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Multi-Dimensional Nonlinear Knapsack Problem (MNKP) for Crashing CPM/PERT Networks

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Abstract: In this paper, a solution method of multi-dimensional nonlinear knapsack problem (MNKP) is proposed for solving the crashing path in critical path method (CPM)/ program evaluation and review technique (PERT). The constrained nonlinear mathematical model of CPM/PERT method is developed by using Taylor's second order expansion and could be solved by MNKP. The numerical results show that the time of completing project will be reduced and the probability of completing project is increased.

Key words: Multi-Dimensional Nonlinear Knapsack problem · Crashing Networks · CPM/PERT · Taylor's series

INTRODUCTION

Nowadays, many problems in real applications naturally result in optimization problems in a form of non-linear integer programming or mixed integer programming. Non-linear integer programming has been one of the great challenges for the optimization research community for many years. However, due to its computational difficulties there exists the exponential growth in its computational complexity with respect to the problem dimension. For this reason, the research efforts of the past few years have leads to development of method to solve the non-linear integer programming (NLIP).

The theory and solution methodologies for nonlinear integer programming are substantially different from the linear integer programming. Application of nonlinear can be found in various areas such as engineering, management science and operation research. From [1], nonlinear knapsack problem (NKP) was classified as subclasses of the NLIP. The general form of NKP described as solutions of the following problem:

Maximize $f(x)$
Subject to $g(x) \le b$

where $x = (x_1, x_2..., x_n) \in R^n$, $f(x)$ and $g(x)$ are continuous and differentiable functions on R^n, b is a real constant and $D \subseteq R^n$. There is a huge variety of problems which can construct based on this general definition. The NKP has a variety of applications, including, financial models, production and inventory management, stratified sampling and the optimal design of queueing network models in manufacturing, computer systems and health care.

In this paper, we present a survey of algorithms and applications for the MNKP which is bounded NKP, or known as the nonlinear resource allocation problem [2]. MNKP has been developed from NKP due to the difficulties arising from the multiple constraints. Extensive computational results show that the proposed method is efficient for solving large scale MNKP. By the way from [3], the MNKP can be solved even if the objective and constraint functions of the original NLIP are non-convex and undifferentiable.

This remainder of this paper is organized as follows: The general form of MNKP are reviewed in section 2, while in section 3 the model construction of CPM/PERT are developed and expand by using Taylor's second order expansion. Numerical results are reported in section 4 and 5, concludes the paper.

Multi-Dimensional Nonlinear Knapsack Problem: According to [1], the nonlinear knapsack problem has only a single constraint, if there are multiple constraints in a nonlinear knapsack problem and then the problem is called a MNKP.

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Hence the MNKP describes as following form [2]:

Maximize $f(x) = \sum_{j=1}^{n} f_j(x_j)$

Subject to $g_i(x) = \sum_{j=1}^{n} g_{ij}(x_j) \le b,\ i = 1,\ldots,m$

$$x \in X = \{x \in Z^n | l_j \le x_j \le u_j, j = 1\ldots,n\}$$

where $l_j < u_j$, $_j$ and u_j are integer numbers for $j = 1,\ldots,n$. And both f_j and g_j are continuous functions that satisfy the monotonicity assumptions, which stated that f_j and g_j are increasing functions on $[l_j, u_j]$ and $j = 1,\ldots,n$.

Problem Formulation

Model Construction: From [4], PERT is a management technique to estimate the probability that a project will finish on time. According to the traditional PERT technique the probability of a certain project meeting a specific schedule time T_s, for a particular event is equal to $\phi(Z)$, the probability can be described as follows:

$$Z = \frac{T_S - T_E}{\sigma_{TE}} \tag{1}$$

where the total expected duration of the project T_E, is the sum of the expected time along the critical path.

$$T_E = \sum_{i=1}^{n} t_i = t_1 + t_2 + \ldots + t_n \tag{2}$$

The variability in the activity time estimates is approximated equations (3).

$$\sigma_i = \frac{b_i - a_i}{6},$$
$$\sigma_{TE} = \sqrt{\sum_{i=1}^{n} \sigma_i^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2} \tag{3}$$

σ_i represents the standard deviation for the activity and σ_{TE} represents the standard deviation for the project along the critical path. While the expected time and variance of the activity on the critical path is computed by the following formula:

$$t_i = \frac{a_i + 4m_i + b_i}{6},\ \sigma_i^2 = \frac{(b_i - a_i)^2}{36} \tag{4}$$

where a is a optimistic activity time, m is a most likely activity time, b is a pessimistic activity time. An amount of money will be invested in each critical path activity is r_i, where $i = 1,2,\ldots,n$ and n is the total number of activities which lie on the critical path. The amount of money invested in that critical path activities are M units of money.

$$M = \sum_{i=1}^{n} r_i = r_1 + r_2 + r_3 + \cdots + r_n \tag{5}$$

Then the pessimistic time of the activity will reduce from b to $\hat{b}$ and the total expected project duration will reduce from T_E to $\hat{T}_E$ and similar with the standard deviation of the project reduce from σ_{TE} to $\hat{\sigma}_{TE}$ lie on the critical path. The new of expected time, variance of the activity and standard deviation for the activity that lie on the critical path will be as follows

$$\hat{t}_i = \frac{a_i + 4m_i + \hat{b}_i}{6},\ \hat{\sigma}_i^2 = \frac{(\hat{b}_i - a_i)^2}{36},\ \hat{\sigma}_i = \frac{\hat{b}_i - a_i}{6} \tag{6}$$

After investing the amount of money in the activities which lie on the critical path, the new total expected duration of the project and standard deviation for the project along the critical path is equal to

$$\hat{T}_E = \sum_{i=1}^{n} \hat{t}_i,\ \hat{\sigma}_{TE} = \sqrt{\sum_{i=1}^{n} \hat{\sigma}_i^2} \tag{7}$$

Then the new probability of realizing the terminal node will be

$$\hat{Z} = \frac{T_s - \hat{T}_E}{\hat{\sigma}_{TE}} = T_s - \sum_{i=1}^{n} \hat{t}_i \Big/ \sqrt{\sum_{i=1}^{n} \hat{\sigma}_i^2} \tag{8}$$

Since the reduction of the expected time and the variance of the activities that lie on the critical path are depend on the amount of the money that will invest in each activity. Therefore, it is means that the decrease in both expected time and variance of the activities will be a function ϕ and ψ of the additional investment. Then the new expected duration and variance of the activities for a certain activity will be

$$\hat{t}_i = t_i - \phi(r_i),\ \hat{\sigma}_i^2 = \sigma_i^2 - \psi(r_i) \tag{9}$$

From [5], the equation of new expected duration and variance of the activities becomes:

$$\hat{t}_i = t_i + q_i r_i \qquad -\frac{t_i}{r_i} < q_i < 0 \tag{10}$$

$$\hat{\sigma}_i^2 = \sigma_i^2 + s_i r_i \qquad -\frac{\sigma_i^2}{r_i} < s_i < 0 \tag{11}$$

where $S_i = q_i$. By substitute the equations (10) and (11) to equation (8), then $\hat{z}$ becomes

$$\hat{Z} = \frac{T_S - \{t_1 + q_1 r_1 + t_2 + q_2 r_2 + \cdots + t_n + q_n r_n\}}{\sqrt{\{\sigma_1^2 + s_1 r_1 + \sigma_2^2 + s_2 r_2 + \cdots + \sigma_n^2 + s_n r_n\}}}$$

$$= \frac{T_S - \left\{\sum_{i=1}^{n} t_i + \sum_{i=1}^{n} q_i r_i\right\}}{\sqrt{\left\{\sum_{i=1}^{n} \sigma_i^2 + \sum_{i=1}^{n} s_i r_i\right\}}} \tag{12}$$

According to [5], $\sum_{i=1}^{n} t_i$ and $\sum_{i=1}^{n} \sigma_i^2$ are constant because same before and after crashing. Therefore let $\sum_{i=1}^{n} t_i$ be C_1 and $\sum_{i=1}^{n} \sigma_i^2$ be C_2.Then

$$\hat{Z} = \frac{T_S - \left\{C_1 + \sum_{i=1}^{n} q_i r_i\right\}}{\sqrt{C_2 + \sum_{i=1}^{n} s_i r_i}} \tag{13}$$

Let $\overline{r_i}$ is the upper limit for the amount of money to be invested in each activity that lie on the critical path. The range of the r_i will be $0 \le r_i \le \overline{r_i}$. Then the new range of s_i and q_i will develop as below, when the amount of money invested r_i increase until upper limit for the amount of money to be invested in each activity $\overline{r_i}$.

$$-\frac{t_i}{\overline{r_i}} < q_i, \; -\frac{\sigma_i}{\overline{r_i}} < s_i, \; -\frac{\sigma_i^2}{\overline{r_i}} < s_i \tag{14}$$

Equation (13) is the required mathematical model, which represents the objective function and the constraints. The value of q_i and S_i are specified by expert having high knowledge in the nature of project activities. The whole model can be rewritten as follows.

$$Max\,\hat{Z} = \frac{T_S - C_1 - \sum_{i=1}^{n} q_i r_i}{\sqrt{C_2 + \sum_{i=1}^{n} s_i r_i}}$$

such that $0 \le r_i \le \overline{r_i}$ and $\sum_{i=1}^{n} r_i \le M$ (15)

Expansion of the Mathematical Model: The mathematical model from equation (15) extends to be nonlinear programming by using the Taylor's series of 2nd order expansion. According to [5], the first term of the mathematical model is constant and depends on $\overline{r_i}$ and then let first term be α as analogous to [5]. Since α is constant and then the $Max\,\hat{Z}$ can be written as follows subject to

$$Max\,\hat{Z} = \left(-\frac{1}{2}\frac{\left(T_s - C_1 - \sum_{i=1}^{n} q_i \overline{r_i}\right)\left(\sum_{i=1}^{n} s_i\right)}{\left(C_2 + \sum_{i=1}^{n} s_i \overline{r_i}\right)^{\frac{3}{2}}} - \frac{\sum_{i=1}^{n} q_i}{\sqrt{C_2 + \sum_{i=1}^{n} s_i \overline{r_i}}} \right)(r_i - \overline{r_i}) + \left(\frac{3}{8}\frac{\left(T_s - C_1 - \sum_{i=1}^{n} q_i \overline{r_i}\right)\left(\sum_{i=1}^{n} s_i\right)^2}{\left(C_2 + \sum_{i=1}^{n} s_i \overline{r_i}\right)^{\frac{5}{2}}} + \frac{1}{2}\frac{\left(\sum_{i=1}^{n} q_i\right)\left(\sum_{i=1}^{n} s_i\right)}{\left(C_2 + \sum_{i=1}^{n} s_i \overline{r_i}\right)^{\frac{3}{2}}} \right)(r_i - \overline{r_i})^2 \tag{16}$$

Subject to $0 \le r_i \le \overline{r_i}$ and $\sum_{i=1}^{n} r_i \le M$

Illustrative Example: The case study consisting of 42 activities to demonstrate how the mathematical model can be applied to decrease project duration was referring from [5]. The mathematic model in [5] was solved by using any linear programming package, despite that we using MNKP of nonlinear integer programming to solve the mathematical model in this paper.

Table 1: a, m and b of the 42 activities

No.	a,m,b	No.	a,m,b	No.	a,m,b
1	5,12,35	15	5,8,11	29	11,16,18
2	2,5,6	16	12,14,15	30	30,32,35
3	5,8,9	17	12,14,18	31	12,14,19
4	10,12,15	18	12,15,18	32	23,29,39
5	2,5,9	19	12,15,19	33	25,29,38
6	2,5,8	20	15,18,19	34	12,14,15
7	5,9,14	21	15,16,19	35	22,28,35
8	5,8,9	22	9,10,12	36	23,24,30
9	8,9,16	23	12,13,19	37	10,11,15
10	5,6,8	24	12,15,17	38	23,23,24
11	12,13,15	25	21,22,29	39	30,32,36
12	5,9,16	26	15,17,19	40	30,35,36
13	2,3,5	27	26,29,31	41	30,30,32
14	13,15,18	28	15,19,21	42	25,26,29

Table 2: Predecessors (*) of the 42 activities

No.	*	No.	*	No.	*
1	-	15	7, 8, 13	29	24, 27
2	-	16	9, 10	30	24, 27
3	1	17	9, 10	31	25, 26
4	1	18	12, 14, 15	32	30, 31
5	2	19	12, 14, 15	33	28, 32
6	3	20	16, 19	34	30, 31
7	3	21	11, 18, 20	35	33
8	4, 5	22	16, 19	36	29, 35
9	4, 5	23	17, 22	37	33
10	2	24	21, 23	38	36, 37
11	6	25	21, 23	39	36, 37
12	6	26	17, 22	40	34, 39
13	9, 10	27	25, 26	41	38, 40
14	9, 10	28	24, 27	42	41

Table 3: Expected time, early start and finish time, late start and finish time, total and free float, standard deviation and variance of the 42 activities

Activity No.	Expected Time	Early Start	Early Finish	Late Start	Late Finish	Total Float	Free Float	σ	σ^2
1	14.67	0.00	14.70	0.00	14.70	0.00	0.00	5.00	25.00
2	4.67	0.00	4.70	17.00	21.70	17.00	0.00	0.67	0.44
3	7.67	14.70	22.30	27.20	34.80	12.50	0.00	0.67	0.44
4	12.17	14.70	26.80	14.70	26.80	0.00	0.00	0.83	0.69
5	5.17	4.70	9.80	21.70	26.80	17.00	17.00	1.17	1.36
6	5.00	22.30	27.30	37.50	42.50	15.20	0.00	1.00	1.00
7	9.17	22.30	31.50	34.80	44.00	12.50	8.50	1.50	2.25
8	7.67	26.80	34.50	36.30	44.00	9.50	5.50	0.67	0.44
9	10.00	26.80	36.80	26.80	36.80	0.00	0.00	1.33	1.78
10	6.17	4.70	10.80	30.70	36.80	26.00	26.00	0.50	0.25
11	13.17	27.30	40.50	71.70	84.80	44.30	44.30	0.50	0.25
12	9.50	27.30	36.80	42.50	52.00	15.20	15.20	1.83	3.36
13	3.17	36.80	40.00	40.80	44.00	4.00	0.00	0.50	0.25
14	15.17	36.80	52.00	36.80	52.00	0.00	0.00	0.83	0.69
15	8.00	40.00	48.00	44.00	52.00	4.00	4.00	1.00	1.00
16	13.83	36.80	50.70	53.30	67.20	16.50	16.50	0.50	0.25
17	14.33	36.80	51.20	73.00	87.30	36.20	26.20	1.00	1.00
18	15.00	52.00	67.00	69.80	84.80	17.80	17.80	1.00	1.00
19	15.17	52.00	67.20	52.00	67.20	0.00	0.00	1.17	1.36
20	17.67	67.20	84.80	67.20	84.80	0.00	0.00	0.67	0.44
21	16.33	84.80	101.20	84.80	101.20	0.00	0.00	0.67	0.44
22	10.17	67.20	77.30	77.20	87.30	10.00	0.00	0.50	0.25
23	13.83	77.30	91.20	87.30	101.20	10.00	10.00	1.17	1.36
24	14.83	101.20	116.00	138.20	153.00	37.00	37.00	0.83	0.69
25	23.00	101.20	124.20	101.20	124.20	0.00	0.00	1.33	1.78
26	17.00	77.30	94.30	107.20	124.20	29.80	29.80	0.67	0.44
27	28.83	124.20	153.00	124.20	153.00	0.00	0.00	0.83	0.69
28	18.67	153.00	171.70	196.20	214.80	43.20	43.20	1.00	1.00
29	15.50	153.00	168.50	257.30	272.80	104.30	104.30	1.17	1.36
30	32.17	153.00	185.20	153.00	185.20	0.00	0.00	0.83	0.69
31	14.50	124.20	138.70	170.70	185.20	46.50	46.50	1.17	1.36
32	29.67	185.20	214.80	185.20	214.80	0.00	0.00	2.67	7.11
33	29.83	214.80	244.70	214.80	244.70	0.00	0.00	2.17	4.69
34	13.83	185.20	199.00	316.20	330.00	131.00	131.00	0.50	0.25
35	28.17	244.70	272.80	244.70	272.80	0.00	0.00	2.17	4.69
36	24.83	272.80	297.70	272.80	297.70	0.00	0.00	1.17	1.36
37	11.50	244.70	256.20	286.20	297.70	41.50	41.50	0.83	0.69
38	23.17	297.70	320.80	341.20	364.30	43.50	43.50	0.17	0.03
39	32.33	297.70	330.00	297.70	330.00	0.00	0.00	1.00	1.00
40	34.33	330.00	364.30	330.00	364.30	0.00	0.00	1.00	1.00
41	30.33	364.30	394.70	364.30	394.70	0.00	0.00	0.33	0.11
42	26.33	394.70	421.00	394.70	421.00	0.00	0.00	0.67	0.44

Table 4: The probability of completing the project by 415 days when M ={0,5000,10000,15000,20000,25000,30000,35000}

No.	M	$\sum_{i=1}^{n}\sigma_i^2$	T_S	T_E		σ_{TE}	Z		(%)
1	0	53.996	415	421	≈421	7.348197	-0.81653	≈-0.82	20.61
2	5000	45.66		417.957	≈418	6.757218	-0.44397	≈-0.44	33.00
3	10000	39.468		416.075	≈417	6.282356	-0.31835	≈-0.32	37.45
4	15000	37.409		414.493	≈415	6.11629	0	≈ 0	50.00
5	20000	35.957		413.461	≈414	5.996416	0.166766	≈ 0.17	56.75
6	25000	33.159		412.932	≈413	5.758385	0.347320	≈ 0.35	63.68
7	30000	30.337		412.538	≈413	5.507903	0.363115	≈ 0.36	64.06
8	35000	30.112		412.338	≈413	5.48744	0.364469	≈ 0.36	64.06

Table 5: The $\overline{r_i}$ of critical activities

i	$\overline{r_i}$	i	$\overline{r_i}$	i	$\overline{r_i}$
1	5000	21	1200	35	1500
4	1000	25	1800	36	1100
9	3000	27	800	39	2200
14	2000	30	750	40	500
19	1000	32	3200	41	850
20	2000	33	2000	42	6000

Table 6: The value q_i and S_i of critical activities

i	$-\frac{t_i}{\overline{r_i}}$	$-\frac{\sigma_i^2}{\overline{r_i}}$	$-\frac{\sigma_i}{\overline{r_i}}$	$S_i = q_i$
1	-0.00293	-0.00500	-0.00100	-0.00010
4	-0.01217	-0.00069	-0.00083	-0.00030
9	-0.00333	-0.00059	-0.00044	-0.00011
14	-0.00758	-0.00035	-0.00042	-0.00023
19	-0.01517	-0.00136	-0.00117	-0.00041
20	-0.00883	-0.00022	-0.00033	-0.00005
21	-0.01361	-0.00037	-0.00056	-0.00033
25	-0.01278	-0.00099	-0.00074	-0.00036
27	-0.03604	-0.00087	-0.00104	-0.00018
30	-0.04289	-0.00093	-0.00111	-0.00045
32	-0.00927	-0.00222	-0.00083	-0.00036
33	-0.01492	-0.00235	-0.00108	-0.00055
35	-0.01878	-0.00313	-0.00144	-0.00066
36	-0.02258	-0.00124	-0.00106	-0.00070
39	-0.01470	-0.00045	-0.00045	-0.00020
40	-0.06867	-0.00200	-0.00200	-0.00025
41	-0.03569	-0.00013	-0.00039	-0.00030
42	-0.00439	-0.00074	-0.00011	-0.00004

By substituting the values from Table 1 and 2, the both forward and backward calculations of 42 activities are constructed as show in Table 3. According to [6], forward calculation for early start and finish time and backward calculation for late start and finish time.

From Table 3, we found that 18 activities have a total float of zero which are 1, 4, 9, 14, 19, 20, 21, 25, 27, 30, 32, 33, 35, 36, 39, 40, 41 and 42. Clearly then there have 18 activities lie on the critical path or as known as critical activities.

From Table 3 also, we found that the total expected time and variance of the 18 critical activities are 421 and 54. If the scheduled duration of the project is 415, then the probability of finishing the project within the completion date 415 days can be computed as $Z = -0.82$. From the tables of areas under the standardized normal curve from $-\infty$ to $-z$ or PERT Factor Tables referred to [7], a Z value -0.82 gives the probability of 0.2061, which means there is a 20.6% chance of completing the project on or before 415 days. Indeed, the result from above is identical with [5, 8, 9].

Hence, in order to increase the probability of completing the project on or before 415 days, the total amount of money, M, will invest to for all critical activities in the project, so that we can demonstrate the increasing in the investment of the project will tend to increase the probability of completing the project within 415 days. Therefore in this paper, 7 different total amounts to be invested to the project were \$5000, \$10000, \$15000, \$20000, \$25000, \$30000 and \$35000

The amounts of money invest to each critical activities, $\overline{r_i}$, specified by experts on the Table 5 as show below.

In the same way, the value of q_i and s_i of the critical activities are also specify by experts who having high knowledge in the nature of project activities. The values of both q_i and s_i satisfy the equation (14) as showed below on the Table 6.

By substituting the values of q_i, s_i and $\overline{r_i}$ in the developed mathematical model, then it can be written as follow:

$$
\begin{aligned}
Max\,\hat{Z} = {} & 0.0000128515 r_1 + 0.0000385567 r_4 + 0.0000141374 r_9 \\
& + 0.0000295588 r_{14} + 0.0000526927 r_{19} + 0.0000064263 r_{20} \\
& + 0.0000424113 r_{21} + 0.0000462623 r_{25} + 0.0000231346 r_{27} \\
& + 0.0000578346 r_{30} + 0.0000462458 r_{32} + 0.0000706566 r_{33} \\
& + 0.0000847956 r_{35} + 0.0000899484 r_{36} + 0.0000257035 r_{39} \\
& + 0.0000321315 r_{40} + 0.0000385571 r_{41} + 0.0000051410 r_{42} \\
& + 1.17933\times10^{-11} r_1^2 + 1.05261\times10^{-10} r_4^4 + 1.41694\times10^{-11} r_9^2 \\
& + 6.22832\times10^{-11} r_{14}^2 + 1.97505\times10^{-10} r_{19}^2 + 2.89981\times10^{-12} r_{20}^2 \\
& + 1.27875\times10^{-10} r_{21}^2 + 1.53789\times10^{-10} r_{25}^2 + 3.76500\times10^{-11} r_{27}^2 \\
& + 2.37207\times10^{-10} r_{30}^2 + 1.57080\times10^{-10} r_{32}^2 + 3.65838\times10^{-10} r_{33}^2 \\
& + 5.24371\times10^{-10} r_{35}^2 + 5.84432\times10^{-10} r_{36}^2 + 4.70558\times10^{-11} r_{39}^2 \\
& + 7.25702\times10^{-11} r_{40}^2 + 1.05065\times10^{-11} r_{41}^2 + 1.86666\times10^{-11} r_{42}^2
\end{aligned}
$$

subject to

$$
\begin{array}{lllll}
r_1 \le 5000 & r_4 \le 1000 & r_9 \le 3000 & r_{14} \le 2000 & r_{19} \le 1000 \\
r_{20} \le 2000 & r_{21} \le 1200 & r_{25} \le 1800 & r_{27} \le 800 & r_{30} \le 750 \\
r_{32} \le 3200 & r_{33} \le 2000 & r_{35} \le 1500 & r_{36} \le 1100 & r_{39} \le 2200 \\
r_{40} \le 500 & r_{41} \le 850 & r_{42} \le 6000 & &
\end{array}
$$

$$
\begin{aligned}
& r_1 + r_4 + r_9 + r_{14} + r_{19} + r_{20} + r_{21} + r_{25} + r_{27} + r_{30} \\
& + r_{32} + r_{33} + r_{35} + r_{36} + r_{39} + r_{40} + r_{41} + r_{42} \le M
\end{aligned}
$$

CONCLUSION

As Table 4 indicates, the probability of completing the project within 415 days was increased from 20.61% to 33%, 37.45%, 50%, 56.75%, 63.68%, 64.06% and 64.06%, upon the increment of investment amount of money. The expected duration of the project was successfully reduced from 421 days to 413 days upon the increased in the amounts of money from $5000 to $35000 to the project. Thus, the result of the probability of the completing the project within 415 days before any investment to the project is identical with the [5, 8, 9] which is 20.6%, where [5] was using any linear programming package to solve the mathematical model, [8] was solved by using simplex method and [9] which solved by using knapsack problem of integer programming.

The results on the probability of completing the project within 415 days for the additional investment amount of money from $5000 to $35000 are identical with the [8, 9]. However the investment of $30000 in this paper is differing with [5] and the reason of difference was already stated in [8, 9].

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