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A New Method for Estimating the Parameters of Kostikov and Modified Kostiakov Infiltration Equations

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Abstract: Infiltration is the process of water movement from the ground surface into the soil and numerous equations have been proposed over years to express infiltration as a function of time. Among the proposed equations, the Kostiakov and Modified Kostiakov are the most commonly used infiltration equations in surface irrigation applications because of their simplicity and capability of fitting most infiltration data. In this study, logarithmic characteristic of infiltration data is used and new exact methods are developed to estimate the parameters of Kostiakov and Modified Kostiakov infiltration equations. Published infiltration data are used to evaluate the proposed methods. The result of this study showed that the parameters of Kostiakov and Modified Kostiakov infiltration equations can be estimated with high precision using the proposed method.

Key words: Infiltration · Kostiakov · Irrigation · Infiltrometer · Final infiltration rate

INTRODUCTION

Infiltration is the process of water movement from the ground surface into the soil and is an important process in the hydrological cycle. Surface runoff and groundwater recharge can be linked with infiltration [1, 2]. Many equations such as Green and Ampt [3], Kostiakov [4], Horton [5], Modified Kostiakov or Mezencev [6], Philip [7], etc., have been developed for monitoring the infiltration process. Among these models, the Kostiakov and Modified Kostiakov are the most commonly used infiltration equations in surface irrigation applications because of their simplicity and capability of fitting most infiltration data. Kostiakov [4] expressed infiltration equation as:

$$F = at^b \quad (1)$$

Where F=cumulative infiltrated depth (cm); t=infiltration opportunity time (min); a(cm/min^b) and b(-)=fitting parameter. It is generally accepted that the power b is greater than zero but less than 1. The first derivative of Eq. (1) yields the infiltration rate equation. This equation is a decreasing function with an infinite initial value and zero final value at infinite time. Observation of long irrigation events as well as theoretical considerations showed that infiltration rate declined not to zero, but to a positive minimum value. This led to developing of Modified Kostiakov infiltration equation [8]. The Modified Kostiakov infiltration equation expressed as

$$F = at^b + f_o t \quad (2)$$

Where F, t, a and b are defined same as Eq. (1); and f_o=final infiltration rate (cm/min). The first derivative of Eq. (2) gives the Modified Kostiakov infiltration rate equation as:

$$f = abt^{b-1} + f_o \quad = (3)$$

The Green and Ampt infiltration model expressed as

$$f = \frac{K\left(H - ?_{\text{f}}\right)}{L} + K \quad (4)$$

Where f = infiltration rate (cm/min); K = hydraulic conductivity of wetted zone (cm/min); H = depth of water

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above the soil surface (cm); L = depth of the wetting front (cm); and Ψ_f = wetting front capillary pressure head (cm). Based on Mohammadzadeh-Habili and Heidarpour [9], the final infiltration rate in Modified Kostiakov equation is equal to hydraulic conductivity coefficient K in Green and Ampt equation. By comparing Eq. (3) with Eq. (4), it can be easily inferred that $f_o=K$. Raoof *et al*. [10] measured and estimated the saturated and unsaturated hydraulic conductivity in steady and transient states on sloping lands. Vieira and Ngailo [11] characterized the spatial variability of soil hydraulic properties of a poorly drained soil.

Many phenomena of water engineering such as groundwater surface drawdown due to pumping [12], reservoir depth-capacity data [13], infiltration [4], etc., have logarithmic characteristic. In this study, logarithmic characteristic of infiltration data is used and new exact methods are developed to estimate the parameters of Kostiakov and Modified Kostiakov infiltration equations. Published infiltration data are used to evaluate the proposed methods.

MATERIALS AND METHODS

Estimation of Kostiakov Equation Parameters: If cumulative infiltrated depth F is plotted as ordinate, against the time t as abscissa on log-log paper, it will be a straight line with slope b [4]. Because of the logarithmic scale of coordinate axes, the dimensionless infiltration data (dimensionless cumulative infiltrated depth F^* versus dimensionless time t^*) will be a straight line with slope b on log-log coordinate and across from $t^*=1$ and $F^*=1$. F^* and t^* are defined as:

$$t^* = \frac{t}{t_e} \tag{5}$$

$$F^* = \frac{F}{F_e} \tag{6}$$

Where t_e = end time of infiltration test; and F_e = cumulative infiltrated depth at the end time of infiltration test. Based on logarithmic characteristic, the relationship between t^* and F^* can be expressed as

$$\mathrm{Log}(F^*) = b\mathrm{Log}(t^*) \tag{7}$$

Eq. (7) can be expressed as

$$\mathrm{Log}(F^*) = \mathrm{Log}(t^{*b}) \tag{8}$$

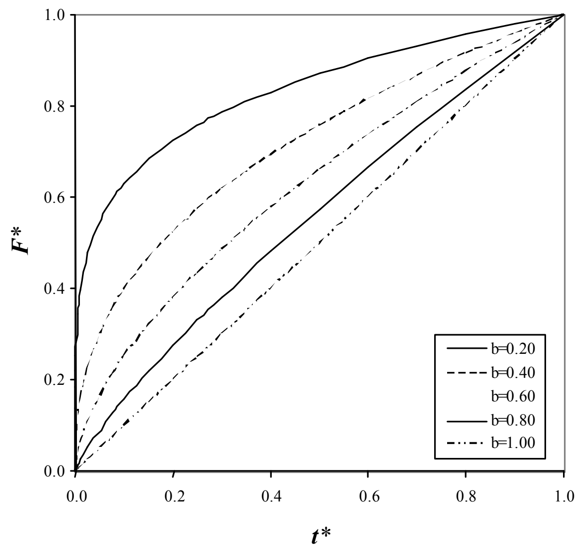

Fig. 1: The curves of Eq. (9) for various values of b in cartesian coordinate

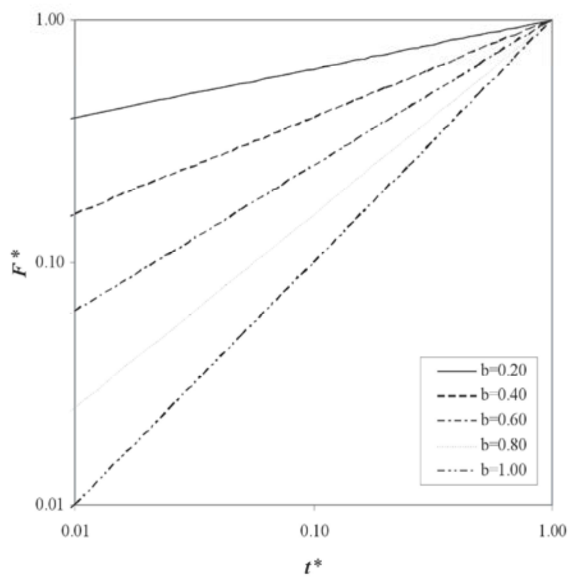

Fig. 2: The curves of Eq. (9) for various values of b in log-log coordinate

Taking antilog from both sides of Eq. (8) gives

$$F^* = t^{*b} \tag{9}$$

Figs. 1 and 2 show the curves of Eq. (9) for the various values of b in cartesian and log-log coordinates, respectively.

Substituting t^* and F^* from Eqs. (5) and (6) into Eq. (9) and simplifying gives

$$F = \frac{F_e}{t_e^b} t^b \tag{10}$$

Comparison of Eq. (10) with Kostiakov equation (Eq. 1) gives

$$a = \frac{F_e}{t_e^b} \tag{11}$$

In Kostiakov infiltration equation, the slope of the cumulative infiltration data on log-log coordinate is taken as b. In practice, infiltration data are not completely fitted along straight line in log-log coordinate and estimation of b from this method is not exact. Furthermore, in logarithmic scale, the same weights are not given to all data and more weights are given to initial values of F. While, in cartesian coordinate, the same weights can be given to all data and more precise value of b will be obtained by minimization of root-mean-square error RMSE between dimensionless infiltration data and the curve of Eq. (9).

Estimation of Modified Kostiakov Equation Parameters: Dividing both sides of Eq. (2) by time t and transferring f_o to the left hand side gives

$$\frac{F}{t} - f_o = at^{b-1} \tag{12}$$

Taking logarithm from both sides of Eq. (12) and simplifying gives

$$\log\left(\frac{F}{t} - f_o\right) = (b-1)\log(t) + \log(a) \tag{13}$$

It follows from Eq. (13) that the relationship between $\log((F/t)\text{-}f_o)$ with $\log(t)$ is linear with slope b-1 and width from origin $\log(a)$. Therefore, the linear relationship between $\log((F/t)\text{-}f_o)$ and $\log(t)$ can be obtained for the actual value of f_o. This characteristic can be used to estimate the exact value of f_o from infiltration test data. After obtaining f_o through a trial and error procedure, b will be equal to 1 plus to the slope of the obtained linear equation and a will be equal to the antilogarithm of log a.

Testing of the Proposed Methods: The performance of the proposed methods is verified with 5 infiltrometer tests data T1, T2, T3, T4 and T5. T1, T2, T3 and T4 were taken from Merriam and Keller [14] and T5 was taken from Walker [15]. The agreement between measured and estimated (from equation) values of F was evaluated by the Index of agreement I_a as

$$I_a = 1.0 - \frac{\sum_{i=1}^{N}(M(i) - E(i))^2}{\sum_{i=1}^{N}(\left|E(i) - \overline{M}\right| + \left|M(i) - \overline{M}\right|)^2} \tag{14}$$

Where $M(i)$=observed value; $E(i)$=estimated value; $\overline{M}$ =average of observed values; and N=number of data points. The root-mean-square error RMSE is also calculated from the following equation

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}[M(i) - E(i)]^2}{N}} \tag{15}$$

The value of I_a ranges from 0.0 to 1.0 and is a widely used dimensionless indicator for goodness of fit [16]. The higher value of I_a indicates a better agreement between the measured and estimated data. I_a = 1.0 indicates a perfect agreement [17]. To evaluate the agreement between measured and estimated data, I_a has also been used in many researches [18, 19, 20, 9].

DISCUSSION OF RESULTS

Using infiltration data of T1, T2, T3, T4 and T5 tests, the dimensionless cumulative infiltrated depth F^* are shown against dimensionless time t^* in Fig. 3. The curve of Eq. (9) is also plotted together with dimensionless data. For each test data, the value of b is obtained by minimization of RMSE between dimensionless data and the curve of Eq. (9). Replacing t_e, F_e and obtained value for b in Eq. (11), the value of a is calculated for the each test. For the 5 tests data, the obtained parameters for Kostiakov equation are presented in Table 1.

To introduce the proposed method for estimating the parameters of Modified Kostiakov equation, the values of 0.0001, 0.0023 and 0.0042 cm/min are selected for the final infiltration rate of T5 data. For the each value of f_o, $\log((F/t)\text{-}f_o)$ and $\log(t)$ are calculated and results are presented in Fig. 4. Linear lines are also fitted to the each set of data.

Table 1: Obtained parameters for Kostiakov equation using the proposed method

Test	a (cm/min^b)	b (-)
T1	0.25	0.64
T2	0.26	0.71
T3	0.23	0.64
T4	0.49	0.62
T5	0.17	0.52

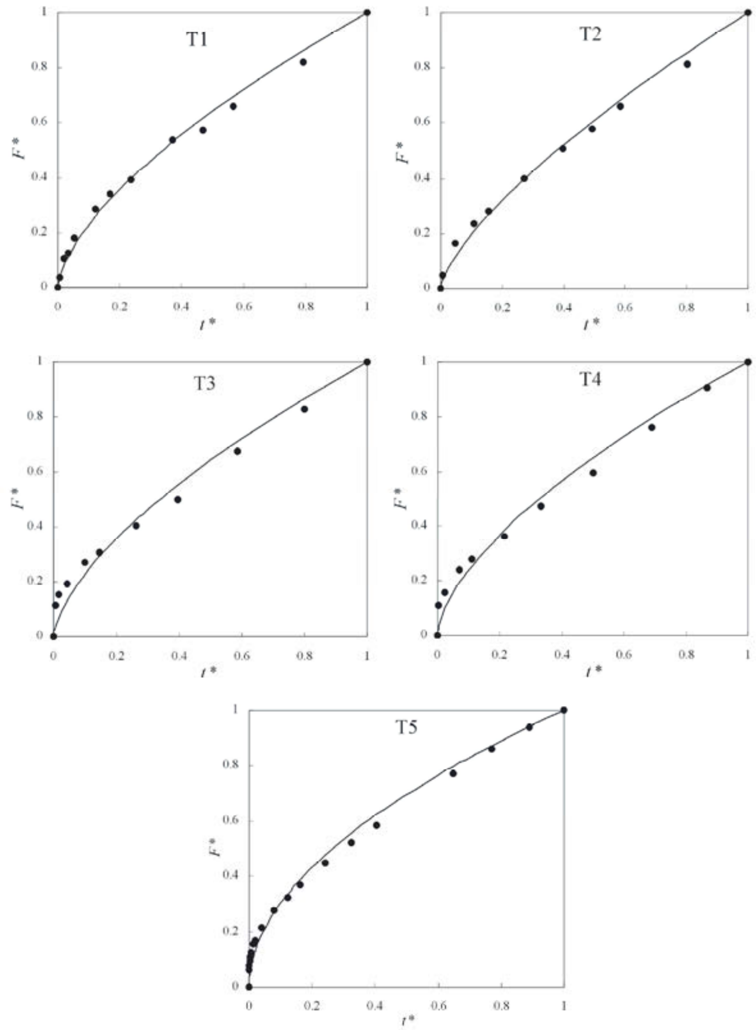

Fig. 3: Dimensionless cumulative infiltrated depth F^* versus dimensionless time t^* with the curve of Eq. (9)

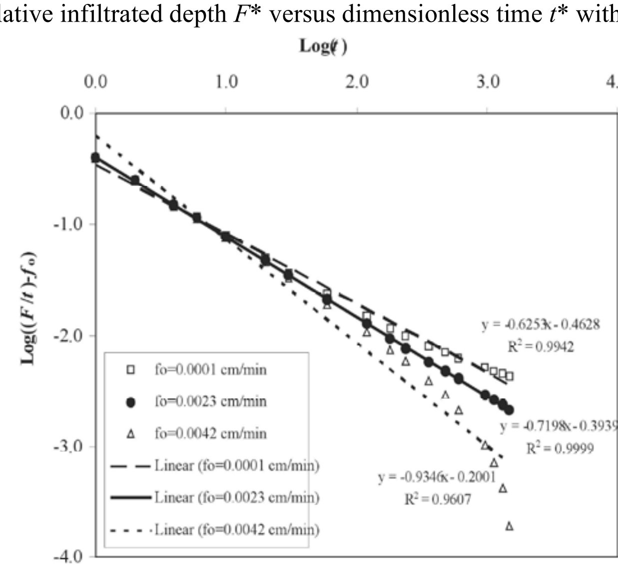

Fig. 4: $\log((F/t)-f_o)$ versus $\log(t)$ for T5

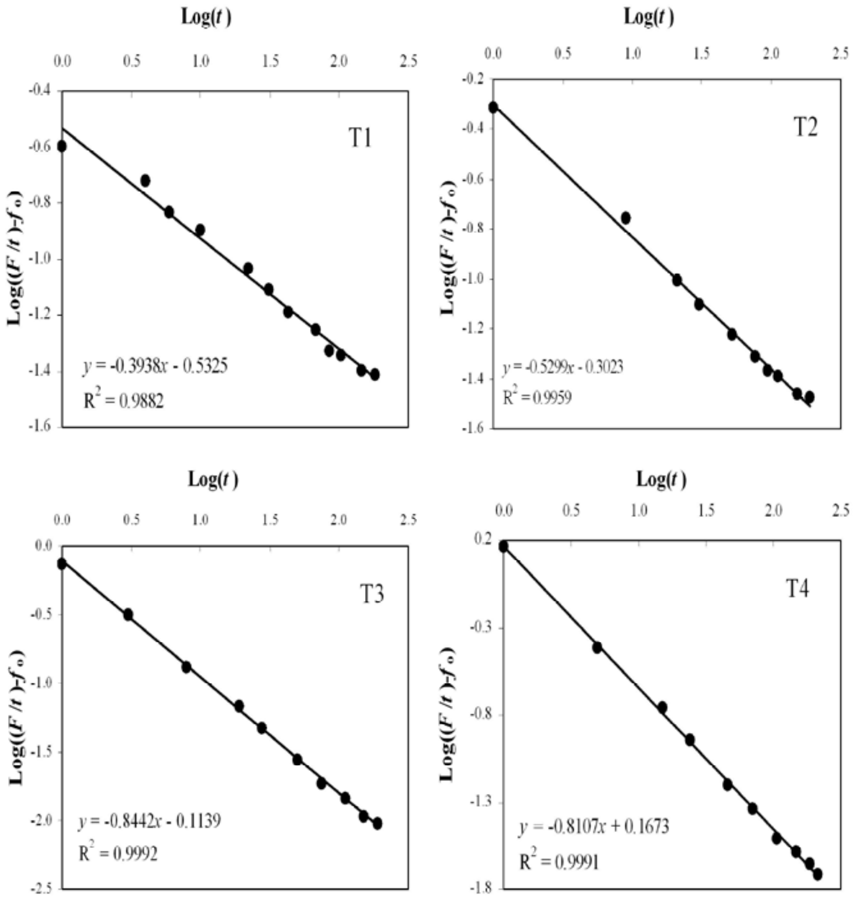

Fig. 5: log((F/t)-f_o) versus log(t) for T1-T4

As seen from Fig. 4, for f_o=0.0023 cm/min, the relationship between log((F/t)-f_o) and log(t) becomes linear with slope -0.7198 and width from origin -0.3939. Therefore, for this test

$$\begin{cases} f_o = 0.0023 \text{ cm/min} \\ a = 1 + (-0.7198) = 0.280 \text{ cm/min}^{0.404} \\ b = \text{Antilog}(-0.3939) = 0.404 \end{cases}$$

For the other tests, through a trial and error procedure, the final infiltration rate f_o are obtained and then log((F/t)-f_o) and log(t) are calculated and results are presented in Fig. 5.

For the each test, the obtained parameters for the Modified Kostiakov infiltration equation are presented in Table 2.

Using obtained parameters for the each test data, the curves of Kostiakov and Modified Kostiakov infiltration equations are plotted with the measured data of T1-T5 in Fig. 6.

Table 2: Obtained parameters for the Modified Kostiakov infiltration equation using the proposed method

Test	a (cm/min^b)	b (-)	f_o (cm/min)
T1	0.293	0.606	0.0001
T2	0.499	0.470	0.0240
T3	0.769	0.156	0.0254
T4	1.470	0.189	0.0450
T5	0.404	0.280	0.0023

Table 3: Calculated Index of agreement I_a for the each test

Test	I_a (Kostiakov equation)	I_a (Modified Kostiakov equation)
T1	0.998	0.999
T2	0.998	0.999
T3	0.994	0.999
T4	0.996	0.999
T5	0.997	0.999

The Index of agreement I_a between measured data and Kostiakov and Modified Kostiakov infiltration equations are calculated and results are presented in Table 3.

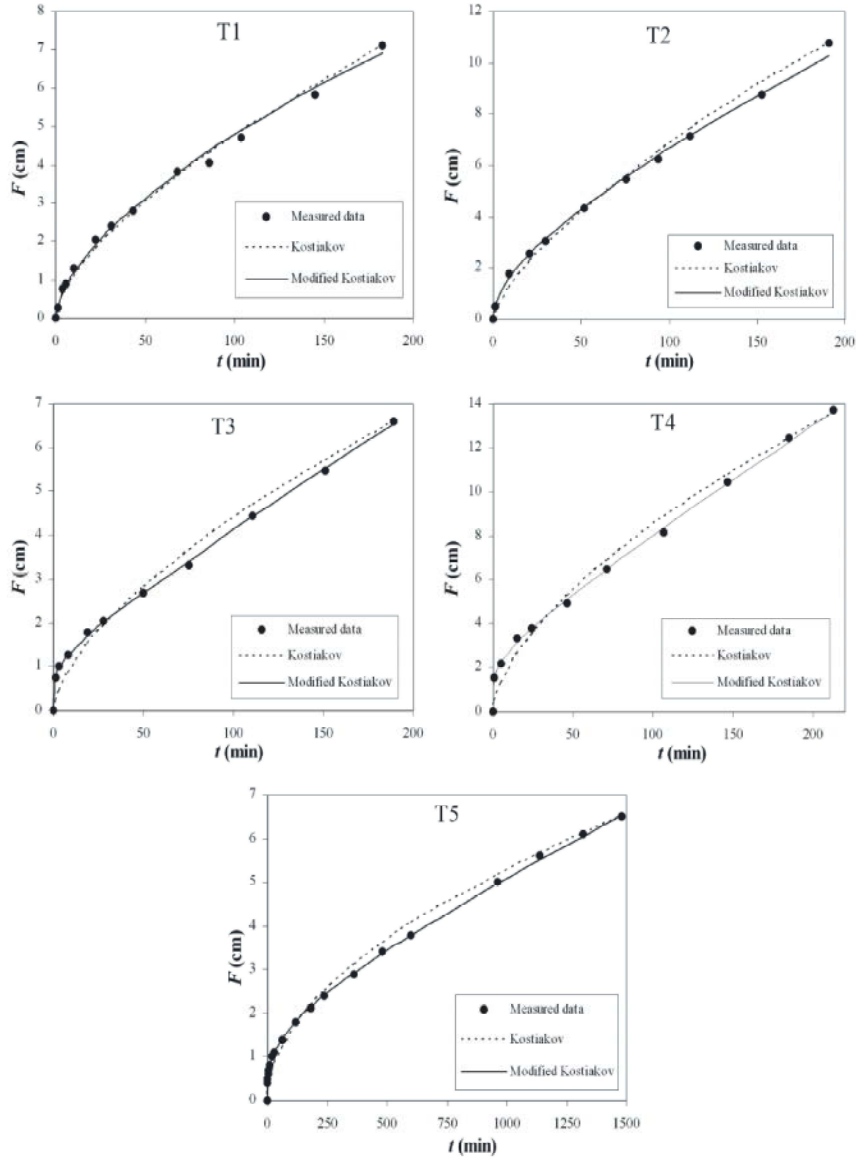

Fig. 6: Cumulative infiltrated depth *F* versus time *t*

Fig. 6 and data of Table 3 show a good agreement between measured data and Kostiakov and Modified Kostiakov infiltration equations. Also, for all the tests I_a is nearly equal to 1. It reveals that parameters of the Kostiakov and Modified Kostiakov infiltration equations are estimated with the high precision. Fig. 6 and data of Table 3 are also show that the agreement between Modified Kostiakov infiltration equations and measured data is better than Kostiakov equation. The reason is due to the fact that in Modified Kostiakov infiltration equation, the final infiltration rate f_o is considered. For T1 final infiltration rate is nearly equal to zero and therefore, 2 models show the same results. But for other test data, Modified Kostiakov equation show the better results in comparison with Kostiakov equation.

CONCLUSIONS

In this study, logarithmic characteristic of infiltration data is used and new exact methods are developed to estimate the parameters of Kostiakov and Modified Kostiakov infiltration equations. Published infiltration data are used to evaluate the proposed methods. The result of study showed that the parameters of Kostiakov and Modified Kostiakov infiltration equations can be estimated with high precision using the proposed method.

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