

Complex Solutions of the Coupled BLP System

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Abstract: The aims of this work searching for construct the exact complex solutions for nonlinear coupled BLP system by using of direct algebraic method. The results emphasize the power of the methods used.

Key words: Coupled BLP system • Direct algebraic method • Complex solutions

INTRODUCTION

Searching for exact solutions of non-linear PDEs have been investigated by many authors. Since the world around us is inherently nonlinear, evolution equations are widely used to describe complex phenomena in various fields of sciences, especially in physics such as, plasma physics, fluid mechanics, optical fibers, solid state physics and nonlinear optics and so on. One of the most exciting advances of nonlinear science and theoretical physics has been a development of methods to look for exact solutions for nonlinear evolution equations. A search of directly seeking for exact solutions of nonlinear equations has been more interest in recent years because of the availability of symbolic computation Mathematica or Maple. Recently, many powerful methods to construct exact solutions of nonlinear PDEs have been established and developed, among these methods we mainly cite, such as the famous Hirota's method [1], the Backlund and Darboux transformation [2-4], Painleve expansions [5], homogeneous balance method [6], Jacobi elliptic function [7, 8], extended tanh-function methods [9-11], extended, F-expansion methods [12-16], variational iteration methods [17-20].

Description of Direct Algebraic Method: For a given partial differential equation

$$G(u, u_x, u_t, u_y, u_{xx}, u_{xt}, u_{yy}, \dots), \quad (1)$$

Our method mainly consists of four steps:

Step 1: We seek complex solutions of Eq. (1) as the following form:

$$u = u(z), \quad z = ik(x + y - ct), \quad (2)$$

Where k and c are real constants. Under the transformation (2), Eq. (1) becomes an ordinary differential equation

$$N(u, iku', -ikcu', iku', -k^2u'', \dots), \quad (3)$$

Where $u' = \frac{du}{dz}$.

Step 2: We assume that the solution of Eq. (3) is of the form

$$u(z) = \sum_{i=0}^n a_i F^i(z), \quad (4)$$

Where $a_i (i = 1, 2, \dots, n)$ are real constants to be determined later. $F(z)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(z) = b + F^2(z), \quad (5)$$

Eq. (5) admits the following solutions:

$$\begin{aligned} F(z) &= \begin{cases} -\sqrt{-b} \tanh(\sqrt{-b}z), & b < 0 \\ -\sqrt{-b} \coth(\sqrt{-b}z), & b < 0 \end{cases} \\ F(z) &= \begin{cases} \sqrt{b} \tanh(\sqrt{b}z), & b > 0 \\ \sqrt{b} \coth(\sqrt{b}z), & b > 0 \end{cases} \\ F(z) &= -\frac{1}{z}, \quad b = 0 \end{aligned} \quad (6)$$

Integer r in (4) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of $u(z)$ in Eq. (3).

Step 3: Substituting (4) into (3) with (5), then the left hand side of Eq. (3) is converted into a polynomial in $F(z)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k, c .

Step 4: Solving the algebraic equations obtained in Step 3 and substituting the results into (4), then we obtain the exact travelling wave solutions for Eq. (1).

Application for Coupled BLP System: In this section we consider the coupling Boiti-Leon-Pempinelli system as

$$\begin{cases} u_{ty} = (u^2 - u_x)_{xy} + 2v_{xxx} \\ v_t = v_{xx} + 2uv_x \end{cases} \quad (7)$$

We make the transformation (2) and change Eq. (23) into an ODE

$$\begin{cases} cu'' = -(u^2 - iku')'' - 2ikv''' \\ -icv' = -kv + 2iuv' \end{cases}$$

By integrating twice of the first equation above and substituting into the second equation above we obtain

$$k^2u'' + 2u^3 + 3cu^2 + c^2u = 0 \quad (8)$$

And

$$\begin{aligned} v_1 &= \frac{1}{2}u + \frac{i}{2k} \int (u^2 + cu) dz \\ z &= ik(x + y - ct) \Rightarrow dz = ik(dx + dy - cdt) \\ v &= \pm \frac{1}{2}ik\sqrt{-b} \tanh(\sqrt{-b}ik(x + y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b} + \\ &\quad \frac{i}{2k} \int ((\pm ik\sqrt{-b} \tanh(\sqrt{-b}ik(x + y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b})^2)(ik(dx + dy - cdt)) + \\ &\quad \frac{i}{2k} \int c(\pm ik\sqrt{-b} \tanh(\sqrt{-b}ik(x + y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b})(ik(dx + dy - cdt)) = \\ &\quad \pm \frac{1}{2}ik\sqrt{-b} \tanh(\sqrt{-b}ik(x + y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b} - \\ &\quad \frac{1}{2} \left[-\frac{1}{2} \ln(\tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}) - 1) - \frac{1}{2} \ln(\tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}) + 1) + 2\sqrt{kbt}x - \right. \\ &\quad \left. \frac{1}{2} \ln(\tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}) - 1) - \frac{1}{2} \ln(\tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}) + 1) + 2\sqrt{kbt}y - \right. \\ &\quad \left. \frac{c}{4} \frac{\ln(\tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}) - 1)}{\sqrt{kbt}} - \frac{c}{4} \frac{\ln(\tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}) + 1)}{\sqrt{kbt}} - 2c\sqrt{kbt} + \right. \\ &\quad \left. \frac{2ikbt \tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}))}{\sqrt{-b}} - \frac{4b^2 \ln(\tanh(\sqrt{-b}ik(x + y - 2\sqrt{kbt}) - 1)}{\sqrt{-b}} - \right. \end{aligned}$$

$$v = \frac{1}{2}u + \frac{i}{2k} \int (u^2 + cu) dz$$

Balancing u^3 and u'' in equation (8), it gives $n = 1$. Therefore we may choose

$$u(z) = a_1 F + a_0 \quad (9)$$

Where $F(z)$ expresses the solution of Eq. (5), a_1, a_0 are real constants to be determined later. Note that the complex conjugate of F in (6) is equal to $\bar{F} = -F$. Hence, substituting (9) into (8) and setting coefficients of $F^i (i = 0, 1, 2, 3)$ to zero, we obtain:

$$\begin{aligned} F^3 : \quad &2a_1 k^2 + 2a_1^3 = 0 \\ F^2 : \quad &6a_1^2 a_0 + 3ca_1^2 = 0 \\ F^1 : \quad &2k^2 a_1 b + 6a_1 a_0^2 + 6ca_1 a_0 + c^2 a_1 = 0 \\ F^0 : \quad &2a_0^3 + 3ca_0^2 + c^2 a_0 = 0 \end{aligned}$$

By solving equations above we obtain

$$a_1^2 = -k^2, \quad a_0 = \mp 2k\sqrt{b}, \quad c^2 = 4k^2 b$$

Substituting (10) into (9) with (6), respectively, we obtain new exact complex solutions for Eq. (7) as follows:

$$u_1 = \pm ik\sqrt{-b} \tanh(\sqrt{-b}ik(x + y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b}$$

$$\begin{aligned}
 & 4\sqrt{kb} \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))-1) + \frac{ikb \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))-1)}{\sqrt{-b}} + \\
 & \frac{4b^2 \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))+1)}{\sqrt{-b}} - 4\sqrt{kb} \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))+1) - \\
 & \frac{ikb \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))+1)}{\sqrt{-b}} + \frac{c}{2} \frac{i\sqrt{k} \tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))}{\sqrt{-b}} - \\
 & \frac{cb \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))-1)}{\sqrt{-bki}} + c \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))-1) + \\
 & \frac{c}{4} \frac{i\sqrt{k} \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))-1)}{\sqrt{-b}} + \frac{cb \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))+1)}{\sqrt{-bki}} - \\
 & c \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))+1) - \frac{c}{4} \frac{i\sqrt{k} \ln(\tanh(\sqrt{-b}ik(x+y-2\sqrt{kb}t))+1)}{\sqrt{-b}}
 \end{aligned}$$

Where $b < 0$ and k is an arbitrary real constant.

$$u_2 = \pm ik\sqrt{-b} \coth(\sqrt{-b}ik(x+y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b}$$

Where $b < 0$ and k is an arbitrary real constant. In this case we obtain v as a previous section.

$$u_3 = \mp ik\sqrt{b} \tan(\sqrt{b}ik(x+y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b}$$

Where $b > 0$ and k is an arbitrary real constant.

$$u_4 = \pm ik\sqrt{b} \cot(\sqrt{b}ik(x+y \mp 2k\sqrt{bt})) \mp 2k\sqrt{b}$$

Where $b > 0$ and k is an arbitrary real constant.

$$u_4 = \mp \frac{1}{x+y}$$

$$v_4 = \frac{1}{2}u + \frac{i}{2k} \int (u^2 + cu) dz$$

$$z = ik(x+y-ct) \Rightarrow dz = ik(dx+dy-ctdt)$$

$$\begin{aligned}
 v_4 &= \frac{1}{2} \left(\pm \frac{1}{x+y} \right) + \frac{i}{2k} \int \left(\left(\frac{1}{x+y} \right)^2 \pm c \left(\frac{1}{x+y} \right) \right) d(x+y-ct) = \\
 &= \pm \frac{1}{2} \frac{1}{x+y} + \frac{i}{2k} \left[-\frac{2}{x+y} \pm 2c \ln(x+y) - c \left(\left(\frac{1}{x+y} \right)^2 \pm c \left(\frac{1}{x+y} \right) \right) t \right]
 \end{aligned}$$

Where $b = 0$

CONCLUSION

We could obtain new kinds of exact complex travelling wave solutions, if we choose various auxiliary ordinary differential equations in our method.

This method can be used to many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculations on computer.

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