World Applied Sciences Journal 13 (2): 300-303, 2011
ISSN 1818-4952

The Dominating Polynomial of $LE(Q_3)$ and $GLE(Q_3)$

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Abstract: To determine domination number and dominating polynomial of molecular graph were always important for mathematical-chemistry scientists, one of the most important graph in chemistry is cube graph (Q_3) and the other graphs that are made from on it. In this paper we want to compute the dominating polynomial of LE (Q_3), that is one the derivatives of (Q_3).

Key words: Cube graph · Stellation · DUAL · Map · Domination number · Dominating polynomial
AMS Subject Classifications: 05C31

INTRODUCTION

The domination number and dominating polynomial of molecular graph are used vastly in mathematical chemistry, specially about the derivatives of (Q_3). In order to obtain of this polynomial we have to define some of concepts in graph theory and for notation is not defined here we refer the reader to [6]. A *MAP* that we show it as *M* is a planar and bridgeless graph.

Let *M* be a map with *n* vertex, *m* edge and *f* face, we know that:

$n - m + f = 2$. *(Euler's formula for planar graphs)*

Stellation of *M* or *ST*(*M*) constructed as follow:

A new vertex added inside of any face of *M* and connected it with each boundary vertex of that face. (see Fig.1).

If n_1, m_1, f_1 be the number of vertices, edges and faces of *ST*(*M*) respectively, we can see that:

$$n_1 = n + f,$$
$$m_1 = 3m,$$
$$f_1 = 2m.$$

Also the *Dual* of *M* or *DU*(*M*) is:

To locate a vertex inside of any face of *M* and to join two such vertex if their corresponding faces share a common edge. (see Fig.2).

If n_2, m_2 and f_2 be the number of vertices, edges and faces of *DU*(*M*) respectively, we have:

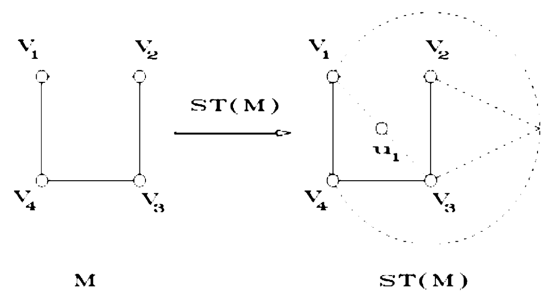

Fig. 1:

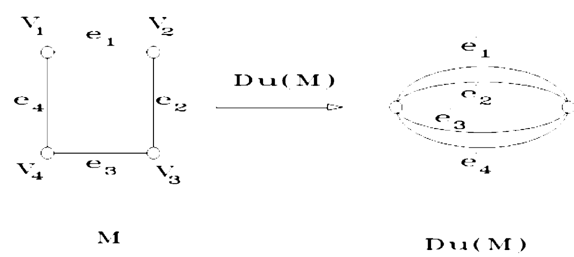

Fig. 2:

$$n_2 = f,$$
$$m_2 = m,$$
$$f_2 = n.$$

Finally the *Leapfrog* of *M* or *LE*(*M*) is defined as follow: (see Fig.3).

$$LE(M) = DU(ST(M)).$$

IF n_3 m_3 and f_3 be the number of vertices, edges and faces of *LE*(*M*) respectively, we can show that:

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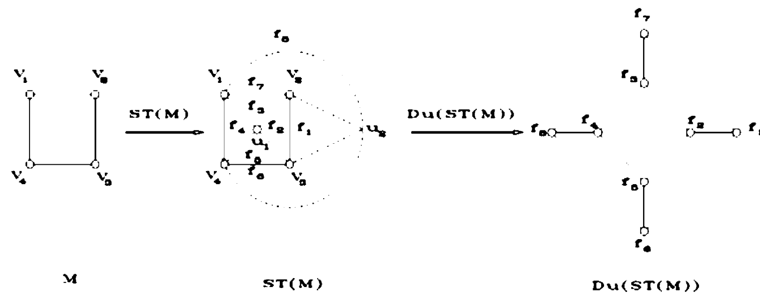

Fig. 3:

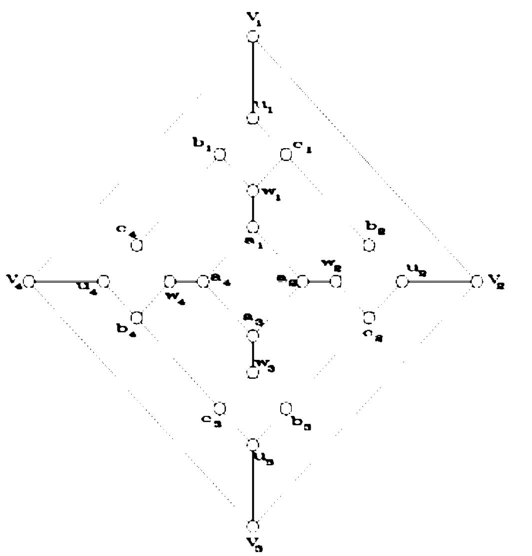

Fig. 4:

$$n_3 = dn$$
$$m_3 = 3m,$$
$$f_3 = f + n.$$

in which d is the degree of vertices of M when M is d-regular.

To Compute Coefficients of Dominating Polynomial $LE(Q_3)$: In this section we may assume that $M = Q_3$ Then the Fig.4 will be $DU(ST(Q_3)) = LE(Q_3)$

Since the vertices $\{a_1,a_2,a_3,a_4\}$ and $\{w_1,w_2,w_3,w_4\}$ are covered by themselves or by the combination of both of them and also the vertices $\{v_1,v_2,v_3,v_4\}$ and $\{u_1,u_2,u_3,u_4\}$ are covered by themselves or by the combination of both of them, therefore for obtaining γ (the domination number) we set:

***A_k* = the Set of *k* Elements of Vertices That Form the Dominating Set:** From above argument and the solving the following system:

$$\gamma = \min \sum_{i=1}^{n} x_i$$
$$\textit{Subject to}: (A + I)X \geq 1,$$
$$x_i \in \{0, 1\}, \quad \textit{where} \quad i = 1,2,...n,$$

in which A *and* I are adjacency and identity matrices respectively. Here $\gamma = 8$. Therefore $|A_i| = 0$ *where* $i = 1,2,...,7$.

In order to compute $|A_8|$ we consider five cases as follow:

- At the first we select four elements from the set $\{a_1,a_2,a_3,a_4\}$ and four elements from the set $\{w_1,w_2,w_3,w_4\}$, in this case all of the vertices covered by these elements and we have:

$$\binom{4}{4}\binom{4}{4} = 1$$

- In the second step we select three elements from the set $\{a_1,a_2,a_3,a_4\}$ for example $a_1, a_2, a_3, \binom{4}{3}$, that caused a_4 covered and for covering the left corner of graph by two elements we have to select w_4, u_4 or b_4, u_4 or c_4, u_4 and or c^4, u_4 that is four cases for left corner and for covering the three other corners we have to select $u_1, u_2, u_3, \binom{3}{3}$, so in this part we have:

$$\binom{4}{3}\binom{3}{3} \times 4 = 16$$

- In this part we select two elements from set $\{a_1,a_2,a_3,a_4\}$ for example $a_1, a_2, \binom{4}{2}$, therefore in order to covering the right and the above corners we have to select u_1 and u_2 also for covering the other parts of the left and the down corners must to select w_4, u_4 or w_4, v_4 or c_4, u_4 and or b_4, u_4 *(Fourcases)*, also for down we have four cases, therefore in this section we have:

$$\binom{4}{2}\times 4\times 4=96$$

- In the fourth step we select one element from set $\{a_1,a_2,a_3,a_4\}$ for example $a_1,\binom{4}{1}$. Therefore a_2 and a_4 covered by a_1 and for covering the other vertices in the above we have to select u_1 and for covering the left and right corners by the same argument in (II), (III) we have 4 × 4 cases and for covering the rest vertices the down corner we have two cases i.e. v_3, u_3 So in this part the number of cases is:

$$\binom{4}{1}\times 4\times 4\times 2=128$$

- Finally, we select four elements from $\{w_1,w_2,w_3,w_4\}\binom{4}{4}$ and for covering the other vertices at the four corners we have to select v_1 or $u_1\binom{2}{1},v_2$ or $u_2\binom{2}{1},v_3$ or $u_3,\binom{2}{1}$ and v_3 or $u_4\binom{2}{1}$ So we have:

$$\binom{4}{4}\times 2\times 2\times 2\times 2=16$$

Since there exist a symmetry between $\{a_1,a_2,a_3,a_4\}$ and $\{v_1,v_2,v_3,v_4\}$, therefore the above argument will be satisfy if we substitute $\{a_1,a_2,a_3,a_4\}$ by $\{v_1,v_2,v_3,v_4\}$, On the other hand selection of 4 elements from $\{a_1,a_2,a_3,a_4\}$ is equivalence to selection of 4 elements from $\{w_1,w_2,w_3,w_4\}$ and is equivalence to the selection 0 element from the set $\{v_1,v_2,v_3,v_4\}$ therefore one case is repeated that must be subtracted from (I) also bellow cases must be subtracted from $|A_8|$, in step (II)$\binom{4}{1}$, in step (III)$\binom{4}{2}$, in step (IV)$\binom{4}{3}$ and finally in step (V)$\binom{4}{4}$. Must be subtracted. Therefore:

$$|A_8| = 2(1 + 16 + 96 + 128 + 16) - 2^4 = 514 - 16 = 498$$

Since every set for covering with cardinality 9 must be contains the 8 above element belong to dominating set, then we must select the one element from 24 – 8 = 16 remind element $\binom{16}{1}$ Hence:

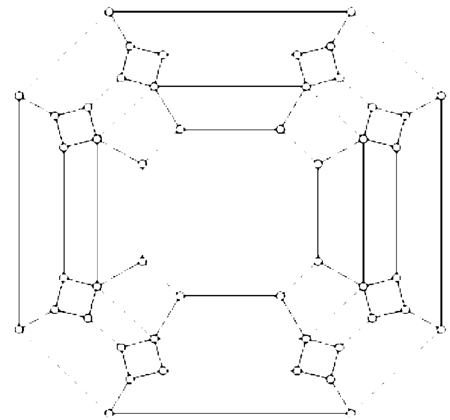

Fig. 5:

$$|A_9| = \binom{16}{1}|A_8| = 16\times 498 = 7968$$

With the same manner we have:

$$|A_i| = \binom{16}{i-8}|A_8| \quad where\ i = 10,11,...,20$$

If we select any 21 vertices of 24 vertices, then all of vertices covered by this 21 elements. So:

$$|A_{21}| = \binom{24}{21}$$

and

$$|A_i| = \binom{24}{i} \quad where\ i = 22,23,24$$

Finally, the dominating polynomial of $LE(Q_3)$ is as follow:

$$D(x)=\sum_{i=8}^{20}\binom{16}{i-8}|A_8|x^i+\sum_{i=21}^{24}\binom{24}{i}x^i \quad in\ which\ |A_8|=498$$

The other graph that we have computed its dominating polynomial is $GLE(Q_3)$ that it is generalize of the molecular graph of $LE(Q_3)$ that shown in Fig.5.

With the same argument in above we have:

$$D(x)=\sum_{i=16}^{48}|A_i|x^i=\sum_{i=16}^{44}\binom{32}{16-i}|A_{16}|x^i+\sum_{i=45}^{48}\binom{48}{i}x^i \quad in which$$

$$|A_{16}| = 147650$$

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