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Complex Solutions of the Regularized Long Wave Equation

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Abstract: In the present work, direct algebraic method with a computerized symbolic computation is used for constructing the travelling wave solutions of regularized long wave equation with variable coefficients arising in physics. The results emphasize the power of the methods used.

Key words: Regularized Long Wave Equation · Direct algebraic method · Complex solutions

INTRODUCTION

A search of directly seeking for exactly solutions of nonlinear equations has been more interest in recent years. One of the most exciting advances of nonlinear science and theoretical physics has been a development of methods to look for exact solutions for nonlinear partial differential equations. among these methods we mainly cite, such as the extended tanh-function methods [1-3], F-expansion methods [4-6], famous Hirota's method [7], the Backlund and Darboux transformation [8-10], Painleve expansions [11], homogeneous balance method [12], Jacobi elliptic function [13-14], extended, variational iteration methods [15-17]. The rest of this paper is arranged as follows.In section 2,we describe the direct algebraic method. In section 3, we apply this method for regularized long wave equation. Finally, in section 4 conclusions are given.

Description of Direct Algebraic Method: For a given partial differential equation

$$G\,(u, u_x, u_t, u_{xx}, u_{tt}, \ldots), \tag{1}$$

Our method mainly consists of four steps:

Step 1: We seek complex solutions of Eq. (1) as the following form:

$$u = u(z) \qquad z = ik(x-ct) \tag{2}$$

Where k and c are real constants. Under the transformation (2), Eq. (1) becomes an ordinary differential equation

$$N(u, iku', -ikcu', -k^2u'', \ldots..), \tag{3}$$

Where $u' = \dfrac{du}{dz}$.

Step 2: We assume that the solution of Eq. (3) is of the form

$$u(z) = \sum_{i=0}^{n} a_i F^i(z), \tag{4}$$

Where a_i (i = 1, 2,....,n) are real constants to be determined later. $F(z)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(z) = b + F^2(z), \tag{5}$$

Eq. (5) admits the following solutions:

$$F(z) = \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\,z), & b \prec 0 \\ -\sqrt{-b}\coth(\sqrt{-b}\,z), & b \prec 0 \end{cases}$$
$$F(z) = \begin{cases} \sqrt{b}\tan(\sqrt{b}\,z), & b \succ 0 \\ -\sqrt{b}\cot(\sqrt{b}\,z), & b \succ 0 \end{cases} \tag{6}$$
$$F(z) = -\frac{1}{z}, \qquad b = 0$$

Integer n in (4) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of $u(z)$ in Eq. (3).

Step 3: Substituting (4) into (3) with (5), then the left hand side of Eq. (3) is converted into a polynomial in $F(z)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k , c,

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Step 4: Solving the algebraic equations obtained in Step 3 and substituting the results into (4), then we obtain the exact travelling wave solutions for Eq. (1).

Application for Regularized Long Wave Equation: In this case we consider the regularized long wave equation as

$$u_t + u_x + r(u^2)_x - su_{xxt} = 0 \quad ,r,s \in R \tag{7}$$

Permits us converting Eq. (7) into an ODE for $u = u(z)$ and $z = ik(x - ct)$ integrating we have

$$ik(1-c)u + riku^2 - sik^3cu'' = 0 \qquad ,r,s \in R \tag{8}$$

Considering the homogeneous balance between u'' and u^2 in Eq. (7), we required that $2m = m+2?m = 2$, so we can write (4) as

$$\begin{aligned}
F^0 &: \quad rika_0^2 - 2sik^3ca_2b^2 + ik(1-c)a_0 = 0\\
F^1 &: \quad 2rika_0a_1 - 2sik^3ca_1b + ik(1-c)a_1 = 0\\
F^2 &: \quad rik(a_1^2 + 2a_0a_2) - 8sik^3ca_2b + ik(1-c)a_2 = 0\\
F^3 &: \quad 2rika_1a_2 - 2sik^3ca_1 = 0\\
F^4 &: \quad rika_2^2 - 6sik^3ca_2 = 0
\end{aligned}$$

By substituting (9) into Eq. (8) and collecting all terms with the same power of F together, the left-hand side of Eq. (8) is converted into another polynomial in F equating each coefficient of this polynomial to zero, yields a set of simultaneous algebraic equations for a_2, a_1, a_0 λ μ c as follows:

$$a_2 = \frac{6k^2cs}{r}, \qquad a_1 = 0, \pm\frac{6\sqrt{b}k^2cs}{r}$$

By solving algebraic equations above we have

$$a_2 = \frac{6k^2cs}{r}, \qquad a_1 = 0, \pm\frac{6\sqrt{b}k^2cs}{r}$$

For $a_1 = 0$ we have

$$a_0 = \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 48s^2k^4c^2b}{k^2rsc}, c = \pm\frac{1}{4sk^2b - 1}$$

And for $a_1 = \pm\frac{6\sqrt{b}k^2cs}{r}$ we obtain

$$a_0 = \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 12s^2k^4c^2b}{k^2rsc},$$

$$c = \pm\frac{1 + 2isk^2b\sqrt{11}}{1 + 44s^2k^4b^2},$$

Family 1: We have

$$a_2 = \frac{6k^2cs}{r}, \qquad a_1 = 0,$$

$$a_0 = \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 48s^2k^4c^2b}{k^2rsc}, c = \pm\frac{1}{4sk^2b - 1}$$

Substituting (10) into (9) with (6), respectively, we obtain new exact complex solutions for Eq. (7) as follows:

$$u_1 = \frac{6k^2cs}{a}\left[\sqrt{-b}\tanh(\sqrt{-b}ik(x \mp \frac{1}{4sk^2b-1}t))\right]^2 + \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 48s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_2 = \frac{6k^2cs}{a}\left[\sqrt{-b}\coth(\sqrt{-b}ik(x \mp \frac{1}{4sk^2b-1}t))\right]^2 + \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 48s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_3 = \frac{6k^2cs}{a}\left[\sqrt{b}\tan(\sqrt{b}ik(x \mp \frac{1}{4sk^2b-1}t))\right]^2 + \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 48s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_4 = \frac{6k^2cs}{a}\left[-\sqrt{b}\cot(\sqrt{b}ik(x \mp \frac{1}{4sk^2b-1}t))\right]^2 + \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 48s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_5 = -\frac{6k^2cs}{a}\left[\frac{1}{ik(x \pm t)}\right]^2 + \frac{1}{2}\frac{-1+c}{r}$$

Where $b = 0$

Family 2: In this case we have

$$a_2 = \frac{6k^2cs}{a}, a_1 = \pm\frac{6\sqrt{b}k^2cs}{r} \quad a_0 = \frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 12s^2k^4c^2b}{k^2rsc}, \qquad c = \pm\frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2},$$

By using relations above we obtain new exact complex solutions for Eq. (7) as follows:

$$u_1 = \frac{6k^2cs}{a}\left[-\sqrt{-b}\tanh(\sqrt{-b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))\right]^2$$
$$\mp\frac{6\sqrt{b}k^2cs}{r}\sqrt{-b}\tanh(\sqrt{-b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))$$
$$+\frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 12s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_2 = \frac{6k^2cs}{a}\left[-\sqrt{-b}\coth(\sqrt{-b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))\right]^2$$
$$\mp\frac{6\sqrt{b}k^2cs}{r}\sqrt{-b}\coth(\sqrt{-b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))$$
$$+\frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 12s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_3 = \frac{6k^2cs}{a}\left[\sqrt{b}\tan(\sqrt{b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))\right]^2$$
$$\pm\frac{6\sqrt{b}k^2cs}{r}\sqrt{b}\tan(\sqrt{b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))$$
$$+\frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 12s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_4 = \frac{6k^2cs}{a}\left[-\sqrt{b}\cot(\sqrt{b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))\right]^2$$
$$\mp\frac{6\sqrt{b}k^2cs}{r}\sqrt{b}\cot(\sqrt{b}ik(x \mp \frac{1+2isk^2b\sqrt{11}}{1+44s^2k^4b^2}t))$$
$$+\frac{1}{12}\frac{-6sk^2c + 6sk^2c^2 + 12s^2k^4c^2b}{k^2rsc}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_5 = -\frac{6k^2cs}{a}\left[\frac{1}{ik(x \pm t)}\right]^2 \mp \frac{6\sqrt{b}k^2cs}{r}\left(\frac{1}{ik(x \pm t)}\right)$$
$$+\frac{1}{2}\frac{-1+c}{r}$$

Where $b = 0$.

CONCLUSION

In this work we have seen that three types of travelling solutions of the regularized long wave equation are successfully found out by using the direct algebraic method. The results emphasize the power of the methods used.

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