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Nil-Armendariz Rings with Applications to a Monoid

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Abstract: For a monoid M, we introduce nil-M-Armendariz rings, which are generalization of nil-Armendariz rings; and we investigate their properties. This article proves that a ring R is nil-M-Armendariz if and only if for any n, $T_n(R)$ is nil-M-Armendariz. We show that if R is a semicommutative and M-Armendariz ring, then R is nil-M×N-Armendariz ring, where N is a u.p.-monoid.

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INTRODUCTION

Throughout this article R denotes an associative ring with identity, nil(R) denotes the set of all nilpotent elements of R and M denotes a monoid with identity e. Rege and Chhawchharia [8] introduced the notion of an Armendariz ring. They define a ring R to be an Armendariz ring if whenever polynomials

$$f(x)=a_0+a_1x^1+\cdots+a_mx^m$$

$$g(x)=b_0+b_1x^1+\cdots+b_nx^n\in R[x]$$

satisfy $f(x)g(x)=0$, then $a_ib_j=0$ for each i and j. (The converse is always true.) The name Armendariz ring was chosen because Armendariz [3] had noted that a reduced ring satisfies this condition. Some properties of Armendariz rings where given in [1, 3-5, 8]. A monoid M is called a u.p.-monoid (unique product monoid) if for any two non empty finite subsets $A,B\subseteq M$, there exists an element $g\in M$ uniquely presented in the form ab where $a\in A$ and $b\in B$. Liu [6] called a ring R M-Armendariz if whenever elements $\alpha=a_1g_1+\cdots+a_mg_m$, $\beta=b_1h_1+\cdots+b_nh_n\in R[M]$ satisfy $\alpha\beta=0$, then $a_ib_j=0$ for all i,j. Which is a generalization of Armendariz rings. We recall that a ring R is called weak M-Armendariz ring [9] if whenever elements

$$\alpha=a_1g_1+\cdots+a_mg_m,\ \beta=b_1h_1+\cdots+b_nh_n\in R[M]$$

satisfy $\alpha\beta=0$, then $a_ib_j\in nil(R)$ for each i and j. Which is a generalization of weak Armendariz rings. Recall that a ring R is said to be nil-Armendariz [2] if whenever two polynomials $f(x),g(x)\in R[x]$ satisfy $f(x)g(x)\in nil(R)[x]$, then $ab\in nil(R)$ for all $a\in coef(f(x))$ and $b\in coef(g(x))$, coef $(f(x))$ denotes the subsets of R of the coefficients of $f(x)$.

In this article we call a ring R a nil-M-Armendariz (an nil-Armendariz ring relative to M) if whenever elements

$$\alpha=a_1g_1+\cdots+a_mg_m,\ \beta=b_1h_1+\cdots+b_nh_n\in R[M]$$

satisfy $\alpha\beta\in nil(R)[M]$, then $a_ib_j\in nil(R)$ for all i,j.

We prove that M-Armendariz rings are nil-M-Armendariz. If $M=\{N\cup\{0\},+\}$, nil-M-Armendariz rings are nil-Armendariz.

Also in Proposition 2.12 we give a suitable answer to this question that a ring R is nil-M-Armendariz if and only if, for any n, $T_n(R)$ is nil-M-Armendariz.

We investigate nil-M-Armendariz properties, also we have:

$$\text{M} - \text{Armendariz} \Rightarrow \text{nil} - \text{M} - \text{Armendariz},$$

$$\text{Armendariz} \Rightarrow \text{nil} - \text{Armendariz} \Rightarrow \text{weak Armendariz}$$

NIL-M-ARMENDARIZ RING

We will assume that all rings are associative with identity. If R is a ring, nil(R) denotes the set of nilpotent elements in R and M denotes a monoid with identity e.

Before stating Proposition 2.2, we need the following:

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Lemma 2.1: [6, Proposition 1.1] Let M be a u.p.-monoid and R a reduce ring. Then R is M-Armendariz.

Proposition 2.2: Let R be a ring such that $nil(R) \trianglelefteq R$ and M be a u.p-monoid and

$$\alpha = a_1g_1 + \cdots + a_mg_m$$

$$\beta = b_1h_1 + \cdots + b_nh_n \in R[M]$$

Then if $\alpha\beta \in nil(R)[M]$, $a_ib_j \in nil(R)$ for all i,j.

Proof: Observe that $\frac{R}{nil(R)}$ is reduced, since M is a u.p-monoid, hence by Lemma 2.1, we have give $\frac{R}{nil(R)}$ is M-Armendariz. Suppose $\alpha\beta \in nil(R)[M]$. Then if we denote by $\overline{\alpha},\overline{\beta}$ the corresponding elements in $\frac{R}{nil(R)}[M]$, $\overline{\alpha}\overline{\beta} = \overline{0}$. Since $\frac{R}{nil(R)}$ is M-Armendariz, so $\overline{a_i}\overline{b_j} = \overline{0}$ for each i,j. Hence, $a_ib_j \in nil(R)$ for all i,j.

Definition 2.3: [9] A ring R is said to be weak M-Armendariz if whenever elements

$$\alpha = a_1g_1 + \cdots + a_mg_m, \beta = b_1h_1 + \cdots + b_nh_n \in R[M]$$

satisfy $\alpha\beta = 0$, then $a_ib_j \in nil(R)$ for each i and j.

Clearly, M-Armendariz rings are weak M-Armendariz. We now present here a stronger condition, given by the property obtained in Proposition 2.2.

Definition 2.4: A ring R is said to be nil-M-Armendariz if whenever elements

$$\alpha = a_1g_1 + \cdots + a_mg_m, \beta = b_1h_1 + \cdots + b_nh_n \in R[M]$$

satisfy $\alpha\beta \in nil(R)[M]$, then $a_ib_j \in nil(R)$ for each i and j.

We recall a ring R is called semicommutative if for all $a,b \in R$, ab = 0 implies aRb = 0.

Lemma 2.5: [7, Lemma 3.1] Let R be a semicommutative ring. Then nil(R) is an ideal of R. By Proposition 2.2 and Lemma 2.5 we have:

Corollary 2.6: Let M be a u.p.-monoid and R a semicommutative ring. Then R is nil-M-Armendariz ring. Recall that if $I \subseteq nil(R)$, then

$$nil(\frac{R}{I}) = \frac{nil(R)}{I}$$

We observe that if M be a u.p.-monoid, then by Proposition 2.2, R is nil-M-Armendariz. More generally we obtain the following.

Proposition 2.7: Let R be a ring and $I \trianglelefteq R$ a nil ideal and M be a monoid. Then R is nil-M-Armendariz if and only if R/I is nil-M-Armendariz.

Proof: We denote $\overline{R} = \frac{R}{I}$. Since I is nil, then $nil(\overline{R}) = \overline{nil(R)}$. Hence $\alpha\beta \in nil(R)[M]$ if and only if $\overline{\alpha}\overline{\beta} \in nil(\overline{R})[M]$. If $a \in coef(\alpha)$ and $b \in coef(\beta)$, then $\alpha\beta \in nil(R)$ if and only if $\overline{a}\overline{b} \in nil(\overline{R})$.

Therefore R is nil-M-Armendariz if and only if $\overline{R}$ is nil-M-Armendariz.

Before stating Proposition 2.9, we need the following:

Lemma 2.8: Let R be a nil-M-Armendariz ring and $n \geq 2$. If $\alpha_1, \alpha_2, \ldots, \alpha_n \in R[M]$ such that $\alpha_1\alpha_2\cdots\alpha_n \in nil(R)[M]$, then if $a_k \in coef(\alpha_k)$ for k = 1,…,n, we have $a_1a_2\cdots a_n \in nil(R)$.

Proof: We use induction on n. The case n = 2 is clear by definition of nil-M-Armendariz ring.

Suppose n>2. Consider $h = \alpha_1\alpha_2\cdots\alpha_{n-1}$. Then $h\alpha_n \in nil(R)[M]$, since R is nil-M-Armendariz $a_ha_n \in nil(R)$ where $a_h \in coef(h)$ and $a_n \in coef(\alpha_n)$. Therefore, for all $a_n \in coef(\alpha_n)$,

$$\alpha_1\cdots\alpha_{n-2}(\alpha_{n-1}a_n) = h\, a_n \in nil(R)[M]$$

and by induction, since the coefficients of $\alpha_{n-1}a_n$ are $a_{n-1}a_n$ where a_{n-1} is a coefficient of α_{n-1}, we obtain $a_1a_2\cdots a_{n-1}a_n \in nil(R)$ for $a_k \in coef(\alpha_k)$, k = 1,…,n.

Proposition 2.9: If R is nil-M-Armendariz, then $nil(R[M]) \subseteq nil(R)[M]$.

Proof: Suppose $\alpha \in nil(R)[M]$ and $\alpha^m = 0$. By Lemma 2.8, we have that $a_1\cdots a_m \in nil(R)$ where a_i is a coefficient of α for i = 1,…,m. In particular, for every $a \in coef(\alpha)$, a^m is nilpotent. Therefore $\alpha \in nil(R)$ for all $a \in coef(\alpha)$ and hence $\alpha \in nil(R)[M]$.

Before stating Proposition 2.11, we need the following:

Lemma 2.10: Let R be a nil-M-Armendariz ring.

(a) If a,b are nilpotent, then ab is nilpotent.
(b) If a,b,c are nilpotent, then (a+b)c and c(a+b) are nilpotent.
(c) If a,b,c are nilpotent, then a+bc is nilpotent.
(d) If a,b are nilpotent, then a-b is nilpotent.

Proof: (a) Suppose a,b are nilpotent and $b^m = 0$. Then

$$(ae - abg)(1e + bg + b^2g^2 + \cdots + b^{m-1}g^{m-1}) = a\,e \in nil(R)[M]$$

Since R is nil-M-Armendariz, hence $ab \in nil(R)$.

(b) Suppose a,b,c are nilpotent, so $a^n = b^m = 0$ for some positive integer m, n. Then

$$(1e + \cdots + a^{n-1}g^{n-1})(1e - ag)(1e - bg)$$
$$(1e + \cdots + b^{m-1}g^{m-1})ce = ce$$

It we multiply the elements in the middle, we obtain

$$(1e + \cdots + a^{n-1}g^{n-1})(1e - (a+b)g + abg^2)$$
$$(1e + \cdots + b^{m-1}g^{m-1})ce = ce$$

Now, since R is nil-M-Armendariz and $ce \in nil(R)[M]$, by Lemma 2.8, we can choose the appropriate coefficients from each element of R[M] to obtain $(a+b)c \in nil(R)$. Similarly we see that $c(a+b) \in nil(R)$.

(c) Suppose a,b,c are nilpotent. By (a), bc is nilpotent and by (b), b(a+bc) is also nilpotent. Hence

$$(1e - bg)(ce + (a + bc)g) = ce + ag - b(a + bc)g^2 \in nil(R)[M]$$

Now, since R is nil M-Armendariz, so 1.(a+bc) = a+bc is nilpotent.

(d) Suppose a,b are nilpotent. Now by applying (c) several times we can see that, since a^2, a and-b are nilpotent, a^2-ab is nilpotent, hence a^2-ab-ba is nilpotent, so a^2-ab-ba+b^2 is nilpotent. Therefore $(a\text{-}b)^2$ is nilpotent, which means that a-b is nilpotent.

From Lemma 2.10 we get.

Proposition 2.11: If R is a nil-M-Armendariz, then nil(R) is a subring of R.

Therefore by Proposition 2.11 we have, if R is an M-Armaendariz ring, then nil(R) is a subring of R.

Proposition 2.12: Let R be a ring and M a monoid. Then R is nil-M-Armendariz if and only if, for any n, $T_n(R)$ is nil-M-Armendariz.

Proof: We note that any subring of nil-M-Armendariz rings is nil-M-Armendariz. Thus if $T_n(R)$ is a nil-M-Armendariz ring, then R is a nil-M-Armendariz. Conversely, let

$$\alpha = A_1g_1 + A_2g_2 + \cdots + A_pg_p$$

and

$$\beta = B_1h_1 + B_2h_2 + \cdots + B_qh_q$$

be elements of $T_n(R)[M]$. Assume that $\alpha\beta \in nil\,T_n(R)[M]$. It is easy to see that there exists an isomorphism of rings $T_n(R)[M] \rightarrow T_n(R[M])$ defined by

$$\sum_{i=1}^{p}\begin{pmatrix} a_{11}^i & a_{12}^i & a_{13}^i & \cdots & a_{1n}^i \\ 0 & a_{22}^i & a_{23}^i & \cdots & a_{2n}^i \\ 0 & 0 & a_{33}^i & \cdots & a_{3n}^i \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn}^i \end{pmatrix} g_i$$

$$\rightarrow \begin{pmatrix} \sum_{i=1}^{p}a_{11}^i g_i & \sum_{i=1}^{p}a_{12}^i g_i & \sum_{i=1}^{p}a_{13}^i g_i & \cdots & \sum_{i=1}^{p}a_{1n}^i g_i \\ 0 & \sum_{i=1}^{p}a_{22}^i g_i & \sum_{i=1}^{p}a_{23}^i g_i & \cdots & \sum_{i=1}^{p}a_{2n}^i g_i \\ 0 & 0 & \sum_{i=1}^{p}a_{33}^i g_i & \cdots & \sum_{i=1}^{p}a_{3n}^i g_i \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \sum_{i=1}^{p}a_{nn}^i g_i \end{pmatrix}.$$

Assume that

$$A_i = \begin{pmatrix} a_{11}^i & a_{12}^i & a_{13}^i & \cdots & a_{1n}^i \\ 0 & a_{22}^i & a_{23}^i & \cdots & a_{2n}^i \\ 0 & 0 & a_{33}^i & \cdots & a_{3n}^i \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn}^i \end{pmatrix}$$

and

$$B_j = \begin{pmatrix} b_{11}^j & b_{12}^j & b_{13}^j & \cdots & b_{1n}^j \\ 0 & b_{22}^j & b_{23}^j & \cdots & b_{2n}^j \\ 0 & 0 & b_{33}^j & \cdots & b_{3n}^j \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & b_{nn}^j \end{pmatrix}$$

Then we have

$$\begin{pmatrix} \sum_{i=1}^{p} a_{11}^{i} g_i & \sum_{i=1}^{p} a_{12}^{i} g_i & \sum_{i=1}^{p} a_{13}^{i} g_i & \cdots & \sum_{i=1}^{p} a_{1n}^{i} g_i \\ 0 & \sum_{i=1}^{p} a_{22}^{i} g_i & \sum_{i=1}^{p} a_{23}^{i} g_i & \cdots & \sum_{i=1}^{p} a_{2n}^{i} g_i \\ 0 & 0 & \sum_{i=1}^{p} a_{33}^{i} g_i & \cdots & \sum_{i=1}^{p} a_{3n}^{i} g_i \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \sum_{i=1}^{p} a_{nn}^{i} g_i \end{pmatrix} \begin{pmatrix} \sum_{j=1}^{q} b_{11}^{j} h_j & \sum_{j=1}^{q} b_{12}^{j} h_j & \sum_{j=1}^{q} b_{13}^{j} h_j & \cdots & \sum_{j=1}^{q} b_{1n}^{j} h_j \\ 0 & \sum_{j=1}^{q} b_{22}^{j} h_j & \sum_{j=1}^{q} b_{23}^{j} h_j & \cdots & \sum_{j=1}^{q} b_{2n}^{j} h_j \\ 0 & 0 & \sum_{j=1}^{q} b_{33}^{j} h_j & \cdots & \sum_{j=1}^{q} b_{3n}^{j} h_j \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \sum_{j=1}^{q} b_{nn}^{j} h_j \end{pmatrix} \in \mathrm{nil}\, T_n(R[M])$$

Because $T_n(R)[M] \cong T_n(R[M])$ and

$$\mathrm{nil}(T_n(R)) = \begin{pmatrix} \mathrm{nil}(R) & R & R \\ 0 & \ddots & R \\ 0 & 0 & \mathrm{nil}(R) \end{pmatrix}$$

then we have

$$\left(\sum_{i=1}^{p} a_{ss}^{i} g_i\right)\left(\sum_{j=1}^{q} b_{ss}^{j} h_j\right) \in \mathrm{nil}(R)[M],$$

for $s = 1,2,\ldots,n$

Since R is nil-M-Armendariz, there exists $m_{ijs} \in N$ such that $(a_{ss}^{i} b_{ss}^{j})^{m_{ijs}} = 0$ for any s,i and j. Let $m_{ij} = \max\{m_{ij1}, m_{ij2}, \ldots, m_{ijs}\}$, then

$$(A_i B_j)^{m_{ij}} = \begin{pmatrix} a_{11}^{i} b_{11}^{j} & * & * & \cdots & * \\ 0 & a_{22}^{i} b_{22}^{j} & * & \cdots & * \\ 0 & 0 & a_{33}^{i} b_{33}^{j} & \cdots & * \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & a_{nn}^{i} b_{nn}^{j} \end{pmatrix}^{m_{ij}} = \begin{pmatrix} 0 & * & * & \cdots & * \\ 0 & 0 & * & \cdots & * \\ 0 & 0 & 0 & \cdots & * \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Thus $((A_i B_j)^{m_{ij}})^n = 0$. This shows that $T_n(R)$ is a nil-M-Armendariz ring.

Corollary 2.13: Let M be a monoid. If a ring R is a M-Armendariz ring, then for any n, $T_n(R)$ is a nil-M-Armendariz ring.

Given a ring R and a bimodule ${}_R M_R$, the trivial extension of R by M is the ring $T(R,M) = R \oplus M$ with the usual addition and the multiplication:

$$(r_1, m_1).(r_2, m_2) = (r_1 r_2, r_1 m_2 + m_1 r_2)$$

This is isomorphic to the ring of all matrices $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$ and $m \in M$ and the usual matrix operations are used.

Proposition 2.14: Let M be a monoid. Then R is nil-M-Armendariz if and only if the trivial extension T(R,R) is nil-M-Armendariz ring.

Proof: It follows from Proposition 2.12.

Proposition 2.15: Let M be a cancellative monoid and N be an ideal of M. If a ring R is a nil-N-Armendariz, then R is a nil-M-Armendariz.

Proof: Let

$$\alpha = a_1 g_1 + \cdots + a_m g_m$$

$$\beta = b_1 h_1 + \cdots + b_n h_n$$

in R[M] with $\alpha\beta \in \mathrm{nil}(R)[M]$. Set $g \in N$, then $gg_1, gg_2, \ldots, gg_m, h_1 g, h_2 g, \ldots, h_n g \in N$ and $gg_i \neq gg_j$ and

$h_i g \neq h_j g$ when $i \neq j$. Now from $(\sum_{i=1}^{m} a_i g_i g)(\sum_{j=1}^{n} b_j h_j g) \in nil(R)[N]$ and the hypothesis that R is nil-N-Armendariz, it follows that $a_i b_j \in nil(R)$ for all i and j. Thus R is nil-M-Armendariz.

Let M be a monoid. If R is semicommutative ring and M-Armendariz ring, then R[M] is semicommutative. Hence we have:

Proposition 2.16: Let M be a monoid and N a u.p.-monoid. If R is a semicommutative and M-Armendariz ring, then R[M] is nil-N-Armendariz ring.

Proof: Since R is a semicommutative and M-Armendariz ring, R[M] is semicommutative, the assertion holds according to corollary 2.6.

Lemma 2.17: Let R be a semicommutative ring and M a monoid. If $a_1 \cdots a_n \in nil(R)$, then

$$a_1 g_1 + \cdots a_n g_n \in nil(R[M])$$

Proof: The proof is similar to that of [7, Lemma 3.7].

Proposition 2.18: Let M be a monoid and N a u.p.-monoid. If R is a semicommutative and M-Armendariz ring, then R[N] is nil-M-Armendariz ring.

Proof: It is easy to see that there exists an isomorphism of rings $R[N][M] \rightarrow R[M][N]$ defined by

$$\sum_p (\sum_i a_{ip} n_i) m_p \rightarrow \sum_i (\sum_p a_{ip} m_p) n_i$$

Now suppose that $\alpha_i, \beta_j \in R[N]$ are such that

$$(\sum_i \alpha_i m_i)(\sum_j \beta_j m'_j) \in nil(R[N])[M]$$

We will show that $\alpha_i \beta_j \in nil(R[N])$ for all i,j, assume that

$$\alpha_i = \sum_p a_{ip} n_p$$

and

$$\beta_j = \sum_q b_{jq} n'_q$$

where $n_p, n'_q \in N$ for all p,q. Then

$$(\sum_i (\sum_p a_{ip} n_p) m_i)(\sum_j (\sum_q b_{jq} n'_q) m'_j) \in nil(R[N])[M]$$

Thus, in R[M][N] we have

$$(\sum_p (\sum_i a_{ip} m_i) n_p)(\sum_q (\sum_j b_{jq} m'_j) n'_q) \in nil(R[M])[N]$$

by Proposition 2.16, R[M] is nil-N-Armendariz,

$$(\sum_i a_{ip} m_i)(\sum_j b_{jq} m'_j) \in nil(R[M])$$

for all p,q. Since R is M-Armendariz, $a_i b_j \in nil(R)$ for all i,j,p,q according to [6, Proposition 1.6]. Hence $\alpha_i \beta_j \in nil(R[N])$, by Lemma 2.17. This means that R[M] is nil-M-Armendariz.

Corollary 2.19: Let M be a monoid and R be a semicommutative ring. If R is M-Armendariz ring, then R[x] and $R[x,x^{-1}]$ are nil-M-Armendariz ring.

Proof: Note that $R[x] \cong R[N \cup \{0\}]$ and $R[x,x^{-1}] \cong R[Z]$.

In [6], Liu showed that if R is reduced and M-Armendariz, then R is M×N-Armendariz, where N is a u.p.-monoid. For nil-M-Armendariz rings, we have following result.

Proposition 2.20: Let M be a monoid and N be a u.p.-monoid. If R is a semicommutative and M-Armendariz ring, then R is nil-M×N-Armendariz ring.

Proof: Suppose that $\sum_{i=1}^{s} a_i (m_i, n_i)$ is in R[M×N]. Without loss of generality, we assume that

$$\{n_1, n_2, \ldots, n_s\} = \{n_1, n_2, \ldots, n_t\}$$

with $n_i \neq n_j$ when $1 \leq i \neq j \leq t$. For any $1 \leq p \leq t$, denote

$$A_p = \{i \mid 1 \leq i \leq s, n_i = n_p\}$$

Then

$$\sum_{p=1}^{t} (\sum_{i \in A_p} a_i m_i) n_p \in R[M][N]$$

Note that $m_i \neq m_{i'}$ for any $i, i' \in A_p$ with $i \neq i'$. Now it is easy to see that there exists an isomorphism of rings $R[M \times N] \rightarrow R[M][N]$ defined by

$$\sum_{i=1}^{s} a_i (m_i, n_i) \rightarrow \sum_{p=1}^{t} (\sum_{i \in A_p} a_i m_i) n_p$$

Suppose that

$$(\sum_{i=1}^{s} a_i(m_i, n_i))(\sum_{j=1}^{s'} b_j(m'_j, n'_j)) \in nil(R)[M \times N]$$

in R[M×N]. Then from the above isomorphism, it follows that

$$(\sum_{p=1}^{t}(\sum_{i\in A_p} a_i m_i) n_p)(\sum_{q=1}^{t'}(\sum_{j\in B_q} b_j m'_j) n'_q) \in nilR[M][N]$$

By Proposition 2.16, R[M] is nil-N-Armendariz, thus we have

$$(\sum_{i\in A_p} a_i m_i)(\sum_{j\in B_q} b_j m'_j) \in nil(R[M])$$

for all p,q. Since R is M-Armendariz, $a_i b_j \in nil(R)$ for any $i\in A_p$ and $j\in B_q$ by [6, Proposition 1.6]. Hence, $a_i b_j \in nil(R)$ for all $1\le i\le s$ and $1\le i\le s'$.

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REFERENCES

1. Anderson, D.D. and V. Camillo, 1998. Armendariz rings and Gaussian rings. Comm. Algebra, 26 (7): 2265-2272.
2. Antoine, R., 2008. Nilpotent elements and Armendariz rings. Journal of Algebra, 319: 3128-3140.
3. Armendariz, E.P., 1974. A note on extension of Bear and PP-rings. Austral. Math Soc., 18 (2): 470-473.
4. Huh, C., Y. Lee and A. Smoktunowicz, 2002. Armendariz rings and semicommutative rings. Comm. Algebra, 30 (2): 751-761.
5. Kin, N.K. and Y. Lee, 2000. Armen dariz rings and reduced rings. J. Algebra, 223 (2): 477-488.
6. Liu, Z., 2005. Armendariz rings relative to a monoid. Comm. Algebra, 33 (3): 649-661.
7. Liu, Z. and Z. Renyu, 2006. On Weak Armendariz rings. Comm. Algebra, 34: 2607-2616.
8. Rege, M.B. and S. Chhawchhria, 1997. Armendariz rings. Proc. Jpn. Acad. Ser. A Math. Sci., 73 (1): 14-17.
9. Zhang, C. and J. Chen, 2009. Weak M-Armendariz rings. Southeast University, 25: 142-146.