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Equilibrium in Economic Development: A Perspective of Social Capital

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Abstract: Each system owns different resources which need to be prioritized for bringing economic equilibrium in the society. This study identifies equilibrium in economic development and its relationship with social capital. Inter temporal accumulation of social capital of certain quantum takes place when cycle of economic activities is started at different levels of the systems; individuals, group, organization and State. In each system different combinations of resource allocation in spending and welfare activities leads to level game. A number of level games emerge from these resource combinations in the four systems. Markov process makes it possible to observe whether the systems reach their respective equilibrium positions. Therefore Markov process validates equilibrium of spending and welfare in respect of social capital for each system.

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INTRODUCTION

There are many indispensable and swelling mechanism for social capital that magnify cohesion, in the sense of soothe of others and civic activities in society to get common interests. Most of the researchers have identified and estimated dissimilar components like dependence, reciprocity, norms, values, set of relations, swap of information, joint effort or bother and disbelieve (Carroll [1] Castle [2] Bjomskov [3] Flora and Robinson [4] Putnam [5] Cox [6]). Nevertheless, Cox [6] incorporates that interaction based on trust along with individuals go forward to increase democracy and raise in production. Putnam [7] states that, the social capital embodied in customs and relationships of civic tryst, which seems as must for government effectiveness and economic spreading out. On the similar lines, Bastelaer and Grootaert [8], have the vision that virtually all of the contacts amid individuals are controlled by the values, attitudes and institutions link that cause social and economic progress. With cognitive and structural capital in the society, the associated organizations may have better straps with the state. State-society synergy may furnish superior end results and powerfully delivery in attaining communal dreams [9]. In addition, Putnam [5] authenticates that institutions entail interpersonal talents and hope for positive mutual aid, which reciprocally strikes and support these abilities and belief. As a result, state, institutions and organizations want to deal out suitable assets/spending for buildup of social capital.

In [10] it is endorsed that "*Economic development causes accumulation of social capital*" through the algebraic formation developed in [11] for social capital matrix. By [10], the economic growth is related to productions gained by expenditure in assorted sectors in not like ways and combinations, which affects social capital of individuals, communities, groups/organizations, institutions and state.

In this work we carry on to go after the algebraic representation of Social Capital Matrix of [11] in which the state S is represented by the Boolean algebra $F_2 = \{0,1\}$ with 2 active S-vectors. Furthermore organization O is represented by a higher dimensional linear space F_2^2 with 4 O-vectors. Similarly the

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community C by linear spaces F_2^3 and individual L by F_2^4 with 8 C-vectors and with 16 L-vectors respectively.

Continuation to [11, 12] in [10], we experienced that the imbeddings $F_2 \to F_2^2 \to F_2^3 \to F_2^4$ of linear spaces (systems) is like a cone. Whenever this cone is revolving (a push-start), the economic actions are in receipt of begin and thus the vectors (with spending and welfare components) of a subsystem are ascertaining social capital as: In a vector $(\beta_1, \beta_2) \in F_2^2$ (respectively $(\beta_1, \beta_2, \beta_3) \in F_2^3$ and $(\beta_1, \beta_2, \beta_3, \beta_4) \in F_2^4$), $a_1 \in F_2$ replicates the part of (β_1, β_2) (respectively $(\beta_1, \beta_2) \in Z_2^2$ the components of (β_1, β_2, β_3) and $(\beta_1, \beta_2, \beta_3) \in F_2^3$ the components of (β_1, β_2, β_3, β_4)). This exposes that, in reality in the organization $O = F_2^2$, community $C = F_2^3$ and individual $L = F_2^4$, each vector has ending coordinate from the respective system but the preceding one's are taken from its subsystem. This shows that for a vector in any system there is presence of welfare or spending of its previous subsystem.

The welfare level in a number of forms facilitate to do utmost services to others, as a result by this belief it is inferred that in any vector β of any system, the maximum number of welfare indicators explain that the vector β has high welfare level and consequently social capital [10].

In continuation to [10], in [12] we have established that the measurement for social capital in a given system F_2^k can be defined as; if $\beta = (\beta_1, \beta_2, \ldots, \beta_k) \in F_2^k$, where $1 \le k \le 4$, then social capital (sc) of the vector β (represented $sc(\beta)) = \frac{\#1s}{\#0s}$ if $\#0s \neq 0$. This means $0 < sc(\beta) < k-1$ but if $\#1s = k$ (respectively $\#0s = k$), then $sc(\beta)^{\uparrow} = W_k^T = k$ (respectively $sc(\beta)_{\downarrow} = I_k = 0$).

The welfare function $w: F_2^k \times F_2^k \to F_2^k$, $i \le k \le 4$, motivates to estimate the social capital of a class $\gamma \in F_2^k$ obtained by the interaction of two classes $a, \beta \in F_2^k$ under w. This addresses only the main diagonal of the social capital matrix, that is interaction of Individual ($L = F_2^4$) with Individual ($L = F_2^4$), Community($C = F_2^3$) with Community ($C = F_2^3$), Organization ($O = F_2^2$) with Organization ($O = F_2^2$) and State ($S = F_2$) with State ($S = F_2$). During interaction of each system with itself, the distinct classes of each system interact each other through welfare function w. Furthermore the class obtained by the interaction of the same class with itself represents the status of spending, that leads to minimum level of social capital. However, if corresponding components of two interactive classes are opposite to each other, the class obtained as a result would be having optimum level of social capital.

Social capital is related to frequency and quantum of spending and welfare activities. Inter temporal accumulation of social capital of certain amount may take place during cycle of economic activities at different levels of the systems; individuals, group, organization and State. In each system different combinations of resource allocation in spending and welfare activities may lead to level game. Resources owned by any system needs to be prioritized for economic development. The resource allocation plan may affect economic equilibrium, thereby; economic development in the society. This study attempts to identify equilibrium in economic development and its relationship with social capital. It observes that a number of level games emerge from various resource combinations in the four systems. In this regards Markov process is used to makes it possible to observe whether the systems reach their respective equilibrium positions or not. Thus Markov process validates equilibrium of spending and welfare in respect of social capital for each system.

The next section presents a review of theory of social capital in perspective of within and across interactions of state, organization, community and individual. In section 3 we give a short introduction to algebra, which is under consideration. Section 4 explains a link between algebra and social capital matrix. Section 5 interpret welfare economics due to interactions of systems. How the social capital works in the interactive economics? it is elaborated in section 6. Furthermore it is established that how social capital is measurable relative to the algebraic representation. Finally through Markov process, we validates equilibrium of spending and welfare in respect of social capital for each system.

REVIEW OF THOERY OF SOCIAL CAPITAL

Following Robinson and Flora [4], social capital emerges through an individual's sacrifice organized in an shot to maintain solidarity with others and social cohesion is a characteristic of society depends on the composed social capital. According to sociologist and economists social capital is marked with respect to its moral fiber, role and facets. Social capital is categorized into cognitive social capital and structural social capital [13].

The following indicate theory which interpret the trades of unlike systems and interaction inside a system.

Intra-action: Coleman [14] states that social capital embraces a capital gain for the individual and it consists of some trait of society and crafts simple certain action of individuals who are within the arrangement. While, Sobel [15] puts in that, these interactions rivet a little number of negotiators who know each other and interact time after time.

Social capital fabricates reciprocity flanked by organizations in turn to develop their joint trust. Inter-organizational/institutional interaction happens through their members, in which individuals engineer investments by dealings and reciprocity with other organizations.

Cognitive social capital and structural social capital make possible and control prototypes of joint interface of institutions. As said by Turner [16] that the word institution "indicates the style that members of a citizens are organized in order to appear essential problems of coordinating their activities to be alive on within an approved environment". Ostrom's [17] role relating to extensive interests has been reinforced by Sobel [25], that common-property assets underline the worth of institutions.

Across interaction: Robinson and Flora [4] verify that individuals and groups can deliberately effort to make tougher social capital. Castle [2] and Sobel [15] note that although self-centeredness is an principal motivator, it does not put off, indeed it may require, contribution in groups. Therefore across interaction of communities and individuals also results in accrual of social capital.

Individuals act together with organization by their friends by allocation of possessions that loudening social capital between them. Individuals build investment through contact and reciprocity with organization that creates social capital, thus a rank of trust grows amid individual and institutions/ organizations. The verdict of Sobel [15] is; altitude of trust on institutions point out the settled consent to them for attractive and influencing over the individuals' welfare.

Both types of social capital effortlessness prototypes of their interface with each others in the form of congnito-structural affairs. Social capital is rise up linking a state and an individual. Individuals ability investment through reciprocity with state directly or crookedly through its institutions/organization, which enlarges their mutual trust. Facts in text clarify that encouraging behavior of citizenship is the blot of the individualistic on state, which leads to stable reciprocity between state and individuals.

Relations among communities by allocation of time and money fabricates social capital between them, as the individuals and groups can intentionally work to reinforce the social capital [2, 5, 15]. Likewise, Woolcock [18] says that substantial capital and human capital are on the whole the chattels of individuals, even as social capital and annex inheres in groups. The social capital is personified restricted by communities, which bypasses on to the relations in a group, in addition social norms and okays, trust, mutual bonds and communiqué of information [14, 19].

A group/community interrelates with institutions/ organizations by allotment of assets that builds up social capital between them. Castle [2] and Sobel [15] have mentioned subsistence of social capital amongst groups because of common interests. Alike,Woolcock [18] infers that social capital is belongings of group. Individuals in a straight line or in a roundabout way on behalf of group create investment through dealings and reciprocity with state or its institutions or organization that collects social capital, which increases their reciprocity and reliance in each others. According to Evans [9]" Active government and mobilized communities can enhance each other's development efforts". Similarly, Harris [20] favours Putnam's [5] judgment that `networks of civic engagement is a key determinant of government presentation.

Individuals on behalf of institutions/organizations make investment and extend reciprocity by means of a state directly or indirectly through its institutions or organizations that support trust in each others. Cognitive and structural social capital makes easy examples of interaction of institutions/organizations with state. Evans [9] is of the spectacle that for growth raison d'êtres, additionally to improvement micro-level capital, state-society synergy can provide better results. Social trust, norms of reciprocity networks of civic engagement and successful teamwork are frequently sustaining. Putnam [5] indicated that for valuable joint venture, institutions desire interpersonal abilities and trust, which are also firm by organized alliances. Institutions, organizations and state may dole out possessions for accretion of social capital to get better value.

ALGEBRAIC STRUCTURES UNDER CONSIDERATION

$$F_p = \{[0]_p, [1]_p, [2]_p, .., [p-1]_p\} = \{0, 1, 2, \ldots, p-1\}$$

ion and multiplications defined as follow is the field of residue modulo the prime integer p. We have taken p = 2, therefore

$$F_2 = \{[0]_2, [1]_2\} = \{0, 1\}$$

A vector space V over a field F is an algebra if it is a ring and $\alpha(vw)=(\alpha v)w=v(\alpha w)$. A field is a one dimensional algebra over itself, i.e., F_2 is an algebra with dimension 1. Furthermore for a positive integer n,

$$F_2^k = \{a = (\alpha_1, \alpha_2, .., \alpha_k) : \alpha_1, \alpha_2, .., \alpha_k \in F_2\}$$

is an algebra over F_2 with dimension k.

Linear transformation (vector space homomorphism) is a map $h:V \rightarrow W$ of finite dimensional vector spaces over the same field F which satisfies

$$h(v_1 + v_2) = h(v_1) + h(v_2) \text{ and } h(\alpha v_1) = \alpha h(v_1)$$

for all $v_1, v_2 \in V$, $\alpha \in F$). A one one, onto and bijective homomorphism h is said to be monomorphism, epimorphism and isomorphism respectively. $V \cong W$ means V and W are isomorphic. [21, 22, 23, 24, 25] are referred for more better understanding.

ALGEBRAIC REPRESENTATION OF SOCIAL CAPITAL MATRIX

Here we have given a little introduction of algebraic representation of social capital matrix [11].

Algebraic representation of systems of social capital matrix: The algebraic representation of State, organization, community and individuals in [11] has been dexterity due to economics of spending and comfort. This algebraic representation [11] has capability to present multi period examination but the composition in [26] is faulty to more than one period information concerning economic activism and construction of social capital.

In [11] we carve the Social Capital Matrix by F_2 with the supposition that the energetic vectors of the state I comprise of the costs activities (e.g. infrastructure improvement, health care, transportation, roads etc.) in order to strain a welfare vector W_S (e.g. poverty reduction, economic growth, etc.). We start it by considering the correspondence $W_s \leftrightarrow 1$ and $I_s \leftrightarrow 0$. So linear space F_2 represents the state S, which contains the vectors I_S and W_S, called S-vectors (S-Categories). We suppose that linear space F_2^2 represents organization with four O-vectors (O-categories) Similarly the linear spaces F_2^3 and F_2^4 represent community with eight C-vectors (C-categories) and individual with sixteen L-vectors (L-categories) respectively.

In [11] for algebraic representation of the social capital matrix [26] we take SOCL, a reverse set-up to LCOS considered in [26], it might be view as the interactions $F_2 \times F_2, F_2^2 \times F_2^2, F_2^3 \times F_2^3, F_2^4 \times F_2^4$, lie on the main diagonal and $F_2^2 \times F_2, F_2^3 \times F_2, F_2^4 \times F_2, F_2^4 \times F_2^2, F_2^4 \times F_2^3, F_2^3 \times F_2^2$, on lower diagonal of social capital matrix. However the interactions $F_2 \times F_2^2, F_2 \times F_2^3, F_2 \times F_2^4, F_2^2 \times F_2^4, F_2^3 \times F_2^4, F_2^2 \times F_2^3$ lie on upper diagonal.

Algebra of categories of the systems: The components of S-categories, O-categories, C-categories and L-categories of all 4 systems are represented as

$$\{a_i : a_i \in F_2\}, \ \{a_i b_j : a_i, b_j \in F_2\}$$

$$\{a_i b_j c_l : a_i, b_j, c_l \in F_2, i, j, l \in \{1,2\}$$

and

$$\{a_i b_j c_l d_m : a_i, b_j, c_l, d_m \in F_2\}$$

for i, j, l, m$\in\{1,2\}$, respectively.

INTERACTIONS OF SYSTEMS AND WELFARE ECONOMICS

Intra-action of categories of system in SOCL: By [11] the interaction of a system with itself is called an intra-active function δ and it is defined as: $\delta : F_2^m \times F_2^m \rightarrow F_2^m$, where $1 \le m \le 4$ by $\delta(a_1..a_m, b_1.b_m) = c_1.c_m \in F_2^m$, for any $a_1.a_m, b_1.b_m \in F_2^m$, where $c_i = a_i + b_i$, $1 \le i \le 4$, δ is construed as the economic exchange between the categories of a system. On the other hand, in resulting one can attain again a category of the same system.

By [11], it has been observed that during intra-action the total assets of interactive categories of a system are totally operational and no part left for its own continued existence which shows the utilization and spending of all assets/ resources in one period, a towering level of trust amongst the categories of the same system is observed, which cause economic functionality and hence social capital of the particular system.

Across interaction of the categories of different systems in SOCL: $F_2^2 \times F_2, F_2^3 \times F_2, F_2^4 \times F_2, F_2^3 \times F_2^2, F_2^4 \times F_2^2$, $F_2^4 \times F_2^3$ with their reverse symmetries indicate twelve across interactions of the systems.

The zero vector space is containing only 0 vector. So, $F_2^l \rightarrow F_2^k$ is imbedding of F_2^l in F_2^k, for $l \le k$, i.e. $F_2^l \cong F_2^l \times 0 \times .. \times 0 \subset F_2^k$, this means $a_1..a_l = a_1..a_l 0_{l+1}..0_k \in F_2^k$. Similarly $k \le l$, $F_2^k \rightarrow F_2^l$ is imbedding of F_2^k in F_2^l, i.e. $F_2^k \cong 0_1 \times .. \times 0_{l-k} \times F_2^k \subset F_2^l$, this means $a_1.a_l = 0_1.0_{l-k} a_1.a_l \in F_2^l$.

Following [11] the across interactive functions $\delta_{l\pm k}$ and $\delta_{l\mp k}$ are defined respectively as follow:

$$\delta_{l\leftarrow k}: F_2^l \times F_2^k \to F_2^k$$

Where $1 \le k \le 4$ and $k \le l$ by

$$\text{by } \delta_{l\leftarrow k}(a_1..a_l, b_1..b_k, b_{k+1}..b_l) = c_1..c_k, c_{k+1}..c_l \in F_2^k,$$
$$c_j = a_j + b_j, 1 \le j \le k,$$

for any

$$a_1..a_l \in F_2^l, b_1..b_k \in F_2^k \text{ and } b_{k+1} = .. = b_l = 0$$

and

$$\delta_{l\to k}: F_2^l \times F_2^k \to F_2^k$$

Where $1 \le k \le 4$ and $1 \le k$

by

$$\delta_{l\to k}(a_1..a_l a_{l+1}..a_k, b_1..b_k) = c_1..c_l c_{l+1}..c_k \in F_2^k,$$
$$c_i = a_i + b_i, 1 \le i \le l,$$

$$\text{for any } a_1..a_l \in F_2^l, b_1..b_k \in F_2^k \text{ and}$$
$$a_{l+1} = .. = a_k = 0,$$

where $c_i = a_i + b_i$, $1 \le i \le 4$, $\delta_{l\to k}$ and $\delta_{l\leftarrow k}$ are recognized as the economic barter with the categories of unlike systems. Nonetheless as an outcome of this barter, a category is obtained, departs to the larger system of across inter-active systems [11].

By [21], for $l \le k$, $F_2^l \circ F_2^k$ is inserting of F_2^l in F_2^k, i.e. $F_2^l \cong F_2^l \times 0 \times .. \times 0 \subset F_2^l \times F_2^{k-1} \cong F_2^k$, the k-l components are inactive during interaction of F_2^l and F_2^k This shows that genuine having of a category of a larger system is completely protected by the vendor category during interaction with a category of a smaller system.

System S has 2 categories in its design, that is of costs I_S and welfare W_S. The same in System O (respectively C and L) has 4 categories (respectively 8 and 16) in its design and in each category 2 (respectively 3 and 4) assure of spending I_S or welfare W_S might happen. In this system distinct combinations of investment and welfare might give different outputs. Economic development is linked to outputs achieved from spending in poles apart sectors/areas in divergent manners and groupings. Their crash on individuals, communities, groups/organizations, institutions and state could determine height of economic improvement.

Push-start a beginning of level game: In each S,O,C and L we constituted in [10] that how a slighter system is imbedded in superior one, which justification that each vectors in a superior system holds either spending components or welfare components. It can be observed as: F_2, F_2^2, F_2^3, F_2^4 are F_2 – spaces.

The study in [10] provides a base for defining the social capital in all systems. That is I_S, I_O, I_C and I_L are spending vectors of state, organization, community and individual respectively such that in which each component is in spending position but $I_O^{Pure}, I_C^{Pure}, I_L^{Pure}$ are purely spending vectors of organization, community and individual respectively in which the state, organization, community components are in welfare status respectively (or $I_O^{Pure}, I_C^{Pure}, I_L^{Pure}$ inheres W_S, W_O^T, W_C^T respectively). Whereas W_S is the welfare vector of state and the vectors $W_O^{Pure}, W_C^{Pure}, W_L^{Pure}$ are purely welfare vectors of organization, community and individual respectively in which the state organization, community components are in spending position respectively but W_O^T, W_C^T, W_L^T are fully (optimum) welfare vectors of organization, community and individual respectively in which the state, organization, community components are in welfare status respectively (or W_O^T, W_C^T, W_L^T inheres W_S, W_O^T, W_C^T respectively).

By [10] the vectors of the systems S, O, C and L are establishing different levels with different periods of time, that is: For $S: [0 \quad 0]^t, [0 \quad 1]^t, [1 \quad 1]^t, [1 \quad 0]^t$. This shows four levels of being activism.

For O:

$$[00 \quad 01 \quad 11 \quad 10], [00 \quad 10 \quad 11 \quad 01],$$
$$[00 \quad 11 \quad 01 \quad 10], [00 \quad 01 \quad 10 \quad 11]^t,$$
$$[01 \quad 00 \quad 11 \quad 10], [11 \quad 01 \quad 00 \quad 10],$$
$$[11 \quad 01 \quad 10 \quad 00]^t ...$$

By the the push fuel (I_S and W_S) of the state these systems start to running their economic activities. This could be realized as a rotating cone [10].

Consequently the varied exhibits of vectors of these systems play a level game on every occurrence cycle of inborn economic activities are on path. In the same way one can obtain the levels for the systems $F_2 \times F_2^2, F_2 \times F_2^3, F_2 \times F_2^4, F_2^2 \times F_2^4, F_2^3 \times F_2^4$. Contemplation of these systems in fact hard-edged the economic cone, that is they fill the chinks of the cone.

HOW THE SOCIAL CAPITAL WORKS IN THE INTERACTIVE ECONOMICS?

In this section first we shall refers the notion developed regarding measurement of the social capital from [13].

Preservation of spending $\underline{I}_S$ and welfare $\underline{W}_S$ in any system: Behind any category of any system there exist a position of state, that is either the vector of a system need spending from the state or it has a push of welfare by the state [13, Proposition 1]. The [13, Proposition 2] explains that in fact any system of SOCL except S has a need of spending.

Social capital in systems

The welfare indicator: In [13, Proposition 3] it is established that the welfare function $w: F_2^k \times F_2^k \to F_2^k$ defined by

$$w(a, \beta) = (\alpha_1 + \beta_1, \alpha_2 + \beta_2, ..., \alpha_k + \beta_k) \in F_2^m$$

where $1 \le k \le 4$ is a system in SOCL and

$$a, \beta \in F_2^k,\ a = (\alpha_1, \alpha_2, ..., \alpha_k),\ \beta = (\beta_1, \beta_2, ..., \beta_k)$$

Then $W_k^{min} < w(a, \beta) < W_k^T$ if $\alpha_i + \beta_i \neq 0$ for some $1 \le k \le 4$, where W_k^T, the optimum level of the welfare and W_k^{min}, the least level of the welfare.

By [13] when the economic activity cone start to rotate, mechanically inside the system, the components are in accord to change their positions from side to side the welfare function w. Consequently the functionality of the economic activity in each system is realized. It is when we submit an application the function w on any pair of the vectors of a system, the vector of the same system is obtained. In other words w figures the diverse levels of the system.

Welfare indicator and measurement of social capital

How social capital emerges from a vector of a system?: A motivation came from Grootaert and Bastelaer [8] that, "the organizations/institutions relationship, attitudes and values that govern interaction among people and contribute to economic and social development" . Although the measuring social capital is one of the difficult part in the study of social capital but in [13] we define social capital in a given system F_2^k as follow.

Definition 1: Let $a = (\alpha_1, \alpha_2, ..., \alpha_k) \in F_2^k$,, where $1 \le i \le k$, with $\alpha_1, \alpha_2, ..., \alpha_k \in \{0,1\}$. Define social capital (sc) of the vector α (represented $sc(\alpha)$) as, $sc(a) = \dfrac{\#1s}{\#0s}$ if $\#0s \neq 0$. This means,

(a) $0 < sc(a) < m - 1$.

(b) If #1s=m and #0s=0, then $sc(a)^{\uparrow} = W_m^T = m$

(c) If #0s=m and #1s=0, then $sc(a)_{\downarrow} = I_m = 0$.

Potential social capital between vectors of a system: In any of system of SOCL if the economic activity is started, certainly it is normal to know about dissimilarity of two of players interacting or sharing their having. So we may define the distance function which assert that whether or not these players are come nearer and interact each other. In continuation as social capital is accumulated between two individuals through one individual's interaction and reciprocity with the other individuals and Coleman [14] indicated that "social capital constitutes a capital asset for the individual and it consists of some aspect of social structure and facilitates certain action of the individuals who are within the structure". This relationship in turn develops trust between individuals that enable them to generate returns in future.

On the basis of these observations in [13, Proposition 4] we indicated the distance as follows:

Let F_2^k, where $1 \le k \le 4$, is a system in SOCL and $a, \beta \in F_2^k$ be any vectors. If the mapping $d: F_2^k \times F_2^k \to \{0,1,2,...,k\}$ defined by $d(\alpha,\beta)$ = The number of components differ. Then (1) (potential social capital of α,β) psc $(\alpha,\beta^{\uparrow})$ if $d(\alpha,\beta) \to k$, (2) (potential social capital of α,β) psc $(\alpha,\beta)_{\downarrow}$ if $d(\alpha,\beta) \to 0$.

The less value of $d(\alpha,\beta)$ creates the mistrust and uncertainty occur, an disproportion reciprocity which decreases the social capital and vice versa. That is it will give a variety of spending and welfare that may share to divergence and convergence of the system in respect of vulnerability of social capital buildup. The difference/spread of spending and welfare gives range of social capital accumulation [13].

EQUILIBRIUM IN SYSTEMS

Generally none of the system in SOCL is measurable in its levels in different periods of time. So there is a need to monitor that at which period of time a level of a system is stable or ideal one in respect of higher social capital (respectively in optimum welfare level). Since we do not deal with quantified inputs and out puts, therefore there is a need to assess the sustainability of the process regarding spending and welfare. Consequently we are in need to adopt the predictive type analyzer and for this we chosen the Markov process which is more reliable and preferable owing to the reasons: (1) It is compatible with algebraic model of [11] and tools and mechanism for quantifiable social capital relative to the welfare indicator, which causes the economic development as described in [10] and [13]; (2) Supportive in policy making significantly by spend one forecast future welfare projects, returns to political economy.

Markov chain: Following [24] a Markov chain is a process in which the probability of the system being in a particular level at a given observation period depend only on its level at the immediately preceding observation period. Suppose that the system has n possible levels. For each $1 \le i, j \le n$ let P_{ij} be the probability that if the system is in level j at a certain observation period, it will be in level i at very next observation period; P_{ij} is called a transition probability. Moreover, P_{ij} applies to every time period. Of course $0 \le P_{ij} \le 1, 1 \le i,j \le n$. If the system is in level j at a certain observation period, then it must be in one of the n levels (it may remain in level j) at the next observation period. Therefore we have $P_{1j} + P_{2j} + \ldots + P_{nj} = 1$.

We arrange these transition probabilities as an $n \times n$ matrix $T = [P_{ij}]$, which is known as transition matrix (probability matrix) of the Markov chain. In other words T is a stochastic or Markov matrix whose elements are probabilities and whose columns add up to 1. A Markov matrix T is called doubly Markov matrix if sum of the rows of T is 1. These matrices are very important in the study of random phenomena where is the exact outcome is not known but probabilities can be determined [27].

If A and B are Markov matrices of same size, then AB is a Markov matrix [27, Theorem 2.10]. Of course $A^2, \ldots, A^n, \ldots$ are also Markov matrices if A is a Markov matrix.

If T is a transition matrix and L_0 is the initial level vector, then

$$\begin{aligned} TL_0 &= L_1 \\ TL_1 &= T^2 L_0 = L_2 \\ &. \\ &. \\ TL_{n-1} &= T^n L_0 = L_n \end{aligned}$$

If whenever

$$TL_s = L_s$$

then we say the system or the Markov process reaches equilibrium and the level vector L_s is known as the steady-level vector. More clearly speaking L_s is the eigen vector of transition matrix T with corresponding eigen value 1.

The vector $L = \begin{bmatrix} l_1 & l_2 & . & . & l_n \end{bmatrix}^t$ is called a probability vector if $l_i \ge 0$, $1 \le i \le n$ and $\sum_{i=1}^{n} l_i = 1$.

A transition matrix T is said to be regular if there exist a positive integer m such that the all entries of T^k are strictly positive. A Markov process is called regular if its transition matrix is regular.

If T is the transition matrix of a regular Markov process, then

1. As $n \to \infty$, $T^n \to A$, where

$$A = \begin{bmatrix} l_1 & l_1 & . & . & l_1 \\ l_2 & l_2 & . & . & l_2 \\ . & . & & & . \\ . & . & & & . \\ l_n & l_n & . & . & l_n \end{bmatrix}$$

with all of columns of A are identical. Every column

$$L = [l_1, l_2 \, . \, . \, l_n]^t$$

is a probability vector such that $l_i > 0$, $1 \le i \le n$.

2. For any probability vector X, $T^n X \to L$ as $n \to \infty$, so that L is a steady level vector, which is unique and satisfy $TL = L$.

Let V be the set of all possible level positions (vectors) of any of the system F_2^k in SOCL and take any two consecutive level vectors L_t and L_{t+1} at a period of time with probability values of their respective components, that is for example

$$L_t = \begin{bmatrix} l_{t1} & l_{t2} & . & . & l_{tk} \end{bmatrix}^t = \begin{bmatrix} P_{t1} & P_{t2} & . & . & P_{tk} \end{bmatrix}^t$$

and

$$\begin{aligned} L_{t+1} &= \begin{bmatrix} l_{t+1,1} & l_{t+1,2} & . & . & l_{t+1,k} \end{bmatrix}^t \\ &= \begin{bmatrix} P_{t+1,1} & P_{t+1,2} & . & . & P_{t+1,k} \end{bmatrix}^t \end{aligned}$$

where $l_{ti}, l_{t+1,i} \in F_2^k$ and $P_{ti}, P_{t+1,i} \in [0,1] \subset R, 1 \le i \le k$.

The probability vectors L_t, L_{t+1} constitute a transition matrix

$$T = \begin{bmatrix} P_{11} & P_{12} & .. & P_{1k} \\ P_{21} & P_{22} & .. & P_{2k} \\ & & & . \\ . & . & .. & \\ & & & . \\ P_{k1} & P_{k2} & .. & P_{kk} \end{bmatrix}$$

After this we consider an initial level probability vector

$$L_0 = \begin{bmatrix} l_{01} & l_{02} & . & . & l_{0k} \end{bmatrix}^t = \begin{bmatrix} P_{01} & P_{02} & . & . & P_{0k} \end{bmatrix}^t$$

and then operating T on it to obtain the next probability vector

$$L_1 = [P_{11} \quad P_{12} \quad . \quad . \quad P_{1k}]^t$$

Steady level vectors: By modeling the Social Capital Matrix in terms of distinct interactive systems we also treat them through Markov process to observe whether the systems reaches their respective equilibrium positions. While the cone of economic activities of these systems is rotating, we randomly pick any two consecutive vectors of one of the system and allocate the probability values to the corresponding transition matrix. Besides this, we also give probability values to the components of initial value vector of the system under consideration. Here, we cannot determine as to which particular level vector of a system is the ideal one; it would be more canonical to put it for decision to the Markov process jury. When we initiate this process by assigning probability values to the transition matrix and to the initial level vector, the Markov chain will finally determine whether or not the system reaches its equilibrium position. For higher order transition matrices we have to use Mat lab.

Since rotation of cone began the economic activities in respective systems. The function $w: F_2^k \times F_2^k \to F_2^k$ in fact play a pivotal role to attain equilibrium level in a system F_2^k, where $k \in \{1,2,3,4\}$ due to the interaction of vectors in F_2^k and sets of different levels appears after the equally distributed periods of time. Of course it is hard to decide which combination of vectors of the system F_2^k is in equilibrium level. However the previous observations help to allocate the probability values to each vector in any level of the system F_2^k.

Now let V be the set of all possible level positions (vectors) of any of the system F_2^k in SOCL and take any two consecutive level vectors L_t and L_{t+1} at a period of time t which constitute a transition matrix T. Furthermore choose initial value probability vector L_0 and operate T on L_0 i.e., $TL_0 = L_1$, the next level (probability) vector. If L_1 is steady level vector, then system reaches equilibrium other wise we again operate T on L_1 and obtain L_2 and we continue these iterations till getting equilibrium.

The imbedding and levels of the systems offered that the rotation of cone of economic activities answers that what should be the status of each vector of each system regarding spending and welfare (respectively social capital). Now here a question arises: Which level of any of system in SOCL is most appropriate one regarding collective welfare (respectively social capital)? In the following we establish the response in pronouncement.

Equilibrium of systems

Theory of social capital and algebraic model: The welfare function $w: F_2^k \times F_2^k \to F_2^k$, where k is a positive integer, motivates to relate the all 16 interactions of social capital matrix but in this study we shall focus on interaction of Individual ($L = F_2^4$) with Individual ($L = F_2^4$), Community($C = F_2^3$) with Community ($C = F_2^3$), Organization ($O = F_2^2$) with Organization ($O = F_2^2$) and State ($S = F_2$) with State ($S = F_2$).

Here we interpret that the vectors of state, organization, community, individual respectively represent 2,4,8 and 16 classes in their own, that is each respective system is categorized into distinct blocks which have their own level of welfare. Whenever each system interact with itself, these distinct classes interact each other through w. Furthermore the interaction of the same class with itself represent the status of spending, that is minimum the social capital.

To much investment (spending) 0 components by either sides of vectors is not always optimal. As more number of welfare 1 occur in the vector of a system, then the level of welfare will increase and more number of 0 occur in a vector of a system, welfare level will decrease which cause decline in social capital and divergence in the system took place. However if in the situation, for example if we consider $(1,0,1,0),(0,1,0,1) \in F_2^4$, then $w((1,0,1,0),(0,1,0,1)) = (1,1,1,1)$. This shows optimal level of return (welfare), which indicate the optimal level of trust and cause to highest social capital.

Whenever the cone of economic activities (caused by the welfare function w) begin to work, the levels of each system in SOCL is changing their status in different periods of time. In fact inside each level of each system the components of vectors have variety of combinations in respect of spending and welfare and hence the social capital varies. So it is very correct to ask: Which level of a system has an excellent status regarding optimum welfare (respectively high social capital)? We can obtain a positive response through Markov machine.

In the following we observe the equilibriums in spending and welfare of systems in SOCL by fixing conventional probability values to consecutive level vectors and to initial level vector for each system. Moreover almost all transition matrices are considered to be regular. Thus in fact we are dealing with regular Markov process. Further if there exist a positive integer n for a regular Markov matrix T such that the columns of T^n are identical and sum of column is 1, then this column is the steady level vector and the system reaches equilibrium after n number of iterations (or periods).

Equilibrium in spending and welfare

State vs state, $F_2 \times F_2$

The State F_2 is a system in SOCL and $a, ß \in F_2$, $a = \alpha_1$, $ß = \beta_1$ be any vectors. The welfare function $w: F_2 \times F_2 \to F_2$ is defined by $w(a, ß) = a_1 + b_1 \in F_2$. Then

(1) $w(a, ß) = W_S^T$, the optimum level of the welfare if $\alpha_1 + \beta_1 = 1$.

(2) $w(a, ß) = W_S^{min}$, the minimal level of the welfare if $\alpha_1 + \beta_1 = 0$.

(3) $W_S^{min} < w(a, ß) < W_S^T$ if $\alpha_1 + \beta_1 \neq 0$.

The state interact with itself means that it allocate resources in any form or extend help, assistance or cooperation to each other. This exchange or reciprocity accumulates social capital within the state. This may observe as under:

(i) $0 + 0 = 1 + 1 = 0 = W_S^{min}$: Spending from either end and if either ends at the level of welfare, then consequently the spending power emerges and hence social capital decreases.

(ii) $0 + 1 = 1 + 0 = 1 = W_S^T$: One end is at the level of spending position and the other is already at optimum level of welfare or the situation is vice versa, then ultimately we obtain more welfare position at that component of the obtained vector. This means it gives the maximum social capital 1 for state.

According to our model the state has some of the levels $[I_S \quad W_S]^t$ and $[W_S \quad I_S]^t$ or $[0 \quad 1]^t$ and $[1 \quad 0]^t$. Now in the following we find which one makes the system S to be in equilibrium. So consider $I_S \quad W_S$.

$$T_S = \begin{array}{c} \begin{bmatrix} x & y \\ 1-x & 1-y \end{bmatrix} \begin{matrix} I_S \\ W_S \end{matrix} \end{array}$$

CaseI: Take

$$T_S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, T_S^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$T_S^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = T_S$$

This shows T_S is a transition matrix but in absorbing form.

(a) If $S_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, so $T_S S_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$.

This means system is already in equilibrium. Assumption of constant returns to scale is followed, which means that output or welfare is equal to inputs or spending.

(b) Assumption of decreasing returns to scale is followed, which means that output (welfare) is less than spending or inputs.

Take

$$S_0 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}, \text{ so } T_S S_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = S_1 \text{ and } T_S S_1 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = S_2.$$

Hence

$$S_0 = S_2 = S_4 = \ldots$$

$$S_1 = S_3 = S_5 = \ldots.$$

This means system will never reach at equilibrium level.

(c) Assumption of increasing welfare to scale may also be analyze. The welfare (output) is greater than spending or inputs.

Case II

$$T_S = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad S_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

We use the Matl ab to get further levels of the system.

```
>> T = [0.5 0.5; 0.5 0.5];
>> S0 = [0.5; 0.5];
>> S1 = T*S0
S1 =0.5000
0.5000
>> S2 = T*S1
S2 = 0.5000
0.5000
```

The initial probability vector is itself steady vector, that is the system is already in equilibrium level at initial period of time.

Case III

$$T_{S1} = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix},\ S_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}.$$

We use the Matlab to get further levels of the system.

```
>> T = [0.4 0.6; 0.6 0.4];
>> S0 = [0.4; 0.6];
>> S1 = T*S0
S1 =0.5200
0.4800
.
.
>> S5 = T*S4
S5 = 0.5000
0.5000.
```

So we obtained equilibrium level, that is

$$S_5 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

is steady level vector and hence the system S reaches equilibrium in fifth period of the time.

Organization vs organization, $F_2^2 \times F_2^2$

The organization F_2^2 is a system in SOCL and $a, ß \in F_2^2$, $\alpha = (\alpha_1, \alpha_2)$, $\beta = (\beta_1, \beta_2)$ be any vectors. The welfare function $w : F_2^2 \times F_2^2 \to F_2^2$ is defined by $w(a, ß) = (\alpha_1 + \beta_1, \alpha_2 + \beta_2) \in F_2^2$. Then

(1) $w(a, ß) = W_O^T$, the optimum level of the welfare if $\alpha_i+\beta_i = 1$ for each $1 \le i \le 2$.
(2) $w(a, ß) = W_O^{min}$, the minimal level of the welfare if $\alpha_i+\beta_i = 0$ for each $1 \le i \le 2$.
(3) $W_O^{min} < w(a, ß) < W_O^T$ if $\alpha_i+\beta_i \neq 0$ for some $1 \le i \le 2$.

An institution/organization interacts with another institution/organization directly or indirectly through their members by allocation of resources (in spending and welfare form) that accumulates social capital for them. That is if $\alpha = (0,1)$, $\beta = (1,0)$ be two organizations in O and after interaction we obtain w $(\alpha,\beta) = (1,1) = \gamma$, a new organization in O such that $sc(\gamma) = 2$, the maximum**.** Individuals on behalf of their institution/organization make investment through interaction and reciprocity with other institutions. This means if $\alpha = (0,1,0,1)$, $\beta = (1,0,0,0)$ be two individuals in L in which of course first two components of both α,β are representing spending welfare and welfare spending respectively, i.e. $(0,1) \mapsto a$, $(1,0) \mapsto ß$. Social capital generates reciprocity among institutions in order to develop their mutual trust.

According to our model the organization has the following levels, i.e.

$$[00\ \ 10\ \ 01\ \ 11]',[10\ \ 00\ \ 01\ \ 11]',$$
$$[10\ \ 01\ \ 00\ \ 11]',[10\ \ 01\ \ 11\ \ 00]',$$
$$[01\ \ 00\ \ 10\ \ 11]',[01\ \ 10\ \ 00\ \ 11]',$$
$$[01\ \ 10\ \ 11\ \ 00]',[11\ \ 00\ \ 10\ \ 01]',$$
$$[11\ \ 10\ \ 00\ \ 01]',[11\ \ 10\ \ 01\ \ 00]',$$

Now in the following we find which of these makes the system O to be in equilibrium. So consider

$$T_O = \begin{array}{c} \begin{array}{cccc} 00 & ..01 & ..10 & ..11 \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \end{array}$$

We adjust the probability of one level to other by the assumption that

	00	00	00	00	01	01	01	01	10	10	10	10	11	11	11	11
	00	01	10	11	00	01	10	11	00	01	10	11	00	01	10	11
Prob	0	1/4	1/4	1/2	1/4	0	1/2	1/4	1/4	1/2	0	1/4	1/2	1/4	1/4	0

Hence

$$T_O = \begin{bmatrix} 0 & 1/4 & 1/4 & 1/2 \\ 1/4 & 0 & 1/2 & 1/4 \\ 1/4 & 1/2 & 0 & 1/4 \\ 1/2 & 1/4 & 1/4 & 0 \end{bmatrix}$$

Further we assign the values to the initial level vector as

$$O_0 = \begin{bmatrix} (0,0) \\ (0,1) \\ (1,0) \\ (1,1) \end{bmatrix} = \begin{bmatrix} (0+0)/4 \\ (0+1)/4 \\ (1+0)/4 \\ (1+1)/4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/4 \\ 1/4 \\ 2/4 \end{bmatrix}$$

We initiate the process by $T_O O_O = O_1$ and continue this process as $T_O O_1 = O_2, \ldots$ We obtained O_{13} the steady level vector at which the system O reaches equilibrium.

Community vs community, $F_2^3 \times F_2^3$

The community F_2^3 is a system in SOCL and $a, ß \in F_2^3$,

$$a = (\alpha_1, \alpha_2, \alpha_3),\ ß = (\beta_1, \beta_2, \beta_3)$$

be any vectors. The welfare function $w: F_2^3 \times F_2^3 \to F_2^3$ is defined by

$$w(a, ß) = (\alpha_1 + \beta_1, \alpha_2 + \beta_2, \alpha_3 + \beta_3) \in F_2^3$$

Then

(1) $w(a, ß) = W_C^T$, the optimum level of the welfare if $\alpha_i + \beta_i = 1$ for each $1 \le i \le 3$.

(2) $w(a, ß) = W_C^{min}$, the minimal level of the welfare if $\alpha_i + \beta_i = 0$ for each $1 \le i \le 3$.

(3) $W_C^{min} < w(a, ß) < W_C^T$ if $\alpha_i + \beta_i \ne 0$ for some $1 \le i \le 3$.

Interaction of one community with other community by allocation of time and money (spending) accumulates social capital among them, that is if we consider welfare function $w: F_2^3 \times F_2^3 \to F_2^3$ such that α = (0,1,0), β = (1,0,0) be two communities in C and after interaction we obtain $w(a, ß) = (1,1,0) = \gamma \mapsto (1,1,0,0)$, a new (community in C) individual in L such that $sc(\gamma) = \frac{2}{1}$ but $sc(a) = \frac{1}{2}$ and $sc(ß) = \frac{1}{2}$, this means sc(β) = sc (α)<sc(γ). Robinson and Flora [4] confirm that individuals and groups/communities can consciously work to strengthen the social capital, for example

$$\begin{aligned} W_C^T + W_L^{Pure} &= a = (1,1,1,0) + (0,0,0,1) \\ &= ß = (1,1,1,1) \\ &= \gamma = W_L^T \end{aligned}$$

where

$$W_C^T = a = (1,1,1) \mapsto (1,1,1,0)$$

with sc(α) = 3, $sc(ß) = \frac{1}{4}$ and sc(γ) = 4. Similarly, Woolcock [18] is of the view that physical capital and human capital(spending) are essentially(purely) the property of individuals, while social capital and extension inheres in groups/communities. This can be view as, if $a = (1,1,1,0) = I_L^{Pure}$, be the individual in L, but $W_C^T = a = (1,1,1) \mapsto (1,1,1,0)$, that is sc($\alpha$) = 3 in community.

According to our model the community has the following levels, i.e.

$$[000\ \ 001\ \ 011\ \ 111\ \ 101\ \ 100\ \ 010\ \ 110]^t$$
$$[001\ \ 000\ \ 011\ \ 111\ \ 101\ \ 100\ \ 010\ \ 110]^t$$
$$[001\ \ 011\ \ 001\ \ 111\ \ 101\ \ 100\ \ 010\ \ 110], \ldots.$$

Now in the following we find which of these makes the system C to be in equilibrium. So consider we consider T_C

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & . & . & P_{18} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & . & . & P_{28} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & . & . & P_{38} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} & . & . & P_{48} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & . & . & P_{58} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & . & . & P_{68} \\ P_{71} & P_{72} & P_{73} & P_{74} & P_{75} & . & . & P_{78} \\ P_{81} & P_{82} & P_{83} & P_{84} & P_{85} & . & . & P_{88} \end{bmatrix}$$

The scheme: The scheme of assigning the probability values is as under;

For $a_{i1}, a_{i2}, a_{i3},, b_{1j}, b_{2j}, b_{3j},, a, b, c, \in F_2$

$a_{i1}a_{i2}a_{i3} + b_{1j}b_{2j}b_{3j} = abc$	P_{ij}
abc=000=111	0.05
abc=100=010=010=001	0.1
abc=110=101=101=011	0.2

For example if 101+111 = 010, then $P_{i_0 j_0} = 0.1$, if 111+110 = 0001, then $P_{i_0 j_0} = 0.1$ and if 111+010 = 101, then $P_{i_0 j_0} = 0.2$.

$$T_C = \begin{bmatrix} .05 & .1 & .1 & .1 & .2 & .2 & .2 & .05 \\ .1 & .05 & .2 & .2 & .05 & .1 & .1 & .2 \\ .1 & .2 & .05 & .2 & .1 & .05 & .1 & .2 \\ .1 & .2 & .2 & .05 & .1 & .1 & .05 & .2 \\ .2 & .05 & .1 & .1 & .05 & .2 & .2 & .1 \\ .2 & .1 & .05 & .1 & .2 & .05 & .2 & .1 \\ .2 & .1 & .1 & .05 & .2 & .2 & .05 & .1 \\ .05 & .2 & .2 & .2 & .1 & .1 & .1 & .05 \end{bmatrix}$$

We consider the initial probability vector C_0 as

$$\begin{bmatrix}.05 & .1 & .1 & .1 & .2 & .2 & .2 & .05\end{bmatrix}$$

We use the Matl ab to get further levels of the system.

```
>> T = [0.05 0.1 0.1 0.1 0.2 0.2 0.2 0.05; 0.1 0.05 0.2 0.2 0.05 0.1 0.1 0.2; 0.1 0.2 0.05 0.2 0.1 0.05 0.1 0.2; 0.1 0.2 0.2 0.05 0.1 0.1 0.05 0.2; 0.2 0.05 0.1 0.1 0.05 0.2 0.2 0.1; 0.2 0.1 0.05 0.1 0.2 0.05 0.2 0.1; 0.2 0.1 0.1 0.05 0.2 0.2 0.05 0.1; 0.05 0.2 0.2 0.2 0.1 0.1 0.1 0.05];
>> C0 = [0.05; 0.1; 0.1; 0.1; 0.2; 0.2; 0.2; 0.05];
>> C1 = T*C0.
```

Individual vs individual, $F_2^4 \times F_2^4$

The system individual F_2^4 is a system in SOCL and $a, ß \in F_2^4$, $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$, $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)$ be any vectors. The welfare function $w : F_2^4 \times F_2^4 \to F_2^4$ is defined by

$$w(a, ß) = (\alpha_1 + \beta_1, \alpha_2 + \beta_2, \alpha_3 + \beta_3, \alpha_4 + \beta_4) \in F_2^4$$

Then

(1) $w(a, ß) = W_L^T$, the optimum level of the welfare if $\alpha_i + \beta_i = 1$ for each $1 \le i \le 4$

(2) $w(a, ß) = W_L^{min}$, the minimal level of the welfare if $\alpha_i + \beta_i = 0$ for each $1 \le i \le 4$

(3) $W_L^{min} < w(a, ß) < W_L^T$ if $\alpha_i + \beta_i \neq 0$ for some $1 \le i \le 4$.

Social capital is accumulated between two individuals through one individual's interaction and reciprocity with the other individuals. By Coleman [14], social capital constitutes a capital asset for the individual, it consists of some aspect of social structure and facilitates certain action of the individuals who are within the structure. This relationship in turn develops trust between individuals that enable them to generate returns in future. So we consolidate it through $w : F_2^4 \times F_2^4 \to F_2^4$, for example if α = (0,1,0,1), β = (1,0,0,1) be two individuals in L and after interaction we obtain $w(a, ß) = (1,1,0,1) = \gamma$, a new individual in L such that $sc(\gamma) = \frac{3}{1}$ but $sc(\alpha) = \frac{2}{2}$ and $sc(\beta) = \frac{1}{3}$, this means $sc(\beta) \le sc(\alpha) \le sc(c)$. Sobel [15] look as "these problems involve small numbers of agents who know each other and interact repeatedly. The theory of repeated games explains how self-interested, calculating individuals can reach cooperative, efficient outcomes in this setting, but the same theory permits inefficient outcomes as well". For example if d = (1,1,0,1), e = (1,1,1,1) be two individuals in L and after interaction we obtain $w(d,e) = (0,0,1,0) = f$, a new individual in L such that $sc(f) = \frac{1}{3}$ but $sc(d) = \frac{3}{1}$ and $sc(e) = 4$, this means $sc(f) \le sc(d) \le sc(e)$

In the following we find which of the level makes the system L to be in equilibrium.

First we assign the values to the components of initial level probability vector.

The scheme: The scheme of assigning the probability values is as under;

For $a_{i1}, a_{i2}, a_{i3}, a_{i4}, b_{1j}, b_{2j}, b_{3j}, b_{4j}, a, b, c, d \in F_2$

$a_{i1}a_{i2}a_{i3}a_{i4} + b_{1j}b_{2j}b_{3j}b_{4j} = abcd$	P_{ij}
abcd=0000=1111	.0313
1000=0100=0010=0001	.0156
1110=1101=1011=0111	.0156
1100=1010=1001=0110=0011	.0104

For example if 0101+1110 = 1011, then $P_{i_0 j_0} = 0.0156$, if 1111+1110 = 0001, then $P_{i_0 j_0} = 0.0156$ and if 1111+1010 = 0101, then $P_{i_0 j_0} = 0.0104$.

We consider the initial probability vector as:

$$L_0 = \begin{bmatrix}0000\\0001\\0011\\0111\\1111\\1110\\1100\\1000\\1001\\1011\\1101\\0101\\0110\\0100\\1010\\0010\end{bmatrix} = \begin{bmatrix}0.0313\\0.0156\\0.0104\\0.0156\\0.0313\\0.0156\\0.0104\\0.0156\\0.0104\\0.0156\\0.0156\\0.0104\\0.0104\\0.0156\\0.0104\\0.0156\end{bmatrix}$$

The following is the representation of transition matrix T_L and of initial probability vector T_0 in MATLAB.

>> T = [0.0313 0.0156 0.0104 0.0156 0.0313 0.0156
0.0104 0.0156 0.0104 0.0156 0.0156 0.0104 0.0104
0.0156 0.0104 0.0156; 0.0156 0.0313 0.0156 0.0104
0.0156 0.0313 0.0156 0.0104 0.0156 0.0104 0.0104
0.0156 0.0156 0.0104 0.0156 0.0104; 0.0104 0.0156
0.0313 0.0156 0.0104 0.0156 0.0313 0.0156 0.0104
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>> L0 = [0.0313; 0.0156; 0.0104; 0.0156; 0.0313; 0.0156; 0.0104; 0.0156; 0.0104; 0.0156; 0.0156; 0.0104; 0.0104; 0.0156; 0.0104; 0.0156;];
>> L1 = T*L0.

Findings

System	TransMatrix	#itration	SteadyLev.vector
S	T_S	5	S_5
O	T_O	13	O_{13}
C	T_C	6	C_6
L	T_L	532	L_{532}

whereas

$$\begin{array}{cccc} S_5 & O_{13} & C_6 & L_{532} \\ & & & 9.9e-323* \\ \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} & \begin{bmatrix} 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \end{bmatrix} & \begin{bmatrix} 0.1250 \\ 0.1250 \\ 0.1250 \\ 0.1250 \\ 0.1250 \\ 0.1250 \\ 0.1250 \\ 0.1250 \end{bmatrix} & \begin{bmatrix} 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \\ 0.2500 \end{bmatrix} \end{array}$$

For the sake of convenience we have taken transition matrices T_S, T_O, T_C and T_L as doubly Markov. But we may consider the transition matrix just regular Markov, for example in Z_2 if we consider the transition matrix $T = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}$ and the initial level vector $S_0 = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$, then by using Matl ab, we obtain;

```
T=[0.4 0.3; 0.6 0.7];
S0=[0.2; 0.8];
N=5;
for k=1:N
S1=T94k*S0
end
S1 = 0.3200
0.6800
.
.
S 4 =0.3333
0.6667
```

Hence $S_4 = \begin{bmatrix} 0.3333 \\ 0.6667 \end{bmatrix}$ is the steady level vector, that is at fourth period of time the system reaches at equilibrium.

CONCLUSION

When the cone of economic activities of the systems; individuals, group, organization and State is rotating, inter temporal accumulation of social capital of certain amount may take place based on spending and welfare during cycle of economic activities at different levels. In each system different combinations of resource allocation in spending and welfare activities lead to level game. Resources possessed by any system needs to be prioritized for economic development. The resource allocation plan affects equilibrium, thereby; economic development in the society. This study acknowledged equilibrium in economic development and its relationship with social capital. It observed that a number of level games emerged from various resource combinations in the four systems. Markov process is used to makes it possible to observe whether the systems reach their respective equilibrium positions or not. Thus Markov process authenticates equilibrium of spending and welfare in respect of social capital for each system.

The study is generalized if we augment the state indicators, that is n≥2; then state S should be F_n: Since F_n is a field if n is a prime integer, so more or less similar algebraic construction applies as considered in this paper and the conduct of SOCL vis-à-vis equilibrium can be characterized with complexities. In contrast, F_n behaves as a commutative ring with identity and represented as Z_n, which is not an integral domain whenever n is not prime. This would certainly be more appropriate option in analyzing the equilibrium in economic development in a rational way and this approach may provide a justification concerning non-availability of smooth arrangement of categories of the systems.

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