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Global function in the Generalized Topological Space

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Abstract: In this paper we introduce the concept of Global function ${}^*A$, Globally closure operator ${}^*c$ in ideal generalized topological space. Along with that the generalized topologies ${}^*\mu$ generated by ${}^*c$ will be studied. Some characterizations are given in comparison with local function in ideal generalized topological space introduced by Á. Császár [1] and closure function introduced by Kuratowski [3].

Key words: Generalized topology . global function (${}^*A$) . globally closure operator ${}^*c$. global generalized topology ${}^*\mu$

INTRODUCTION

The subject of ideals was introduced by Kuratowski [3] and since then many mathematicians investigated this conception. Á. Császár [1] introduced the notion of ideals to generate the concept of ideal generalized topological space. Our aim is to introduce a new notion and supplement the properties which were derived from the local function in an ideal generalized topological space. We consider a non-empty set X and $\exp X$ as its power set. The interior and closure are represented by i and c. Then $\mu\subset\exp X$ the collection is said to form generalized topology [1] iff $\varphi\in\mu$ and arbitrary union of elements of μ belongs to μ. In this paper we introduced the concept of Global function ${}^*A$ and Global closure operator ${}^*c$. We will further investigate the Global closure operator in comparison with the Kuratowski's closure operator and generate the new Generalized topology ${}^*\mu$.

PRELIMINARIES

The local function in ideal generalized topology μ was introduced by Á. Császár [1] in 2005.

Definition 1.1: [1] Let $A\subseteq X$, then A^* is defined by $x\in A^*$ if and only if $x\in M\in\mu$ implies that $M\cap A\notin I$. If

$$M_\mu = \cup\{M : M\in\mu\}$$

and $x\notin M_\mu$ then by definition $x\in A^*$. If there is no ambiguity then we write A^* in place of ${}^*A(I,\mu)$. And $x\notin A^*$ implies that $M\cap A\in I$.

Definition 1.2: Let X be a non-empty set and exp(X) be the collection of subsets of X. Then we define a function d: exp (X)$\to$exp(X) which satisfies the following condition [3]

(1) $d(\varphi)=\varphi^*=\varphi$
(2) $d(A\cup B)=d(A)\cup d(B)$
(3) $d(d(A))\subseteq d(A)$

Definition 1.3: The ideals are defined as non-empty collection I of subsets of X which satisy the following two conditions [3]:

(i) $A\in I$ and $B\subseteq A\Rightarrow B\in I$
(ii) $A\in I$ and $B\in I\Rightarrow A\cup B\in I$

GLOBAL FUNCTION

Definition 2.1: Let (X,μ,I) be an ideal generalized topological space and $A\subseteq X$. Then Global function of A is defined as:

$${}^*A(I,\mu)=\begin{cases} x\in X: A\cup M\notin I, \text{for every } M\in\mu(X,x) \\ \text{and if } x\notin M \text{ and } M\in\mu, \text{then } x\in{}^*A; \text{ if } A\neq\varphi \\ \varphi; \text{ if } A=\varphi \end{cases}$$

and if there is no ambiguity then we write ${}^*A$ in place of ${}^*A(I,\mu)$. And $x\notin A^*$ implies that $M\cup A\in I$.

Remarks 2.2

(i) If $I=\varphi$ then ${}^*A(I,\mu)=X$.

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(ii) If $A^*(I,\mu)$ is the local function for $A\subset X$ and ${}^*A(I,\mu)$ is its global function then $A^*\subset {}^*A$.
(iii) Neither $A\subset {}^*A$ nor $A^*\subset A$.

Proofs: The proof of (i) is quite obvious.

(ii) Let

$$x \notin^* A \Rightarrow A \cup M \in I, \text{for each M containing 'x'}$$
$$\Rightarrow A \cap M \in I \Rightarrow x \notin A^*$$
$$\Rightarrow A^* \subset^* A.$$

(iii) Quite obvious.

Example 2.3: Let us consider $X\neq\varphi$,

$$X = \{a,b,c\}$$
$$\mu = \{\varphi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$$
$$I = \{\varphi,\{a\},\{c\},\{a,c\}\}$$

if $A = \{b,c\}$ then ${}^*A = \{a,b,c\}$ and $A^* = \{b\}$. Therefore $A^*\subset {}^*A$

Definition 2.4: Let the ideal I_m be the collection of subsets of X, for which if $A\in I_m$ implies that $A^c\in I_{rm}$.

Theorem 2.5: If $M\in I_{rm}$ then ${}^*(A^c) = ({}^*A)^c$.

Proof: Let

Let $x \notin^* (A^c) \Rightarrow M \cup A^c \in I_{m} \Rightarrow M \in I_{rm}$ and $A^c \in I_m$

$$\Rightarrow M\in I_{m} \text{ and } A\notin I_{m} \Rightarrow M\cup A\notin I_{m}$$
$$\Rightarrow x \in^* A \Rightarrow x \notin ({}^*A)^c$$
$${}^*(A^c) \subseteq ({}^*A)^c \qquad \text{(i)}$$

Converse,

$$\text{Let } x \notin ({}^*A)^c \Rightarrow x \in^* A \Rightarrow M \cup A \in I_m$$

Given in the theorem that $M\in I_{rm}$.
Then

$$A^c \in I_m \Rightarrow M \cup A^c \in I_m \Rightarrow x \notin {}^*(A^c)$$
$$({}^*A)^c \subseteq^* (A^c) \qquad \text{(ii)}$$

Hence from equations (i) and (ii), we get

$${}^*(A^c) = ({}^*A)^c$$

Corollary 2.6: If it is not given that $M\in I$. And either $M\in I$ or $M\notin I$ then in that case ${}^*(A^c) = ({}^*A)^c$.

Theorem 2.7: Let (X,μ) be a generalized topological space with ϑ and I ideals on X and let A and B be subsets of X. Then

(a) $A\subset B \Rightarrow {}^*A \subset {}^*B$
(b) $\vartheta \subseteq I \Rightarrow {}^*A(\vartheta,\mu) \subset {}^*B(I,\mu)$
(c) ${}^*(A\cap B) = {}^*A \cap {}^*B$
(d) ${}^*(A\cup B) = {}^*A \cup {}^*B$
(e) ${}^*({}^*A) \subseteq {}^*A$
(f) $I\in\vartheta \Rightarrow {}^*I \neq \varphi$
(g) ${}^*(A-B) \subseteq {}^*A - {}^*B$

Proof

(a) Let $x\notin {}^*B$, then there exists $M\in\mu$ such that $M\cup B\in I$. Since $A\subseteq B$ and I is ideal on X, therefore $M\cup A\in I$ which implies that $x\notin {}^*A$. Then this proves that ${}^*A\subseteq {}^*B$.
(b) Let $x\notin {}^*A(\vartheta,\mu)$, then there exists $M\in\mu$ such that $M\cup A\in\vartheta$ As $\vartheta\subseteq I$ so, $M\cup A\in\vartheta \Rightarrow M\cup A\in I$ implies that ${}^*A(I,\mu)$.
(c) From (a) we can prove that

$${}^*(A\cap B) \subseteq^* A \cap^* B \qquad \text{(i)}$$

Or we can calculate this in simple computational way,

$$x \notin^* A\cap^* B \Rightarrow x \notin^* A \text{ and } x \notin^* B$$
$$\Rightarrow M\cup A\in\vartheta \text{ and } M\cup B\in\vartheta$$
$$\Rightarrow M\cup(A \text{ and } B)\in\vartheta \Rightarrow x\notin^* (A\cap B)$$
$$\Rightarrow^* (A\cap B) \subseteq^* A \cap^* B \qquad \text{(i)}$$

Now consider that

$$x \notin^* (A\cap B) \Rightarrow M\cup(A\cap B)\in\vartheta$$
$$\Rightarrow (M\cup A)\cap(M\cup B)\in\vartheta$$
$$\Rightarrow M\cup A\in\vartheta \text{ and } M\cup B\in\vartheta$$
$$\Rightarrow x\notin^* A \text{ and } x\notin^* B \Rightarrow x\notin^* A\cap^* B$$
$$\Rightarrow^* (A\cap B) \supseteq^* A\cap^* B \qquad \text{(ii)}$$

From (i) and (ii), we have

$${}^*(A\cap B) = {}^*A\cap {}^*B$$

(d), By (i), we have ${}^*A\cup {}^*B\subseteq {}^*(A\cup B)$. Let $x\notin {}^*(A\cup B)$, then $M\cup(A\cup B)\in I \Rightarrow M\in I$ and $(A\cup B)\in I$ whenever $x\in M\in\mu$. As $A\subseteq A\cup B$ or $B\subseteq A\cup B$, this implies that $(M\cup A)\notin I$ or $(M\cup A)\notin I$. This implies $x\in {}^*A$ or $x\in {}^*B$, that is $x\in {}^*A\cup {}^*B$. Therefore, we have ${}^*(A\cup B)\subseteq {}^*A\cup {}^*B$. Consequently. We have ${}^*(A\cup B) = {}^*A\cup {}^*B$. (e)

Let

$$x\notin^* ({}^*A)$$
$$\Rightarrow M\cup(M\cup A)\in I, \text{ for each } M\in\mu \text{ containing '}x\text{'}.$$
$$\Rightarrow (M\cup M)\cup A\in I \Rightarrow M\cup A\in I \Rightarrow x\notin^* A$$

Therefore this implies that

$$\Rightarrow^* ({}^*A) \subseteq^* A$$

(f) It is quite obvious.
(g) Let

$x \notin {}^*(A - B) \Rightarrow M \cup (A - B) \in I$
$\Rightarrow (M \cup A) \cap (M \cup B^c) \in I$
$\Rightarrow^* (A) \cap^* (B^c) \in I$. As $M \in I$ then
${}^*(B^c) \subseteq ({}^*B)^c$ by Corrollary 2.3,
$\Rightarrow^* (A) \cap ({}^*B)^c \in I \Rightarrow^* A - {}^*B \in I \Rightarrow x \notin^* A -^* B$
$\Rightarrow {}^*(A - B) \subseteq^* A -^* B$

Converse of it can be given in the same way, therefore this implies that ${}^*(A\text{-}B) \supseteq {}^*A\text{-}{}^*B$.
Therefore finally we can say that ${}^*(A\text{-}B) = {}^*A\text{-}{}^*B$.

Theorem 2.8: Let (X,μ) be an ideal generalized topological space and let I and ϑ be the two ideals on *X*. For a subset *A* of *X*, the following statements hold:

(a) ${}^*A(I \cap \vartheta) = {}^*A(I) \cup {}^*A(\vartheta)$
(b) ${}^*A(I \vee \vartheta) = {}^*A(I) \cap {}^*A(\vartheta)$

Proof:
(a) The inclusion ${}^*A(I) \cup {}^*A(\vartheta) \subset {}^*A(I \cap \vartheta)$ follows directly. Let $x \notin {}^*A(I \cap \vartheta)$, then $A \cup M_{(x)} \in I \cap \vartheta$ for every μ-open set $M_{(x)} \in \mu$ containing *x*. Hence $A \cup M_{(x)} \in I$ and $A \cup M_{(x)} \in \vartheta$. This implies that ${}^*A(I) \cup {}^*A(\vartheta) \supset {}^*A(I \cap \vartheta)$ and so ${}^*A(I) \cup {}^*A(\vartheta) = {}^*A(I \cap \vartheta)$.
(b) The proof is quite similar.

GLOBAL OPERATOR

Definition 2.9: Let d: $P(X) \rightarrow P(X)$, whereas P(X) is power function of a non empty set X. And function *d* is defined as $d(A) = {}^*A$,

(i) $d(\varphi) = \varphi$
(ii) $d(A \cap B) =^* (A \cap B) =^* A \cap^* B = d(A) \cap d(B)$
(iii) $d(A \cup B) =^* (A \cup B) =^* A \cup^* B = d(A) \cup d(B)$
(iv) $d(d(A)) \subset d(A)$

So this operator satisfies the four axioms in ideal generalized topology. Hence it goes a step ahead of Kuratowski's closure operator. By using the above operator we can define globally interior operator and globally closure operator. And the in detail study is given in the following discussion.

GLOBALLY CLOSURE OPERATORS

Definition.3.1: The Globally interior operator is given as ${}^*i(A) = A \cap {}^*A$. And the globally closure operator is given by the formula ${}^*c(A) = A \cup {}^*A$.

We five forward the class of generalized topology called global generalized topology,

$${}^*\mu(I) = \{U \in P(X): {}^* c(X\text{-}U) = X\text{-}U\}$$

is generalized topology *c.

Preposition 3.2: Globally closure operator is distributive over the union of two ${}^*\mu$-open sets.

Proof
${}^*c(A \cup B) = (A \cup B) \cup^* (A \cup B)$
$= (A \cup B) \cup ({}^*A \cup {}^*B)$
$= (A \cup^* A) \cup (B \cup {}^*B) \Rightarrow {}^*c(A) \cup {}^*c(B)$
${}^*c(A \cup B) =^* c(A) \cup^* c(B)$.

Remark 3.3: It can be shown that

$${}^*c(A \cap B) \neq (A \cap B) \cup^* (A \cap B) .$$

Preposition 3.4: Show that $X = {}^*X$.

Proof: Let $x \in X^*$ then $M \in \mu$ and $M \neq \varphi$ would imply that $X = M \cup X \notin I$ so that it imply that $X = {}^*X$.

Preposition 3.5: If μ is a generalized topology then $M \in \mu$ implies $M \cup^* A \subset^* (M \cup A)$ for $A \subset X$.

Proof: Let $x \notin {}^*(M \cup A)$ implies the existence of $x \in M_1 \in \mu$ such that

$$M_1 \cup (M \cup A) \in I \Rightarrow x \in M_1 \cup (M \cup A) \Rightarrow x \notin M \cup^* A$$

Therefore $M \cup^* A \subset^* (M \cup A)$.

Lemma 3.6: $A \subset X$ implies that ${}^*(A \cup A) \subset {}^*A$.

Proof: $x \notin A^*$ implies the existence of $M \in \mu$ such that $x \in M$ and $M \cup A \in I$. Such that this implies that $M \cup (A \cup {}^*A) \in I$ implies that $x \notin {}^*(A \cup {}^*A)$. Therefore, ${}^*(A \cup {}^*A) \subset {}^*A$.

Remark 3.7: Let

$${}^*(\ ): \exp X \rightarrow \exp X \text{ then } {}^*cA = A \cup^* A$$

defines an non-idempotent map.

Preposition 3.8: Prove that globally closure operator contains Kuratowski's closure operator

Proof: Let X is non empty set and $A \subseteq X$ then the globally closure operator is given as,

$$c^*(A) = A \cup A^* \subseteq A \cup^* A = {}^*c(A)$$
$$c^*(A) \subseteq^* c(A)$$

Therefore, globally closure operator is μ-closed.

Preposition 3.9: For $A \subseteq B \Rightarrow^* c(A) \subseteq {}^*c(B)$

Proof: By theorem (2.3) (a)

$$A \subseteq B \Rightarrow {}^*A \subseteq {}^*B \Rightarrow A \cup^* A \subseteq B \cup^* B \, {}^*c(A) \subseteq^* c(B)$$

Remark 3.10: Let X is a non empty set $X = {}^*c(X)$.

Preposition 3.11: F is $^*\mu$-closed if $F \subset {}^*F$.

Proof: By [1] F is $^*\mu$-closed iff $F = c^*F \subset {}^*F$ iff $F = F \cup {}^*F$ iff $F \subset {}^*F$.

REFERENCES

1. Császár, Á., Modification of generalized topologies via Hereditary classes, Acta Math. Hungar, 115(1__2)(227): 29-36.
2. Jankovic, D. and T.R. Hamlet, 1990. New topologies from old via ideals. Am. Math Mon, 97 (4): 295-310.
3. Kurtowski, K., 1933. Topology I, Warszawa.