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Numerical Solution of Hirota-Satsuma Couple Kdv and a Coupled MKdv Equation by Means of Homotopy Analysis Method

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Abstract: In this paper, the numerical solution of systems of Hirota-Satsuma coupled Kdv and a coupled MKdv Equation by means of Homotopy Analysis Method (HAM) are presented. The HAM can extremely minimize the volume of computations with respect to traditional techniques and yields the analytical solution of the desired problem in the form of a rapidly convergent series with easily computable components. To illustrate the ability and flexibility of the method some examples are provided. A comparison was also made between HAM and Adomian Decomposition Method (ADM). The results reveal that the method is very effective and simple.

Key words: Hirota-Satsuma coupled Kdv equations . coupled MKdv equation . series solution . homotopy analysis method

INTRODUCTION

Several methods have been suggested to solve partial differential equations. These methods include the Homotopy Perturbation Method (HPM) [1-3], ADM [4-7] and Variational Iteration Method (VIM) [7-10]. The HAM [10-22] is one of a general analytic approach to get series solutions of various types of nonlinear equations ordinary differential equations, partial differential equations, differential-integral equations, differential-difference equation and coupled equations. In 1992, Liao [15] proposed a new analytical technique; namely, the Homotopy Analysis Method based on homotopy of topology. However, in Liao's PhD dissertation [13], the method, which is a coupling of the traditional perturbation method and homotopy in topology, this method has been successfully employed to solve many types of problems in science and engineering [23]. HAM contains an auxiliary parameter h which provides a simple way to adjust and control the convergent region and the rate of convergence of the series solution and deforms continuously to a simple problem. The basic motivation of this work is using the HAM for the coupled Kdv and Coupled Mkvd equation [24]. These solutions may well describe various phenomena in nature, such as vibrations, solitons and propagation with a finite speed [7]. Analytical solutions can also be obtained by different methods such as ADM [4-7] and VIM [7, 8].

This paper investigates for the first time the applicability and effectiveness of HAM on Hirota-Satsuma couple Kdv and A coupled MKdv equation. In this case, the auxiliary parameter h was obtained by plot h-curve for the complex general series of solutions. The result shows that the ADM solution [23] is a special case of the HAM solution.

THE MODEL WITH COUPLED SYSTEM

We consider the generalized Hirota-Satsuma Coupled KdV system as follows [23]:

$$u_t = \frac{1}{2}u_{xxx} - 3uu_x + 3vw_x + 3v_x w_x$$

$$v_t = -v_{xxx} + 3uv_x \tag{2.1}$$

$$w_t = -w_{xxx} + 3uw_x$$

In addition, a new-coupled MKdv equation is given as:

$$u_t = \frac{1}{2}u_{xxx} - 3u^2u_x + \frac{3}{2}v_{xx} + 3uv_x + 3u_x v - 3\lambda u_x$$

$$v_t = -v_{xxx} - 3vv_x - 3u_x v_x + 3u^2 v_x - 3\lambda v_x \tag{2.2}$$

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BASIC IDEA OF HAM

In HAM, the system can be written by

$$N_i[u_i(x,t)] = 0, i = 1,2,3 \qquad (3.1)$$

where N_i is nonlinear operator, $u_i(x,t)$ is unknown function, x and t are the independent variables, $u_{i,0}(x,t)$ is the initial condition, $h \neq 0$ is an auxiliary parameter and L_i is an auxiliary linear operator. The parameter $q \in [0,1]$ is also the embedding parameter.
Let us construct a homotopy

$$(1-q)L[\phi_i(x,t;q) - u_{i,0}(x,t)] = q\, h_i N_i[\phi_i(x,t;q)] \qquad (3.2)$$

so-called zero-order deformation equation.
When q = 0, the zero-order deformation equation become

$$\phi_i(x,t;0) = u_{i,0}(x,t) \qquad (3.3)$$

and when q = 1, since $h_i \neq 0$ the zero-order deformation of equation (3.2) is

$$\phi_i(x,t;1) = u_i(x,t) \qquad (3.4)$$

where the embedding parameter q increases from 0 to 1.

Using Taylor's theorem, $\phi_i(x,t;1)$ can be expanded in a power series of q as follows:

$$\phi_i(x,t;q) = u_{i,0}(x,t) + \sum_{m=1}^{\infty} u_{i,m}(x,t) q^m \qquad (3.5)$$

where

$$u_{i,m}(x,t) = \frac{1}{m!} \frac{\partial^m \phi_i(x,t;q)}{\partial q^m}\bigg|_{q=0} \qquad (3.6)$$

If the initial condition guesses $u_{i,0}(x,t)$, the auxiliary linear operator L_i, the non-Zero auxiliary parameter $h \neq 0$ then the power series in equation (5) is converges at q = 1.
Therefore, we obtain:

$$\phi_i(x,t;q) = u_{i,0}(x,t) + \sum_{m=1}^{\infty} u_{i,m}(x,t) \qquad (3.7)$$

According to the definition of equation (3.6), the governing equation of $u_i(x,t)$ can be derived from the zero-order deformation of equation (2). Using m times differentiating with respective to q from the zero-order deformation equation (2) and setting q=1, we have the so-called m-th-order deformation equation as:

$$L[u_{i,m}(x,t) - \chi_m u_{i,0}(x,t)] = q\hbar_i R_{i,m}\left(\tilde{u}_{i,m-1}(x,t)\right) \qquad (3.8)$$

where

$$R_{i,m}(u_{i,m-1}(x,t)) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi_i(x,t;q)]}{\partial q^{m-1}}\bigg|_{q=0} \qquad (3.9)$$

and

$$\chi_m = \begin{cases} 1, m > 1 \\ 0, m \leq 1 \end{cases} \qquad (3.10)$$

APPLICATIONS

In this Section, the application of HAM for solving coupled Kdv and Coupled MKdv are considered. The HAM provides an analytical solution in terms of an infinite power series. To show the efficiency of the present method for our problems, the obtained results are compared with the ADM solutions.

Homotopy analysis method for coupled Kdv: Let us consider the given system (2.1). We start the application of the HAM for solving this system using the initial conditions. Suppose that:

$$u(x,0) = \frac{1}{3}(\beta - 8k^2) + 4k^2 \tanh^2 kx$$

$$v(x,0) = \frac{-4(3k^2 c_0 - 2\beta k^2 + 4k^4 c_2}{3c_2^2} + \frac{4k^2}{c_2} \tanh^2 kx \qquad (4.1)$$

$$w(x,0) = c_0 + c_2 \tanh^2 kx$$

are the initial approximations of $u(x,t)$, $v(x,t)$ and $w(x,t)$. In continuation, we choose the auxiliary linear operators as:

$$L[\phi_i(x,t;q)] = \frac{\partial \phi_i(x,t;q)}{\partial t}, i = 1,2,3 \qquad (4.2)$$

with the following property

$$L[C_i] = 0 \qquad (4.3)$$

where C_i are integral constants. Now, we define the nonlinear operators

$$N_1[\phi_1(x,t;q)] = \frac{1}{2}\frac{\partial^3 \phi_1(x,t;q)}{\partial x^3} - 3\phi_1(x,t;q)\frac{\partial \phi_1(x,t;q)}{\partial x} + 3\phi_2(x,t;q)\frac{\partial \phi_3(x,t;q)}{\partial x} + 3\phi_3(x,t;q)\frac{\partial \phi_2(x,t;q)}{\partial x}$$

Table 1:

X	u_{ADM}	u_{HAM}	v_{ADM}	v_{HAM}	w_{ADM}	w_{HAM}
-50	0.346645	0.346659	0.0657245	0.065726	1.99978	1.99982
-40	0.346505	0.346613	0.0656678	0.0656793	1.9936	1.99865
-30	0.345487	0.346273	0.0653356	0.0653356	1.99797	1.99006
-20	0.338603	0.343845	0.0628901	0.0628901	1.91451	1.92892
-10	0.310816	0.329861	0.0489609	0.0489609	1.51483	1.58069
0	0.307183	0.306667	0.0257347	0.0257347	1.00996	1.00003
10	0.348992	0.329874	0.0489077	0.0489077	1.64217	1.57936
20	0.348992	0.343836	0.0629354	0.0629245	1.94185	1.92978
30	0.347044	0.346271	0.06534099	0.0653417	1.99192	1.99021
40	0.346719	0.346613	0.0656894	0.0656801	1.99890	1.99867
50	0.346674	0.346659	0.0657274	0.0657261	1.99985	1.99982

$$N_2[\phi_2(x,t;q)] = -\frac{\partial^3\phi_2(x,t;q)}{\partial x^3} + 3\phi_1(x,t;q)\frac{\partial\phi_2(x,t;q)}{\partial x} \quad (4.4)$$

$$N_3[\phi_3(x,t;q)] = -\frac{\partial^3\phi_3(x,t;q)}{\partial x^3} + 3\phi_1(x,t;q)\frac{\partial\phi_3(x,t;q)}{\partial x}$$

The corresponding HAM can be considered as:

$$u_m = \chi_m u_{m-1} + \hbar\int_0^t R_m\left(\tilde{u}_{m-1}(x,t)\right)d\tau$$

$$v_m = \chi_m v_{m-1} + \hbar\int_0^t R_m\left(\tilde{v}_{m-1}(x,t)\right)d\tau \quad (4.5)$$

$$w_m = \chi_m w_{m-1} + \hbar\int_0^t R_m\left(\tilde{w}_{m-1}(x,t)\right)d\tau$$

where

$$R_m\left(\tilde{u}_{m-1}(x,t)\right) = \frac{1}{2}\partial_{xxx}u_{m-1} - 3\sum_{k=0}^{m-1}u_k\partial_x u_{m-k-1} + 3\sum_{k=0}^{m-1}v_k\partial_x w_{m-k-1} + 3\sum_{k=0}^{m-1}u_{m-1-k}\partial_x u_k$$

$$R_m\left(\tilde{u}_{m-1}(x,t)\right) = -\partial_{xxx}v_{m-1} + 3\sum_{k=0}^{m-1}u_k\partial_x v_{m-k-1} \quad (4.6)$$

$$R_m\left(\tilde{u}_{m-1}(x,t)\right) = -\partial_{xxx}w_{m-1} + 3\sum_{k=0}^{m-1}u_k\partial_x w_{m-k-1}$$

We have solved this problem using HAM for $c_0 = 1$, $c_2 = 1$, $k = .1$, $\beta = 1$, $h = -1$ and $t = 1$ and $t = 1$. The obtained results have been compared with ADM. The comparison is given in Table 1.

Homotopy analysis method for coupled MKdv: Consider the system (2.1) and apply the HAM to solve this system. Suppose that the initial conditions are as:

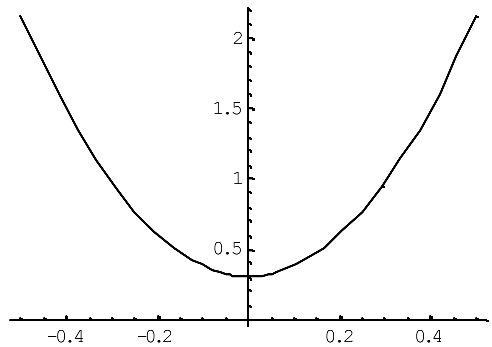


Fig. 1: The h-curve

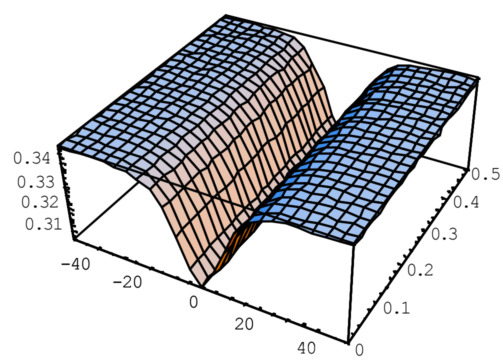


Fig. 2: u_{approx}

$$u(x,0) = k\tanh kx$$

$$v(x,0) = \frac{1}{2}(4k^2+\lambda) - 2k^2\tanh^2 kx \quad (4.7)$$

are the initial approximations of $u(x,t)$, $v(x,t)$ and $w(x,t)$. In continuation, we choose the auxiliary linear operators as:

$$L[\phi_i(x,t;q)] = \frac{\partial\phi_i(x,t;q)}{\partial t}, i = 1,2 \quad (4.8)$$

with the property

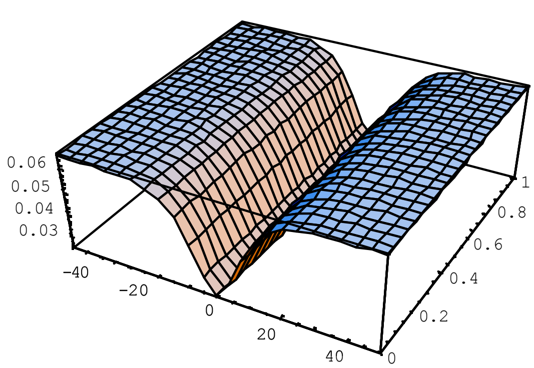


Fig. 2: v_{approx}

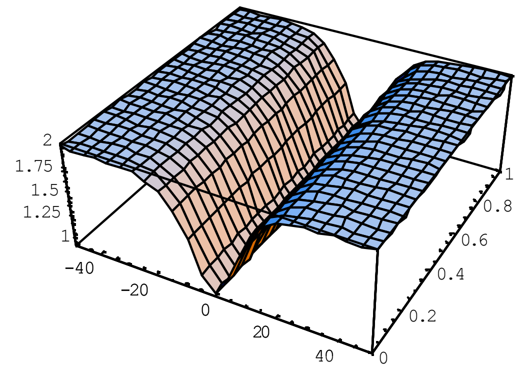
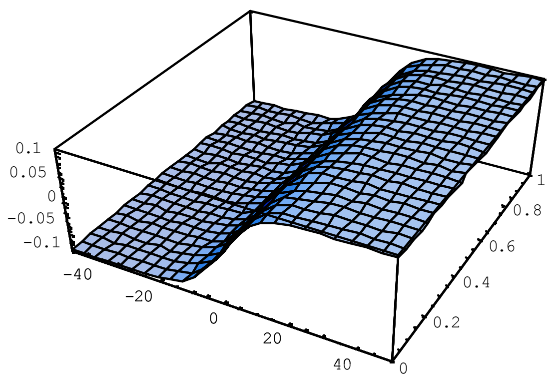
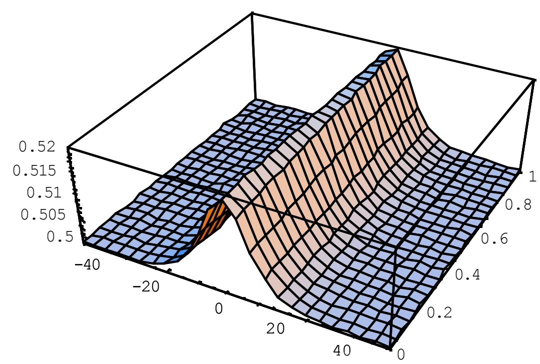


Fig. 2: w_{approx}



u(x,t)



v(x,t)

Fig. 3: The approximate solution of u(x,t) and v(x,t)

$$L[C_i] = 0$$

where C_i are integral constants. We define the nonlinear operators

$$N_1[\phi_1(x,t;q)] = \frac{1}{2}\frac{\partial^3\phi_1(x,t;q)}{\partial x^3} - 3\phi_1^2(x,t;q)\frac{\partial\phi_1(x,t;q)}{\partial x} + \frac{3}{2}\frac{\partial^2\phi_2(x,t;q)}{\partial x^2} + 3\phi_1(x,t;q)\frac{\partial\phi_1(x,t;q)}{\partial x} + 3\phi_2(x,t;q)\frac{\partial\phi_1(x,t;q)}{\partial x} - 3\lambda\frac{\partial\phi_1(x,t;q)}{\partial x}$$

$$N_2[\phi_2(x,t;q)] = -\frac{\partial^3\phi_2(x,t;q)}{\partial x^3} - 3\phi_2(x,t;q)\frac{\partial\phi_2(x,t;q)}{\partial x} - 3\frac{\partial\phi_2(x,t;q)}{\partial x}\frac{\partial\phi_1(x,t;q)}{\partial x} - 3\phi_1^2(x,t;q)\frac{\partial\phi_1(x,t;q)}{\partial x} + 3\lambda\frac{\partial\phi_2(x,t;q)}{\partial x} \quad (4.9)$$

The corresponding HAM can be considered as:

$$u_m = \chi_m u_{m-1} + \hbar\int_0^t R_m\left(\tilde{u}_{m-1}(x,t)\right)d\tau$$

$$v_m = \chi_m v_{m-1} + \hbar\int_0^t R_m\left(\tilde{v}_{m-1}(x,t)\right)d\tau \quad (4.10)$$

where

$$R_m\left(\tilde{u}_{m-1}(x,t)\right) = \frac{1}{2}\partial_{xxx}u_{m-1} - 3\sum_{i=0}^{m-1}\partial_x u_{m-i-1}\sum_{j=0}^{i}u_j u_{i-j} + \frac{3}{2}\partial_{xx}v_{m-1} + 3\sum_{i=0}^{m-1}v_i\,\partial_x u_{m-i-1} - 3\lambda\partial_x u_{m-1}$$

$$R_m\left(\tilde{u}_{m-1}(x,t)\right) = -\partial_{xxx}u_{m-1} - 3\sum_{i=0}^{m-1}v_i\,\partial_x v_{m-i-1} - 3\sum_{i=0}^{m-1}\partial_x v_i\,\partial_x u_{m-i-1} - 3\sum_{i=0}^{m-1}\partial_x v_{m-i-1}\sum_{j=0}^{i}u_j u_{i-j} + 3\lambda\partial_x v_{m-1} \quad (4.11)$$

Table 2:

x	u_{ADM}	u_{HAM}	v_{ADM}	v_{HAM}
-50	-0.0999220	-0.0999895	0.500004	0.500003
-40	-0.0999432	-0.0999228	0.500031	0.500019
-30	-0.0995747	-0.0994310	0.500229	0.500141
-20	-0.09568996	-0.0958693	0.501631	0.501021
-10	-0.0791645	-0.0729886	0.509388	0.506530
0	-0.00753516	-0.0075500	0.519889	0.519889
10	0.0791390	0.0793302	0.510342	0.510342
20	0.0968921	0.0969362	0.501224	0.501833
30	0.0994251	0.0995800	0.500170	0.500258
40	0.0999422	0.0999431	0.500023	0.500035
50	0.09999894	0.0999923	0.500003	0.500005

We have solved this problem using HAM for $c_0 = 1$, $c_2 = 1$, $k =.1$, $\beta = 1$, $h = -1$ and $t = 1$. The obtained results have been compared with ADM. The comparison is given in Table 2.

CONCLUSION

We have described and demonstrated the applicability of the HAM for solving system of Hirota-Satsuma couple Kdv and a coupled MKdv Equation. Our method is a direct method, further it is simple and accurate. It is a practical method and can easily be implemented on computer to solve such problems. We have used the method with initial condition and have tabulated the numerical results as well as the ADM solutions. The tables show that the present method approximates the exact solution very well.

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