World Applied Sciences Journal 12 (4): 519-530, 2011
ISSN 1818-4952

Similarity Solutions of the Effect of Variable Viscosity on Unsteady Mixed Convection Boundary Layer Flow over a Vertical Surface Embedded in a Porous Medium via HAMAD Formulations

[1]M.A.A. Hamad and [2]Muatazz Abdolhadi Bashir

[1]Department of Mathematics, Faculty of Science, Assiut University, Assiut 71516, Egypt
[2]Department of Mathematics, Faculty of Science, Garyounis University, Benghazi 1308, Libya

Abstract: The similarity solutions of the effect of variable viscosity on unsteady mixed convection boundary layer flow over a vertical surface embedded in a porous medium are obtained by using HAMAD formulations. Using HAMAD formulations the partial differential equations governing the flow and the heat transfer transform to ordinary differential equations. The fluid viscosity is assumed to vary as an inverse linear function of temperature *T*. Numerical results for the flow and heat transfer characteristics are obtained for various values of the mixed convection parameter ε, the unsteady parameter *A* and the variable viscosity parameter θ_e. The numerical results for the steady case are calculated and compared with previous work.

Key words: HAMAD method . unsteady mixed convection . boundary-layer flow . similarity solutions

INTRODUCTION

The porous media heat transfer problems have numerous thermal engineering applications such as geothermal energy recovery, crude oil extraction, thermal insulation, ground water pollution, oil extraction, thermal energy storage, thermal insulations, and flow through filtering devices. Excellent reviews on this topic are provided in the literature by Nield and Bejan [1], Vafai [2], Ingham and Pop [3] and Vadasz [4]. Recently, Cheng and Lin [5] examined the melting effect on mixed convective heat transfer from a permeable vertical flat plate embedded in a liquid-saturated porous medium with aiding and opposing external flows. Rashad [6] studied the combined effect of MHD and thermal radiation on heat and mass transfer by free convection over vertical flat plate embedded in a porous medium. EL-Kabeir et al. [7] applied the group theoretical method to solve the problem of coupled heat and mass transfer by natural convection boundary layer flow for water-vapor around a permeable vertical cone embedded in a non uniform porous medium in the presence of magnetic field and thermal radiation effects.

There have been several studies of the effect of temperature-dependent viscosity on free/mixed boundary layer flow. Ratcliff et al. [8] presented a numerical investigation of the effects of a strongly temperature-dependent viscosity on the radial structure, horizontal planform, and heat transfer characteristics of a thermally convecting, infinite Prandtl number, Boussinesq fluid in a spherical shell. Basal and mixed-mode heating are considered wherein viscosity varies with temperature. For basal heating, three convective regimes (mobile lid, sluggish lid, and stagnant lid) are observed in accord with theoretical and numerical studies in cartesian geometry. For mixed-mode heating calculations mobile lid and sluggish lid regimes are studied with changes in horizontal length scales. Hossain et al. [9] studied the natural convection flow about a vertical cone and vertical wavy surfaces, respectively, with the viscosity inversely proportional to the linear function of temperature. Kim and Choi [10] presented theoretical analysis of thermal instability driven by buoyancy forces under a time-dependent temperature field of conduction is conducted in an initially quiescent, horizontal liquid layer. The dependency of viscosity on temperature is considered and the propagation theory is employed for the stability analysis. Soh and Mureithi [11] computed exact and numerical solutions of a fully developed flow of a generalized second-grade fluid, with power-law temperature-dependent viscosity (μ=?$^{-M}$), down an inclined plane.

Analytical solutions are found for the case when M = m+1, m?1, m being a constant that models shear thinning (m<0) or shear thickening (m>0). Cheng [12] examined the natural convection heat transfer from a horizontal isothermal cylinder of elliptic cross section in a Newtonian fluid with temperature-dependent

Corresponding Author: Hamad, M.A.A., Mathematics Department, Faculty of Science, Assiut University, Assiut 71516, Egypt

viscosity. Results for the local Nusselt number and the local skin-friction coefficient presented as functions of eccentric angle for various values of viscosity-variation parameter, aspect ratio, and Prandtl number. The total heat transfer rate and the total skin friction of the elliptical cylinder with slender orientation are higher than those of the elliptical cylinder with blunt orientation. Recently, Pla et al. [13] analyzed convective solutions of a two dimensional fluid layer in which viscosity depends exponentially on temperature. This problem takes in features of mantle convection, since large viscosity variations are to be expected in the Earth's interior. Ahmad et al. [14] studied the non-similar partial differential equations governing the problem of steady laminar mixed convection boundary layer flow past an isothermal horizontal circular cylinder placed in a viscous and incompressible fluid of temperature-dependent viscosity. They found the flow and heat transfer characteristics are significantly influenced by viscosity/temperature parameter.

The effect of the time is very important to consider in physics and engineering problems, such as the boundary-layer flow problems. Some recent papers of unsteady natural convection boundary-layer flow, Hassanien and Hamad [15] introduced new similarity solutions of flow and heat transfer of a micropolar fluid along a vertical plate in a thermally stratified medium. The general analysis is developed in them study for the case of ambient temperature that varies exponentially with time as well as uniform and varies with the position. Rodriguez et al. [16] studied the transient cooling of a fluid initially at rest inside a vertical cylinder submitted to heat losses through the walls. D'Alessio and Perera [17] presented numerical and analytical solutions for the problem of unsteady free convection from an inclined elliptic cylinder. They obtained an analytical solution valid for small times and large Grashof numbers. Hmouda et al. [18] investigated a storage tank with an internal gas flue experimentally and numerically during its long-term cooling process.

In this paper, using HAMAD formulations for PDEs of 3-independent variable (see Hamad [19]) the governing partial differential equations with the boundary conditions reduce to ordinary differential equations with the appropriate boundary conditions, which are solved numerically using a finite deference method. Here we have focused our attention on the effect of the parameters govern the problem on the temperature and the heat transfer coefficient in unsteady case. The agreement between the present results and the previous is excellent.

MATHEMATICAL SIMULATION

Formulation of the problem: Consider a problem of unsteady laminar mixed convection boundary layer flow of a viscous and incompressible fluid of viscosity depends on temperature past a vertical semi-infinite flat plate placed in a porous medium of constant ambient temperature T_∞ (see Fig. 1.). Assume that the plate is maintained at the uniform temperature T_w, where $T_w>T_\infty$ is for a heated plate and $T_w<T_\infty$ corresponds to a cooled plate. We also assume that the porous medium is filled with a fluid of variable viscosity. Consider the Cartesian coordinate system $(\bar{x}, \bar{y})$, the plate is located parallel to the free stream velocity U_∞ oriented in the upward direction. Under these assumptions along with the Boussinesq and boundary layer approximations, then, the governing boundary layer equations of this problem are

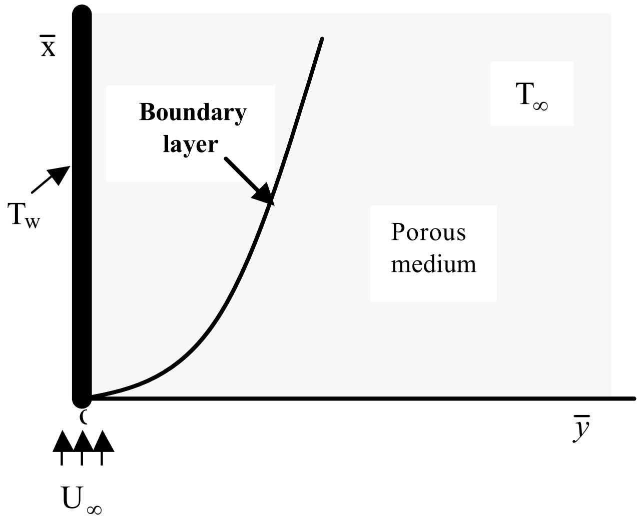

Fig. 1. Physical model and coordinate system

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u} = U_\infty + \frac{\rho_\infty g K \beta}{\mu}(T - T_\infty), \tag{2}$$

$$\frac{\partial T}{\partial \bar{t}} + \bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_m \frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

subject to the boundary conditions:

$$\begin{aligned} &\bar{v} = 0, \quad T = T_w \quad at \quad \bar{y} = 0, \\ &\bar{u} \to U_\infty, \quad T \to T_\infty \quad as \quad \bar{y} \to \infty, \end{aligned} \tag{4}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$-and $\bar{y}$-axes, respectively; $\bar{t}$ is the time; T is the fluid temperature, β and α_m are the coefficients of thermal expansion and thermal diffusivity of the porous medium, respectively; g is to gravitational acceleration, ρ_∞ is the ambient fluid density; K is the permeability of the porous medium and μ is the dynamic viscosity.

We assume that the dynamic viscosity μ has the following form (see Chin et al. [20]).

$$\mu = \frac{\mu_\infty}{1 + \gamma(T - T_\infty)} = \frac{1}{a(T - T_e)} \tag{5}$$

where, γ and μ_∞ are the thermal property of the fluid and the ambient fluid viscosity, respectively, which is a constant. Also, a and T_e are constants, given by

$$a = \frac{\gamma}{\mu_\infty}, \quad T_e - T_\infty = -\frac{1}{\gamma} \tag{6}$$

By introducing the following non-dimensional variables:

$$t=\frac{\bar{t}\,U_\infty}{L},\ x=\frac{\bar{x}}{L},\ y=\frac{Pe^{1/2}}{L}\bar{y},\ u=\frac{\bar{u}}{U_\infty},\ v=\frac{Pe^{1/2}}{U_\infty}\bar{v},$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_e=\frac{T_e-T_\infty}{T_w-T_\infty}=\frac{-1}{a(T_w-T_\infty)}, \tag{7}$$

where,

$$Pe=U_\infty L/\alpha_m$$

is the Peclet number for a porous medium and θ_e is the viscosity/temperature parameter and its value is determined by the viscosity/temperature characteristics of the fluid and the operating temperature difference $\Delta T = T_w - T_\infty$. It is worth mentioning that in the case when the temperature difference ΔT is positive, θ_e must physically be >1 for gases and <1 for liquids. However, the opposite is true if ΔT is negative, where θ_e must physically be >1 for Liquids and <1 for gases since γ has the opposite sign in each of these cases and vice versa (Chin et al. 2007). Substituting (5) and (7) into (1)-(3) we get the non-dimensional form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{8}$$

$$u=1+\varepsilon\,(\theta-\frac{\theta^2}{\theta_e}), \tag{9}$$

$$\frac{\partial\theta}{\partial t}+u\,\frac{\partial\theta}{\partial x}+v\,\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial y^2}, \tag{10}$$

with the boundary conditions:

$$\begin{aligned} &v=0, \quad \theta=1 \quad at \quad y=0,\\ &u\to 1, \quad \theta\to 0 \quad as \quad y\to\infty, \end{aligned} \tag{11}$$

where ε is the constant mixed convection parameter which is given by

$$\varepsilon=\frac{g\,K\,\beta\,(T_w-T_\infty)\,L/(\alpha_m\,\nu)}{U_\infty\,L/\alpha_m}=\frac{Ra}{Pe}, \tag{12}$$

Where,

$$Ra=g\,K\,\beta\,(T_w-T_\infty)\,L/(\alpha_m\,\nu)$$

is the Rayleigh number for porous medium. It should be mentioned that $\varepsilon>0$ corresponds to a heated plate (assisting flow), $\varepsilon<0$ corresponds to a cooled plate (opposing flow) and $\varepsilon=0$ corresponds to the forced convection flow.

Further, we define the stream function ? according to

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \tag{13}$$

Using (13), equations (8)-(10) become

$$\frac{\partial \psi}{\partial y} = 1 + \varepsilon\,(\theta - \frac{\theta^2}{\theta_e}), \tag{14}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2}. \tag{15}$$

with the boundary conditions:

$$\begin{aligned} &\frac{\partial \psi}{\partial x} = 0, \quad \theta = 1 \quad at \quad y = 0, \\ &\frac{\partial \psi}{\partial y} \to 1, \quad \theta \to 0 \quad as \quad y \to \infty, \end{aligned} \tag{16}$$

METHOD OF SOLUTION

The invariant group: The procedure is initiated with the group G, a class of two-parameters a_1 and a_2, of the form (see Moran and Gaggioli [21]).

$$G: \begin{cases} \tilde{t} = C^t(a_1,a_2)\,t + K^t(a_1,a_2), \\ \tilde{x} = C^x(a_1,a_2)\,x + K^x(a_1,a_2), \\ \tilde{y} = C^y(a_1,a_2)\,y + K^y(a_1,a_2), \\ \tilde{\psi} = C^\psi(a_1,a_2)\psi + K^\psi(a_1,a_2), \\ \tilde{\theta} = C^\theta(a_1,a_2)\theta + K^\theta(a_1,a_2), \end{cases} \tag{17}$$

where *C's* and *K's* are real valued and at least differentiable in their real arguments a_1 and a_2.

Transformations of the derivatives are obtained from G via chain-rule operations as

$$\tilde{S}_{\tilde{i}} = (C^S / C^i) S_i, \qquad \tilde{S}_{\tilde{i}\,\tilde{j}} = (C^S / C^i\, C^j) S_{ij}, \quad i, j = x, y, t \tag{18}$$

where S stands for ψ and θ.

Equations (14) and (15) is said to be invariantly transformed under (17) whenever

$$\tilde{\psi}_{\tilde{y}} - \varepsilon(\tilde{\theta} - \frac{\tilde{\theta}^2}{\theta_e}) = 1$$

when (14) holds (19)

$$\tilde{\theta}_{\tilde{y}\,\tilde{y}} + \tilde{\psi}_{\tilde{x}}\,\tilde{\theta}_{\tilde{y}} - \tilde{\psi}_{\tilde{y}}\,\tilde{\theta}_{\tilde{x}} - \tilde{\theta}_{\tilde{t}} = 0$$

when (15) holds (20)

Substitution from (17) and (18) into equation (19) for the independent variables, the functions and their derivatives yields

$$[\frac{C^{\psi}}{C^{y}}]\psi_{y} - \varepsilon\{[C^{\theta}]\theta - [(C^{\theta})^{2}\frac{\theta^{2}}{\theta_{e}} + R_{1}(a_{1},a_{2})\} = 1$$

when (14) holds (21)

where

$$R_{1}(a_{1},a_{2}) = K^{\theta}(1 - 2C^{\theta}\frac{\theta}{\theta_{e}} - \frac{K^{\theta}}{\theta_{e}}). \qquad (22)$$

The invariance of equation (21) implies $R_1(a_1, a_2) = 0$. This is satisfied by putting

$$K^{\theta} = 0 \qquad (23)$$

and

$$C^{\psi} = C^{y}, \quad C^{\theta} = 1 \qquad (24)$$

Similarly from equations (20) we get

$$C^{t} = C^{x} = (C^{y})^{2}. \qquad (25)$$

Moreover, the boundary conditions (16) should be invariant under G, this implies

$$K^{y} = 0. \qquad (26)$$

Finally, the group G which transforms invariantly, the differential Eqs. (14) and (15) and the boundary conditions (16) becomes

$$G = \begin{cases} \tilde{x} = (C^{y})^{2}x + K^{x} \\ \tilde{y} = C^{y}y \\ \tilde{t} = (C^{y})^{2}t + K^{t} \\ \tilde{\psi} = C^{y}\psi + K^{\psi} \\ \tilde{\theta} = \theta \end{cases} \qquad (27)$$

HAMAD Formulations for Similarity Transformations: This section presents the relation between the original independent variables and the new similarity independent variable η. Also, the relations between the original independent/dependent variables and the new dependent variables which are functions of η. These relations come out directly from HAMAD formulations for PDEs of 3-independent variables (see Hamad [19]).

From Eq. (26) and the definitions of

$$\alpha_{ij}, \beta_{ij}, i = 1,2,...,5,\ j = 1,2$$

and by considering

$x_1 = t,\ x_2 = x,\ x_3 = y,\ y_1 = \psi,\ y_2 = \theta$ (see Hamad [19]), then

$$\begin{aligned}&\alpha_{11} = \alpha_{21} = 2\alpha_{31},\ \ \alpha_{41} = \alpha_{31},\ \alpha_{51} = \beta_{31} = \beta_{51} = 0,\\ &\alpha_{12} = \alpha_{22} = 2\alpha_{32},\ \ \alpha_{42} = \alpha_{32},\ \alpha_{52} = \beta_{32} = \beta_{52} = 0,\end{aligned} \tag{28}$$

Now the definitions of $\lambda ijmn$, $\rho ijmn$, $\sigma ijmn$, (Hamad [19]), then

$$\begin{aligned}&\lambda_{1321} = \lambda_{1221} = \lambda_{1421} = \lambda_{1521} = \rho_{1321} = \rho_{5112} = \sigma_{1321} = \sigma_{1521} = 0,\\ &\rho_{2112} = 2\rho_{3112},\ \rho_{1221} = 2\rho_{3221},\ \ \rho_{4112} = \rho_{3112},\ \ \rho_{1421} = 2\rho_{3421},\end{aligned} \tag{29}$$

from which the suitable formulations (see Hamad [19]) which turn out the independent similarity variable, the stream function and the temperature are

$$\begin{aligned}&\eta = \frac{\rho_{3112}\, y}{\sqrt{2\rho_{3112}\, x + 2\rho_{3221}\, t + \sigma_{1221}}},\\ &\psi = \frac{1}{\rho_{3112}}\sqrt{2\rho_{3112}\, x + 2\rho_{3221}\, t + \sigma_{1221}}\ F(\eta) - \frac{2\rho_{3421}\, t + \sigma_{1421}}{\rho_{3112}},\\ &\theta = \theta(\eta),\end{aligned} \tag{30}$$

without lose of generality, we rewrite the similarity transformations (30) as following

$$\begin{aligned}&\eta = \frac{y}{\sqrt{2x + 2At + B}},\\ &\psi = \sqrt{2x + 2At + B}\ F(\eta) - (A_1 t + B_1),\\ &\theta = \theta(\eta),\end{aligned} \tag{31}$$

where

$$A = \frac{\rho_{3221}}{\rho_{3112}},\ B = \frac{\sigma_{1221}}{\rho_{3112}},\ A_1 = \frac{2\rho_{3421}}{\rho_{3112}},\ B_1 = \frac{\sigma_{1421}}{\rho_{3112}},\ \rho_{3112} \neq 0$$

are arbitrary constants and $A = A_1 = 0$ corresponds to the steady case.

Reductions to ordinary differential equations: The similarity transformations (31) maps Eqs. (14) and (15) to the following ODEs

$$F' = 1 + \varepsilon(\theta - \frac{\theta^2}{\theta_e}), \tag{32}$$

$$\theta''+(F+A)\theta'=0, \quad (33)$$

with the boundary conditions

$$\begin{aligned} &F(0)=0, \quad \theta(0)=1, \\ &F'(\infty)=1, \ \ \theta(\infty)=0. \end{aligned} \quad (34)$$

COMPARISON

We notice that when the unsteady parameter A = 0, we have gotten the same similarity representation for the steady case as in Chin et al. [20]. Also, we notice that the effect of variable viscosity can be neglected if θ_e is large (>>1), i.e., if either the constant a or (T_w-T_∞) is small. If θ_e is small then either the fluid viscosity changes with the temperature or the temperature difference is high. It is also noticed that if $\theta_e \to \infty$ equations (34) and (35) reduce to those for a constant viscosity case found by Markin [22] or Aly et al. [23] for an isothermal flat plate in the steady state.

RESULTS AND DISCUSSION

By using HAMAD formulations, a similarity transformations and similarity representation of the effect of variable viscosity on unsteady mixed convection boundary layer flow over a vertical surface embedded in a porous medium is obtained and it shown in equations (32)-(34). The similarity reduction corresponds to constant viscosity can be easy obtain if we put ($|\infty_e| \to \infty$), also the similarity reduction corresponds to the steady case can be obtain by putting the parameters *A* and A_1 (which appear in equations (31) and (33)) equal to zero. We have obtained the same result as in Chin et al. [20] in the steady case. Equations (32) and (33) with the boundary layer (34) have been solved numerically by using finite difference method (MATLAB package). Computations are carried out for various values of the viscosity/temperature parameter θ_e, the mixed convection parameter *?* and the *unsteady* steady parameter *A*. Both assisting (ε>0) and opposing (ε<0) flows are considered.

Table 1 shows the values of heat transfer coefficient, $-\theta'(0)$ for various values of the mixed convection parameter ? the viscosity/temperature parameter θ_e and the unsteady parameter *A*. The result calculated for $\varepsilon = 1,10$, $\theta_e = -8,-4,-1,1,4,8,\infty$ and $\lambda_2 = 0,0.5,1$. Also, in this table the comparisons of the present work with Chin et al. [20] results for unsteady case have been shown. It is found that the agreement between both results is excellent. Therefore, we believe that this comparison supports very well the validity of the present work. The results show that for each θ_e, when *?* or *A* increase the values of $-\theta'(0)$ increases, and the values of $-\theta'(0)$ in the unsteady case is greater than it in the steady case. Physically, this means the temperature of the plate is increased.

Figure 2 shows the temperature profiles for both the assisting and opposing flows $\varepsilon = -10,-5,0,5,10$, when $\theta_e = 1$ and $A = 1$. It can be seen that decreasing from $\varepsilon > 0$ (assisting flow) to $\varepsilon < 0$ (opposing flow) leads to an increase of the thermal boundary layer thickness.?

Figures 3 displays the effect of the unsteady parameter on the temperature in the case of assisting flow $\varepsilon = 1$ the mixed convection parameter $\theta_e = 1$. It is noticed that the thermal boundary layer thickness corresponds to the steady case ($A = 0$) is greater than the thermal boundary layer thicknesses correspond to unsteady cases. Also, it is observed that for increasing the unsteady parameter, the thermal boundary layer decrease.

Figure 4 depicts the effect of the mixed convection parameter *?* on the temperature for the constant viscosity ($\theta_e \to \infty$) when $A = 0$ (steady case) and $A = 1$ (unsteady case). The results are obtained for $\varepsilon = -1.5,-1.3,-1.2,0,1$. For the steady case we have obtained the same results as in Chin et al. 2007. It is noticed that the unsteady thermal boundary layer thickness is smaller than the steady thermal boundary layer thickness in both assisting and opposing flows. Also, in both steady and unsteady cases, the thermal boundary layer thickness is decreased with increasing the mixed convection parameter ? .

Figure 5 displays the variations of the heat transfer coefficient, $-\theta'(0)$ with ? , for various values of θ_e ($\theta_e = -8, -4, -1, \infty, 1, 4, 8$) in both steady ($A = 0$) and unsteady ($A = 1$) cases. The results for steady case is the same results in Chin et al. 2007. It is noticed that the value of heat transfer coefficient for each θ_e in the unsteady case is greater than it for steady case. This means the temperature difference between the plate and the fluid in unsteady case is higher than in steady case. The dash and solid bold two lines indicate the case of constant viscosity ($\theta_e \to \infty$) in comparison with other values of θ_e corresponding to variable viscosity. Also, the behavior of the results show that for fixed values of θ_e, values of $-\theta'(0)$ will increase as ? or A increases. Logically, fluid with higher velocity increases the heat transfer coefficient. This indicates that the temperature of the plate is increased, therefore, the temperature difference between the plate and the fluid will increase, which will lead to an increase of $-\theta'(0)$.

CONCLUTIONS

The similarity representation of the system of partial differential equations govern the effect of variable viscosity on *unsteady* mixed convection boundary layer flow over a vertical surface embedded in a porous medium are obtained using HAMAD method (see HAMAD [19]). Numerical solutions of the ordinary differential equations are also obtained. For steady case, we have obtained the same similarity system as in Chin et al. [20], also we have compared our present results for steady case with those of Chin et al. [20], and the agreement between both results is excellent. From the numerical results, it is observed that the flow and thermal characteristics are significantly influenced by the unsteady parameter. It is also noted that for fixed values of the temperature/ viscosity parameter, decreasing the mixed convection or the unsteady parameters lead to an increase of the thermal boundary layer thickness. Also, in the unsteady case the values of the heat transfer coefficient is greater than of it in steady case.

Table 1: Heat transfer coefficient, $-\theta'(0)$ for various values of ? ? e and the unsteady parameter *A* (*? = 0* for steady case as by Chin et al. 2007)

?	1				10			
	?				?			
? e	1	0.5	0	Chin et al. 2007	1	0.5	0	Chin et al. 2007
∞	1.9832	1.4990	1.0189	1.0191	2.9319	2.5294	2.1446	2.1445
-8	1.9959	1.5138	1.0370	1.0370	3.0062	2.6078	2.2269	2.2268
-4	2.0084	1.5283	1.0547	1.0546	3.0784	2.6836	2.3062	2.3061
-1	2.0812	1.6121	1.1543	1.1542	3.4729	3.0946	2.7323	2.7323
1	1.8779	1.3741	0.8605	0.8603	2.2157	1.7489	1.2908	1.2908
4	1.9577	1.4691	0.9821	0.9821	2.7755	2.3633	1.9691	1.9689
8	1.9706	1.4842	1.0006	1.0008	2.8551	2.4480	2.0589	2.0587

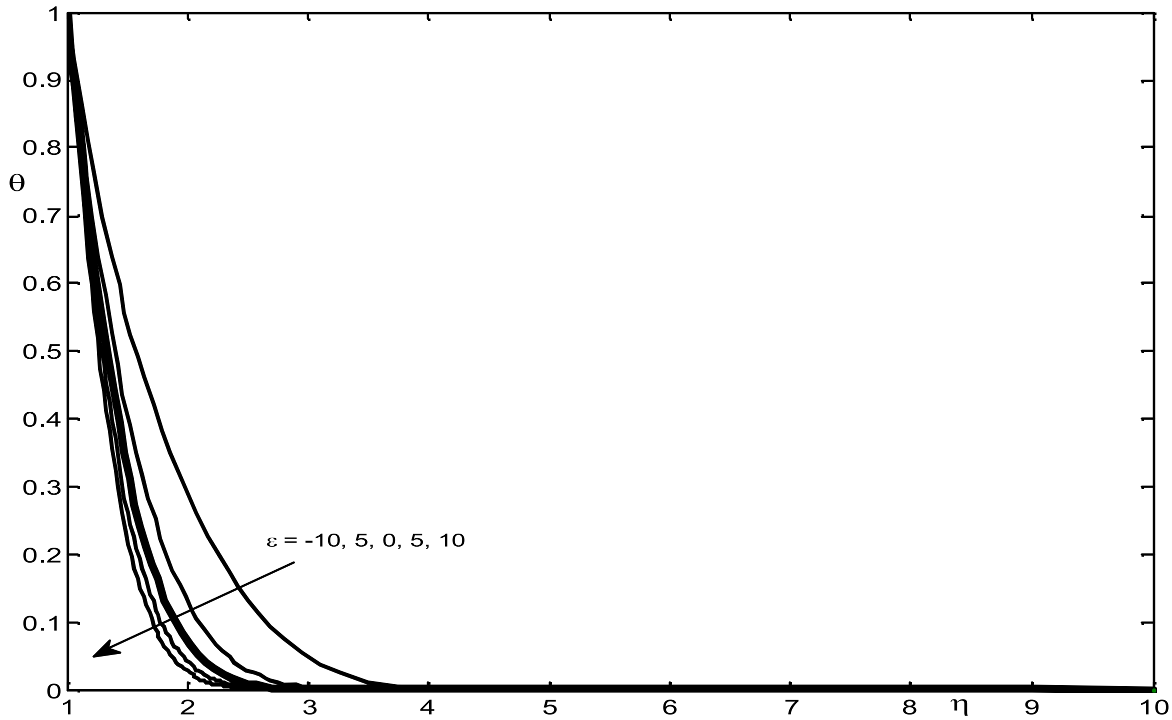

Fig. 2. Temperature profiles for various values of ε when $\theta_e = 1$ and $\lambda_2 = 1$

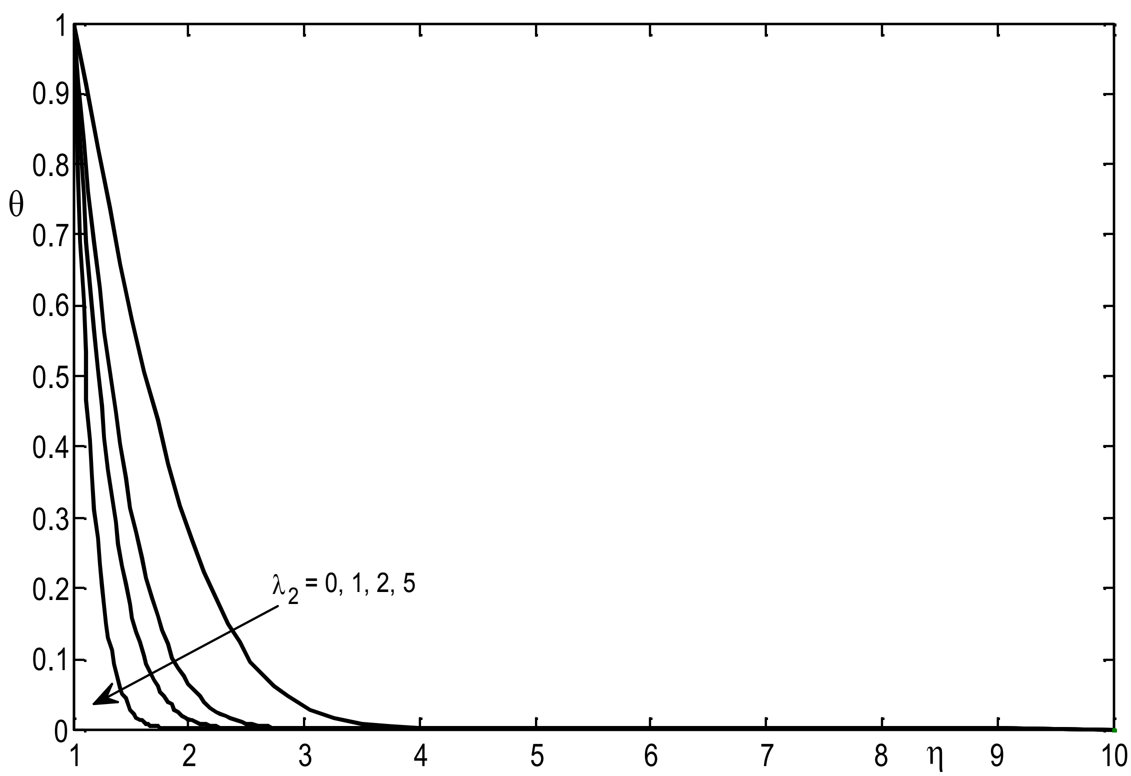

Fig. 3. Temperature profiles for various values of the time parameter λ_2 when $\theta_e = 1$ and $\varepsilon = 1$

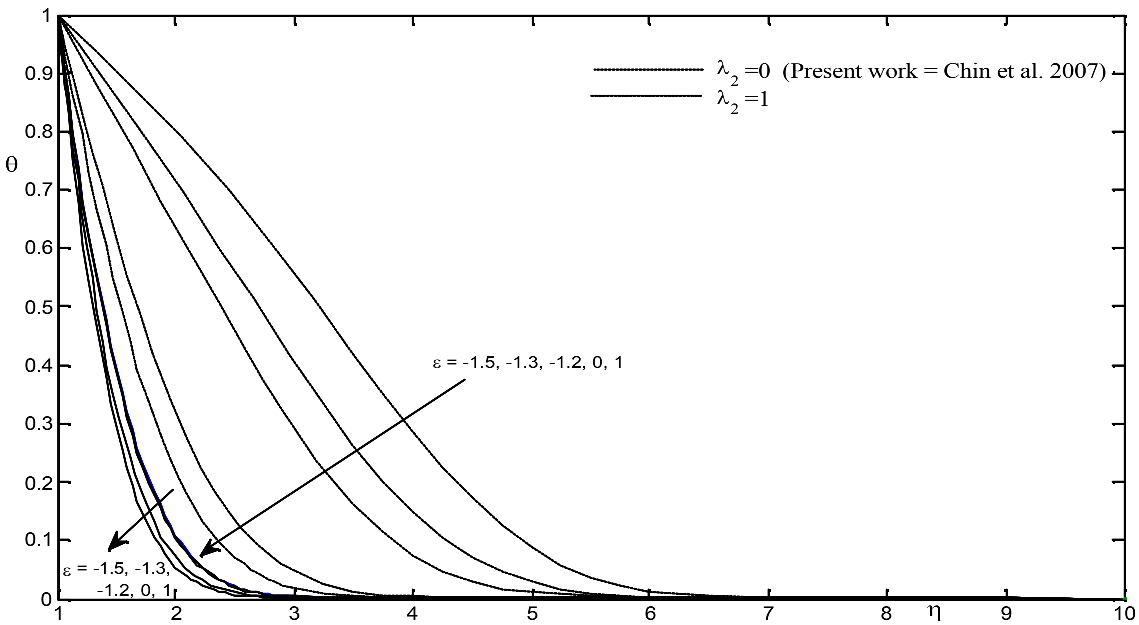

Fig. 4. Temperature profiles for various values of the time parameter ε when $\theta_e = \infty$ (constant viscosity) and λ_2 =0,1

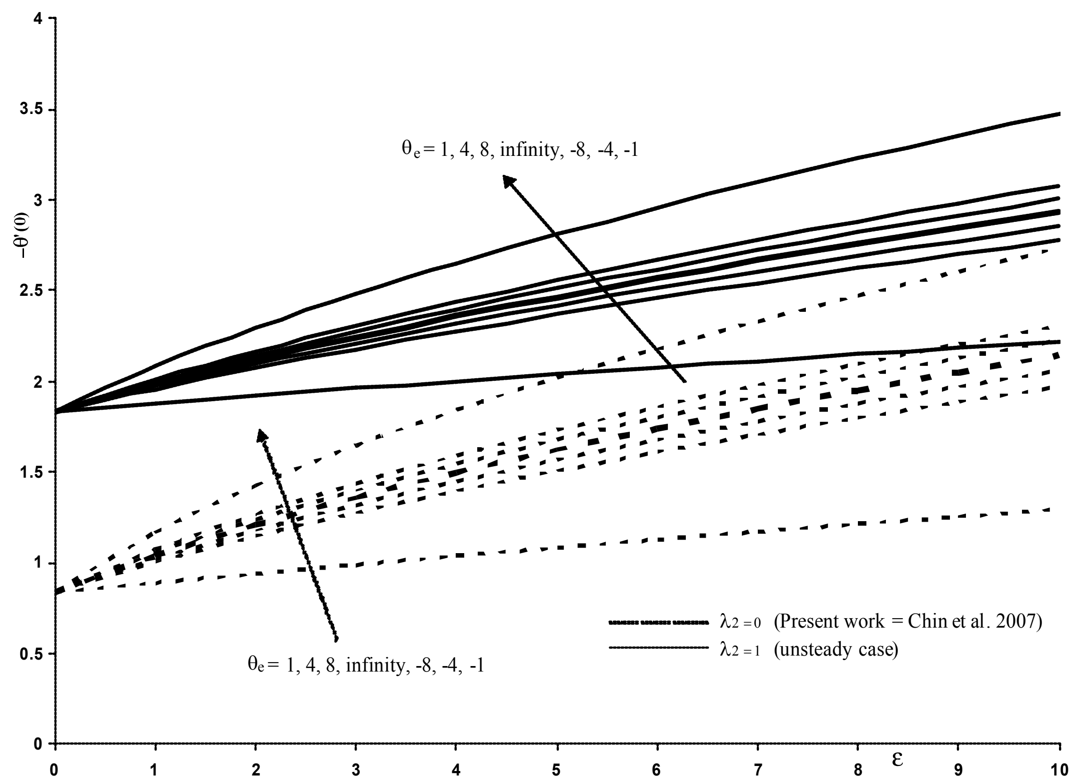

Fig. 5.Variation of the heat transfer coefficient, $-\theta'(0)$ with ε for various values of θ_e in both steady ($\lambda_2 = 0$) and unsteady ($\lambda_2 = 1$) cases.

REFERENCES

1. Nield, D.A. and A. Bejan, 2006. Convection in Porous Media (3rd Edn.). Springer, New York.
2. Vafai, K. 2005. Handbook of Porous Media (2nd Edn.). Taylor & Francis, New York.
3. Ingham, D.B. and I. Pop, 2005. Transport Phenomena in Porous Media III. Elsevier, Oxford.
4. Vadasz, P., 2008. Emerging Topics in Heat and Mass Transfer in Porous Media. Springer, New York.
5. Cheng, W.T. and C.H. Lin, 2007. Melting effect on mixed convective heat transfer with aiding and opposing external flows from the vertical plate in a liquidsaturated porous medium. Int. J. Heat Mass Transfer 50: 3026-3034.
6. Rashad, A.M., 2008. Influence of radiation on MHD free convection from a vertical flat plate embedded in porous medium with thermophoretic deposition particles. Commun Nonlinear Sci Numerical Simulation, 13: 2213-2222.
7. EL-Kabeir, S.M.M., M.A. EL-Hakiem and A.M. Rashad, 2007. Group method analysis for the effect of radiation on MHD coupled heat and mass transfer natural convection flow water vapor over a vertical cone 1through porous medium. Int. J. Appl. Math. Mech., 3: 35-53.
8. Ratcliff, J.T., G. Schubert and A. Zebib, 1996. Effects of temperature-dependent viscosity on thermal convection in a spherical shell. Physica D: Nonlinear Phenomena, 97: 242-252.
9. Hossain, M.A., S. Kabir and D.A.S. Rees, 2002. Natural convection of fluid with temperature dependent viscosity from heated vertical wavy surface. J. Appl. Math. Phys. (ZAMP), 53: 48-52.
10. Kim, M.C. and C.K. Choi, 2006. The onset of buoyancy-driven convection in fluid layers with temperature-dependent viscosity. Physics of the Earth and Planetary Interiors, 155: 42-47.
11. Soh, C.W. and E.W. Mureithi, 2006. Exact and numerical solutions of a fully developed generalized second-grade incompressible fluid with power-law temperature-dependent viscosity. Int. J. of Non-Linear Mechanics, 41: 271-280.
12. Cheng, C., 2006. The effect of temperature-dependent viscosity on the natural convection heat transfer from a horizontal isothermal cylinder of elliptic cross section. Int. Communications in Heat and Mass Transfer, 33: 1021-1028.
13. Pla, F., A.M. Mancho and H. Herrero, 2009. Bifurcation phenomena in a convection problem with temperature dependent viscosity at low aspect ratio. Physica D: Nonlinear Phenomena, 238: 572-580.
14. Ahmad, S., N.M. Arifin, R. Nazar and I. Pop, 2009. Mixed convection boundary layer flow past an isothermal horizontal circular cylinder with temperature-dependent viscosity. Int. J. of Thermal Sciences, 48: 1943-1948.
15. Hassanien, I.A. and M.A.A. Hamad, 2008. Group theoretic method for unsteady free convection flow of a micropolar fluid along a vertical plate in a thermally stratified medium. Applied Mathematical Modelling, 32: 1099-1114.
16. Rodriguez, I., J. Castro, C.D. Perez-Segarra and A. Oliva, 2009. Unsteady numerical simulation of the cooling process of vertical storage tanks under laminar natural convection. Int. J. of Thermal Sciences, 48: 708-721.
17. D'Alessio, S.J.D. and R.N. Perera, 2009. Unsteady free convection from elliptic cylinders at large Grashof numbers. Int. J. of Heat and Mass Transfer, 52: 5940-5953.
18. Hmouda, I., I. Rodriguez, C. Bouden and A. Oliva, 2010. Unsteady natural convection cooling of a water storage tank with an internal gas flue. Int. J. of Thermal Sciences, 49: 36-47.
19. Hamad, M.A.A., 2010. Formulations: General formulations for Exact and Similarity transformations of ODEs and PDEs. Middle-East J. of Scientific Research (online 2010).
20. Chen, K.E., R. Nazar, N.M. Arifin and I. Pop, 2007. Effect of variable viscosity on unsteady mixed convection boundary layer flow over a vertical surface embedded in a porous medium. Int. Comm. In Heat and Mass Transfer, 34: 464-473.
21. Moran, M.J. and R.A. Gaggioli, 1966. A new systematic formalism for similarity analysis with application to boundary-layer flows. US Army Math. Research Center, Tech. Summary Report No. 918.
22. Merkin, J.H., 1980. Mixed convection boundary layer flow on a vertical surface in a saturated porous medium. J. Eng. Math., 14: 301-313.
23. Aly, E.H., L. Elliott and D.B. Ingham, 2003. Mixed convection boundary-layer flow over a vertical surface embedded in a porous medium. Eur. J. Mech.-Fluids, 22: 529-543.