substituting Eq. (28) into Eq. (27) and separating the resulting equation into real and imaginary parts gives

$$\frac{da_n(T_1)}{dT_1} = 0 \tag{29}$$

$$2s_n\frac{d\beta_n(T_1)}{dx} + \frac{Pkm_f^4 a_n^2}{4s_n} = 0 \tag{30}$$

Integrating Eq. (29) and Eq. (30) yields

$$a_n(T_1) = a_{n0} \tag{31}$$

$$\beta_n(T_1) = -\frac{Pkm_f^4 a_{n0}^2}{4s_n}T_1 + \beta_{n0} \tag{32}$$

Substituting Eqs. (31, 32) into Eq. (28) and then inserting the result into Eq. (24), gives the n^{th} lateral natural frequencies of the nonlinear vibration of the shaft as

$$\omega_{d_n}^{NL} = \omega_{d_n} - \varepsilon\frac{Pkm_f^4 a_{n0}^2}{4\omega_{d_n}} \tag{33}$$

The nonlinear lateral natural frequencies are dependent on the axial force, shaft characteristics, small parameter and amplitude.

In order to obtain final response, it is essential to consider effect of spinning speed on the lateral natural frequency. This parameter effects on natural frequency in form of an axial force, which it is [16].

$$P_s = \upsilon\rho I_p\Omega^2, \qquad I_p = \frac{\pi}{2}R^4 \tag{34}$$

Where (R) is outer radius, (Ω) is spinning speed, (ν) and (ρ) are poisson's ratio and mass density of the shaft. (P_s) is pressure caused by spinning speed, which transmit effect of spinning speed to the natural frequency.

Table 1: Nominal values of the parameters in the numerical example

Elasticity modulus (E)	2.07×10^{11}(pa)
Mass density (ρ)	7800(kg/m)
poisson's ratio (ν)	0.33

RESULTS AND DISCUSSION

In this section, we consider numerical examples to examine the effect of axial forces on the lateral natural frequencies of a nonlinear flexible rotating shaft.

The following fixed parameter values were chosen as

For performing calculation and plotting graphs via Matlab Software, effect of spinning speed on the axially force and lateral frequencies to exert.

In Figs. (2-5), the linear and nonlinear lateral natural frequencies, (ω_{d_n}) and ($\omega_{d_n}^{NL}$) are plotted versus the non-dimensional axial force P, fore the first two modes. The Figures are plotted for different values of amplitude (a_{n0}).

It is seen that the lateral natural frequencies are ascending with respect to the axial force.

As expected, the increase of axial force leads to lateral natural frequencies and effect of nonlinear term increasing.

If values of amplitude increase, the curves show stronger effects of nonlinearity. Therefore the nonlinear analysis is necessary for such systems.

Figs. (6-9), illustrate changes of non-dimensional axial force and shaft characteristics on lateral natural frequencies.

As expected, when the non-dimensional axial force increases, lateral natural frequencies and effect of nonlinearity both increase.

In addition, it is seen that if the non-dimensional shaft characteristics (k) values become large, effect of nonlinear term, approach a large value.

It is note that the dimensionless shaft characteristics (k), defined as

$$k = \frac{AL^2}{I}, I = \frac{\pi}{2}R^4 \Rightarrow k \propto (\frac{L}{R})^2 \quad (35)$$

According to Eqs. (33) and (35), increase of square (L/R) cause to increase lateral natural frequency.

The effect of axial force and nonlinearity for two modes show similar behaviors, approximately.

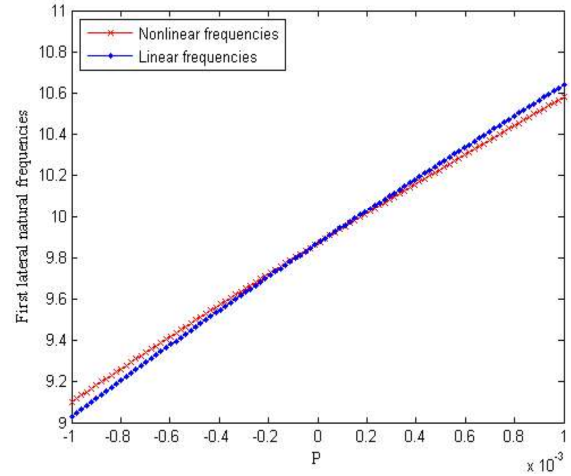

Fig. 2: Change of first lateral natural frequency versus axial force for a pin-pin shaft for $a_{n0} = 0.4$

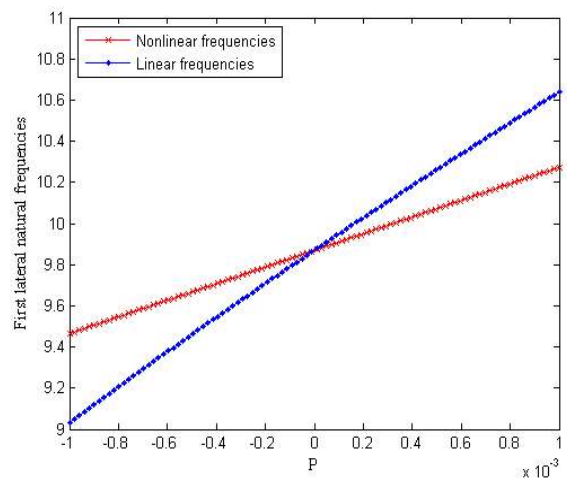

Fig. 3: Change of first lateral natural frequency versus axial force for a pin-pin shaft for $a_{n0} = 1.0$

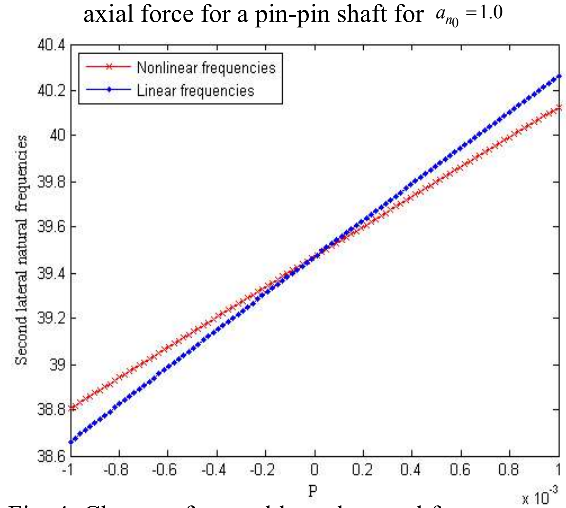

Fig. 4: Change of second lateral natural frequency versus axial force for a pin-pin shaft for $a_{n0} = 0.3$

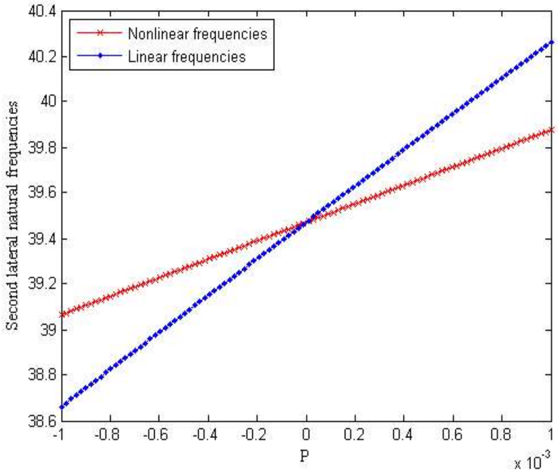

Fig. 5: Change of second lateral natural frequency versus axial force for a pin-pin shaft for $a_{n0} = 0.5$

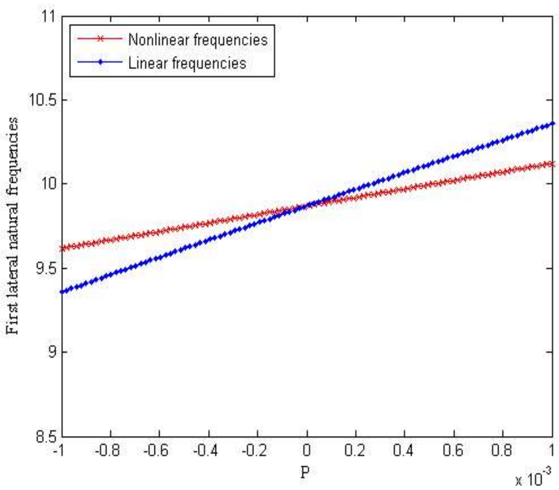

Fig. 6: Change of first lateral natural frequency versus axial force for a pin-pin shaft for k = 1000

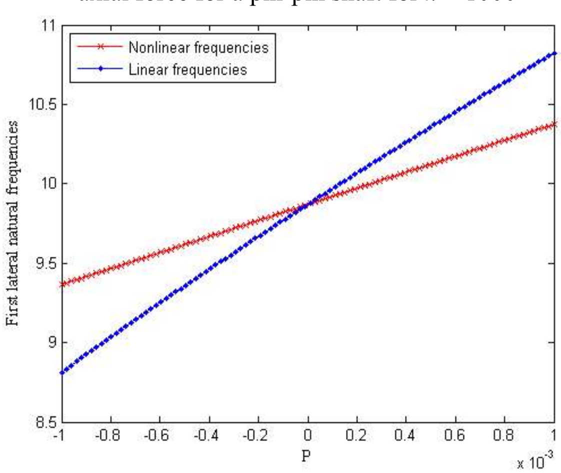

Fig. 7: Change of first lateral natural frequency versus axial force for a pin-pin shaft for k = 2000

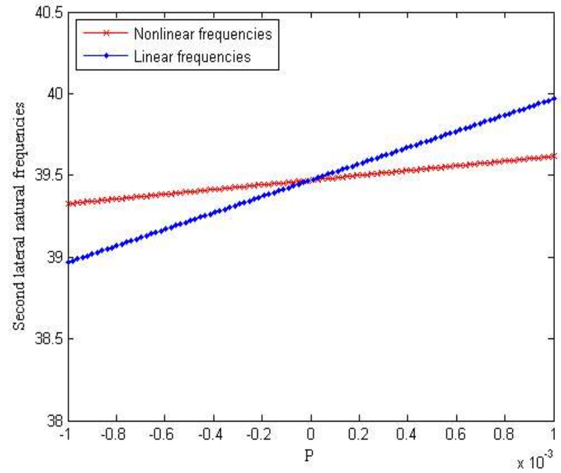

Fig. 8: Change of second lateral natural frequency versus axial force for a pin-pin shaft for k = 1000

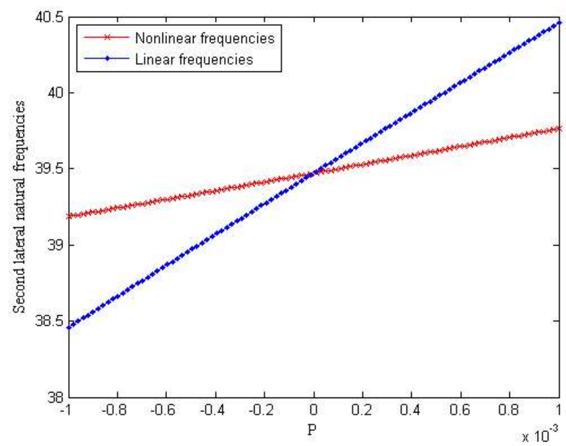

Fig. 9: Change of second lateral natural frequency versus axial force for a pin-pin shaft for k = 2000

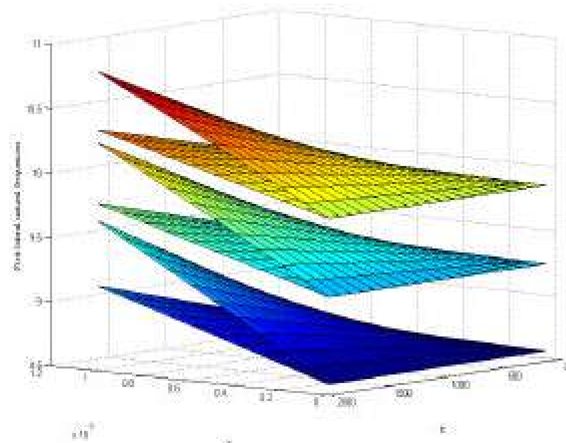

Fig. 10: Change of first lateral natural frequency versus axial force and shaft characteristics for a pin-pin shaft for ζ=0.001, 0.070 and 0.100.

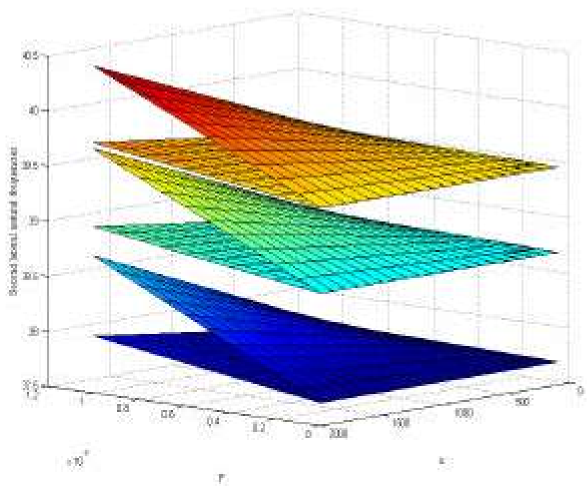

Fig. 11: Change of second lateral natural frequency versus axial force and shaft characteristics for a pin-pin shaft for ζ=0.001, 0.010 and 0.015.

Figs. (10) and (11) show changes of first and second lateral natural frequency of shaft versus dimensionless axial force (*P*) and shaft characteristic (*k*). Also, below curves in pair curves of Figs. (10) and (11) are according to linear solution and upper curves in pair curves are pertain nonlinear solution.

CONCLUSION

The effect of axial forces on the lateral natural frequencies of a flexible rotating shaft was analyzed. The shaft was assumed to be uniform and the nonlinear Euler-Bernoulli theory was used. In the general formulation of the governing equation of motion, internal viscoelastic dampings were included and a circular shaft rotating at a constant speed was admitted. Nonlinear partial differential equation of motion was derived by considering equilibrium equations for an element of the shaft. The ends of the shaft were pined-pined and multiple scales method was applied directly to the partial differential equation of motion. Two linear and nonlinear lateral natural frequencies were a nalyzed. In addition, the natural frequencies are plotted as functions of damping coefficients, shaft characteristics, axial force and amplitude. Also, lateral natural frequencies increases by applying tension axial loading and decreases by applying compression axial loading at the ends of the rotating shaft.

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