# Control of Active Suspension System: An Interval Type-2 Fuzzy Approach

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**Abstract:** In this paper, we introduce a novel interval type-2 Fuzzy Logic System (FLS) for active automobile suspension control in presence of system unmodeling, measurement noises and external disturbances. In order to efficient reduction of noise effects and improving the reliability of suspension system, a fuzzy controller is designed. Interval type-2 FLS is used to improve the disturbance rejection of suspension system which can be caused by road shocks. Therefore, better control performance and car passengers comfort is achieved, in comparison with previous type-1 fuzzy controllers. In addition, it is shown that the proposed controller provide better responses even in comparison with similar type-2 fuzzy controllers. The designing procedure of proposed interval type-2 fuzzy controller is presented in detail and simulation examples are presented to verify the effectiveness of the method. Results are compared with some other methods proposed in the past research.

**Key words:** Active suspension system . interval type-2 fuzzy control . passengers comfort

## INTRODUCTION

Automotive companies are competing to make more developed cars, while comfort of passengers is an important demand and everyone expects from industries to improve it day by day. Therefore, in order to provide a smooth ride and satisfy passengers comfort, designing a modern suspension system is mandatory.

A good and efficient suspension system must rapidly absorb road shocks and then return to its normal position, slowly. However, in a passive suspension system with a soft spring, movements will be high, while using hard springs causes hard moves due to road roughness [1]. Therefore, it's difficult to achieve good performance with a passive suspension system.

In order to improve passengers comfort, motion sickness [2] and head toss [3] must get minimized. Motion sickness is caused by acceleration in a direction perpendicular to the body longitudinal axis, while a sudden roll motion causes head toss. Such sudden motions can be produced when a car wheel drives into a deep hole.

In the last two decades, significant advances have been made in the mechanical suspension systems control theory [4-7]. One of the efficient traditional suspension systems is passive hydraulic ones, which uses high-pressure hydraulics to prestress the springs. However, in this kind of suspension systems, sufficient control performance needs the exact model of the system to be known.

Recently, to satisfy demands for reducing motion sickness, head toss and overcome the restrictive assumption of knowing the exact model; many control schemes based on intelligent technologies, such as active fuzzy logic systems [8-11] have been developed.

Since Zadeh [12] initiated the fuzzy set theory, Fuzzy Logic Control (FLC) schemes have been widely developed and successfully applied to many real world applications [13]. Besides, FLC schemes have been used to control suspension systems. For example, Salem and Aly [14], designed a quarter-car system on the basis of the concept of a four-wheel independent suspension system. They proposed a fuzzy control for active suspension system to improve the ride comfort. Hyniova *et al.* [15] used an FLS to control active hydropneumatic suspension system.

In recent years it is shown that, type-1 fuzzy systems have difficulties in modeling and minimizing the effect of rule and data uncertainties [16-22]. One reason is that a type-1 fuzzy set is certain in the sense that the membership grade for a particular input is a crisp value.

The type-2 fuzzy sets which are characterized by Membership Functions (MF) that are themselves fuzzy was first introduced by Zadeh [23] and has been attracting many interests [16-22]. For such type-2 sets, each input has unity secondary membership grade defined by two type-1 MF, upper MF and lower MF. Recently, type-2 fuzzy sets have been successfully applied on different applications as type-2 fuzzy neural

network [24], image processing [25], embedding intelligent agents [26], pattern recognition [27, 28] mobile robots control [29] and fuzzy controller designs [30, 31]. For example, Cao *et al.* [32] proposed an adaptive fuzzy logic controller based on an interval type-2 fuzzy logic system for vehicle non-linear active suspension systems.

In this article, a novel fuzzy controller is proposed for an active suspension system. Interval type-2 fuzzy rules are designed in order to improve system robustness to noise measurement and external disturbances, in presence of system unmodeling.

The rest of this paper is organized as follows: Problem formulation and type-1 fuzzy method are presented in Section 2. In Section 3, a brief introduction on Interval type-2 fuzzy systems is reviewed. Section 4 introduces the proposed control scheme. Simulation results are included in Section 5. Finally, Section 6 provides the concluding remarks.

# PROBLEM FORMULATION AND TRADITIONAL CONTROL METHODS

**System description:** Let us consider a quarter car suspension model with two degrees of freedom as shown in Fig. 1.

The motion equations of wheel and car body can be written as:

$$\begin{cases} m_{b}\ddot{z}_{b} = f_{a} - k_{f}(z_{b} - z_{w}) - c_{s}(\dot{z}_{b} - \dot{z}_{w}) \\ m_{w}\ddot{z}_{w} = -f_{a} + k_{f}(z_{b} - z_{w}) - k_{f}(z_{w} - z_{r}) \end{cases}$$
(1)

where parameters used in (1) are defined as following:

m<sub>b</sub>: Sprung mass (body mass)

mw: Unsprung mass

k<sub>1</sub>: Spring stiffness coefficient

k<sub>2</sub>: Tire stiffness coefficient

 $f_a$ : Active force

c<sub>s</sub>: Damping coefficient

z<sub>r</sub>: Displacement of road

z<sub>b</sub>: Displacement of the car body

z<sub>w</sub>: Displacement of wheel

Spring stiffness coefficient  $(k_1)$  and the tire stiffness coefficient  $(k_2)$  are considered to be linear in operation range. Displacement of the car body  $(z_b)$  and displacement of wheel  $(z_w)$  are measured from the static equilibrium point, while tires are assumed not to leave the ground. Quarter car model can be transformed to a state space model, with choosing the following state variable equations:

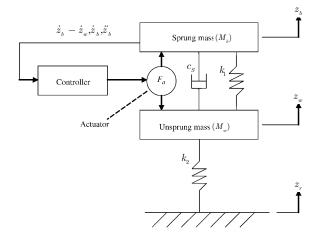


Fig. 1: Suspension system block diagram

$$\begin{cases} \mathbf{x}_1 = \mathbf{z}_b - \mathbf{z}_w \\ \mathbf{x}_2 = \mathbf{z}_w - \mathbf{z}_r \\ \mathbf{x}_3 = \dot{\mathbf{z}}_b \\ \mathbf{x}_4 = \dot{\mathbf{z}}_w \end{cases}$$
 (2)

where  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$  are suspension deflection, tire deflection, vertical car body velocity and the vertical wheel velocity, respectively.

Eq. (2) can be rewritten as

$$\begin{cases} \dot{x}_{1} = x_{3} - x_{4} \\ \dot{x}_{2} = x_{4} - \dot{z}_{r} \\ \dot{x}_{3} = \frac{1}{m_{b}} \left( -k_{1}x_{1} - c_{s}(x_{3} - x_{4}) + f_{a} \right) \\ \dot{x}_{4} = \frac{1}{m_{w}} \left( k_{1}x_{1} + c_{s}(x_{3} - x_{4}) - k_{2}x_{2} - f_{a} \right) \end{cases}$$
(3)

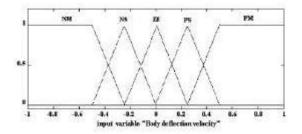
and  $f_a$  is considered as the control input.

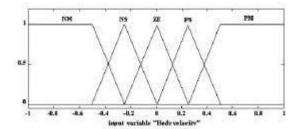
Then the quarter-car dynamics (1)-(3) can be written as:

$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \\ \dot{x}_{3} \\ \dot{x}_{4} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_{1}}{m_{b}} & 0 & -\frac{c_{s}}{m_{b}} & \frac{c_{s}}{m_{b}} \\ \frac{k_{1}}{m_{w}} & -\frac{k_{2}}{m_{w}} & \frac{c_{s}}{m_{w}} & -\frac{c_{s}}{m_{w}} \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \end{bmatrix}$$

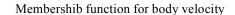
$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_{b}} \\ -\frac{1}{m_{w}} \end{bmatrix} f_{a} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \dot{z}_{r}, \ y_{p} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \end{bmatrix}$$

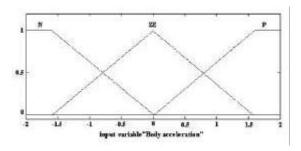
$$(4)$$

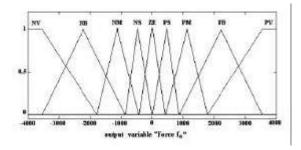




Membership function for body deflection velocity







Membership function for body acceleration

Membership function for desired actuator force

Fig. 2: Membership functions for the four mentioned variables

Table 1: Abbreviations

NV	Negative very big	ZE	Zero
NB	Negative big	PS	Positive small
NM	Negative medium	PM	Positive medium
NS	Negative small	PB	Positive big

where  $y_p$  is car body velocity.

In next subsection a traditional fuzzy method used for active suspension system control [15] is presented.

**Type-1 fuzzy method:** The type-1 fuzzy logic controller used in the active suspension system of [15], has three inputs: body acceleration  $(\ddot{z}_b)$ , body velocity  $(\dot{z}_b)$  and body deflection velocity  $(\dot{z}_b - \dot{z}_w)$ . The desired actuator force  $(f_a)$  is the model output.

Membership fuzzy sets used for representing the aforementioned variables of the active suspension system are illustrated in Fig. 2, where the abbreviations used for describing fuzzy sets are introduced in Table 1.

The fuzzy system is designed in order to minimize the vertical car velocity  $(z_b)$  in response to the road displacement  $(z_r)$ . Therefore acceleration, welocity and deflection velocity of the car is used to conclude the desired suspension system control input  $(f_a)$ . The rule base is represented in the Table 2.

Although, proposed method in [15], can control the active suspension system, in presence of disturbances

and unmodelings, this controller has difficulties in modeling and minimizing the effects of uncertainties. Therefore, in order, to design a high-performance fuzzy control scheme, an interval type-2 fuzzy logic system will be introduced and implemented in next sections.

# INTERVAL TYPE-2 FUZZY LOGIC SYSTEMS (IT2FLS)

Fuzzy Logic Systems (FLS) are known as the universal-approximators and have various applications in identification and control designs. A type-1 fuzzy system consists of four major parts: fuzzifier, rule base, inference engine and defuzzifier. A type-2 fuzzy system has a similar structure, but one of the major differences can be seen in the rule base part, where a type-2 rule base has antecedents and consequents using Type-2 Fuzzy Sets (T2FS). In a T2FS, we consider a Gaussian function with a known standard deviation, while the mean (m) varies between m1 and m2. Therefore, a uniform weighting is assumed to represent a footprint of uncertainty as shaded in Fig. 3. Because of using such a uniform weighting, we name the T2FS as an Interval Type-2 Fuzzy Set (IT2FS).

Utilizing a rule base which consists of IT2FSs, the output of the inference engine will also be a T2FS and hence we need a type-reducer to convert it to a type-1 fuzzy set before defuzzification can be carried out. Figure 4 shows the main structure of type-2 FLS.

$\dot{z}_{_b} - \dot{z}_{_w}$	ż	ïZ <sub>b</sub>	f,	$\dot{z}_{_b} - \dot{z}_{_w}$	$\dot{z}_{_{b}}$	$\ddot{z}_{_{b}}$	f <sub>a</sub>	$\dot{z}_{_b} - \dot{z}_{_w}$	$\dot{Z}_{_{b}}$	ï <sub>b</sub>	f,	$\dot{z}_{_b} - \dot{z}_{_w}$	$\dot{z}_{_{b}}$	ï <sub>b</sub>	f <sub>a</sub>
PM	PM	ZE	ZE	PS	NS	ZE	PM	PS	NS	P or N	PB	PM	PM	P or N	NS
PS	PM	ZE	NS	ZE	NS	ZE	PS	ZE	NS	P or N	PM	PS	PM	P or N	NM
ZE	PM	ZE	NM	NS	NS	ZE	PS	NS	NS	P or N	PM	ZE	PM	P or N	NB
NS	PM	ZE	NM	NM	NS	ZE	ZE	NM	NS	P or N	PS	NS	PM	P or N	NB
NM	PM	ZE	NB	PM	NM	ZE	PB	PM	NM	P or N	PV	NM	PM	P or N	NV
PM	PS	ZE	ZE	PS	NM	ZE	PM	PS	NM	P or N	PB	PM	PS	P or N	NS
PS	PS	ZE	NS	ZE	NM	ZE	PM	ZE	NM	P or N	PB	PS	PS	P or N	NM
ZE	PS	ZE	NS	NS	NM	ZE	PS	NS	NM	P or N	PM	ZE	PS	P or N	NM
NS	PS	ZE	NM	NM	NM	ZE	ZE	NM	NM	P or N	PS	NS	PS	P or N	NB
NM	PS	ZE	NM	ZE	ZE	ZE	ZE	ZE	ZE	P or A	ZE	NM	PS	P or N	NB
PM	ZE	ZE	PS	NS	ZE	ZE	ZE	NS	ZE	P or N	NS	PM	ZE	P or N	PM
PS	ZE	ZE	ZE	NM	ZE	ZE	NS	NM	ZE	P or N	NM	PS	ZE	P or N	PS
PM	NS	ZE	PM					PM	NS	P or N	PB				

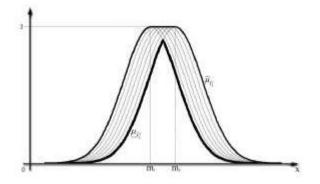


Fig. 3: Interval type 2 fuzzy set (IT2FS)

By using singleton fuzzification, the singleton inputs are fed into the inference engine. Combining the fuzzy if-then rules, the inference engine maps the singleton input  $x = [x_1, x_2,...x_3]$  into a type-2 fuzzy set as the output. A typical form of an if-then rule can be written as:

$$R_i = if x_i is \tilde{F}_1^i$$
 and  $x_i is \tilde{F}_2^i$  and  $\cdots$  and  $x_n is \tilde{F}_n^i$  then  $\tilde{G}^i$  (5)

where  $\tilde{F}_k^i$  s are the antecedents (k = 1, 2, ..., n) and  $\tilde{G}^i$  is the consequent of the ith rule. We use sup-star method as one of the various inference methods. The first step is to evaluate the firing set for ith rule as following:

$$F^{i}(\underline{x}) = \prod_{k=1}^{n} \mu_{\tilde{F}_{k}^{i}}(x_{k})$$
 (6)

As all of the  $\tilde{F}_k^i$  s are IT2FSs, so  $\tilde{F}(\underline{x})$  can be written as  $F^i(\underline{x}) = [\underline{f}^i(\underline{x})]$  where:

$$\underline{\mathbf{f}}^{i}(\underline{\mathbf{x}}) = \prod_{k=1}^{n} \underline{\boldsymbol{\mu}}_{\tilde{\mathbf{f}}_{k}^{i}}(\mathbf{x}_{k}) \tag{7}$$

$$\overline{f}^{i}(\underline{x}) = \prod_{k=1}^{n} \overline{\mu}_{\tilde{F}_{k}^{i}}(x_{k})$$
 (8)

The terms  $\underline{\mu}_{F_k}$  and  $\overline{\mu}_{F_k}$  are the lower and upper membership functions, respectively (Fig. 3).

In the next step, the firing set  $F(\underline{x})$  is combined with the *ith* consequent using the product t-norm to produce the type-2 output fuzzy set.

The type-2 output fuzzy sets are then fed into the type reduction part. The structure of type reducing part is combined with the defuzzification procedure, which uses Center of Sets (COS) method. First, the left and right centroids of each rule consequent is computed using Karnik-Mendel (KM) algorithm [33] as shown in Fig. 5. Let's call them y<sub>1</sub> and y<sub>1</sub> respectively.

The firing sets  $F^i(\underline{x}) = [\underline{f}^i(\underline{x}) \ \overline{f}^i(\underline{x})]$  computed in the inference engine are combined with the left and right centroid of consequents and then the defuzzified output is evaluated by finding the solutions of following optimization problems:

$$y_{l}(\underline{x}) = \min_{\forall f^{k} \in \left[\underline{f}^{k} \, \overline{f}^{k}\right]} \left( \sum_{k=1}^{M} y_{l}^{k} \, f^{k}(\underline{x}) \middle/ \sum_{k=1}^{M} f^{k}(\underline{x}) \right)$$
(9)

$$y_{r}(\underline{x}) = \max_{\forall r^{k} \in [\underline{f}^{k}, \overline{f}^{k}]} \left( \sum_{k=1}^{M} y_{r}^{k} f^{k}(\underline{x}) / \sum_{k=1}^{M} f^{k}(\underline{x}) \right)$$
(10)

define  $f_1^k(\underline{x})$  and  $f_r^k(\underline{x})$  as the functions which are used to solve (9) and (10) respectively and let

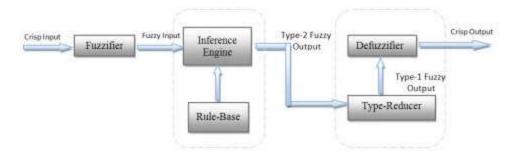


Fig. 4: Main structure of type-2 FLS

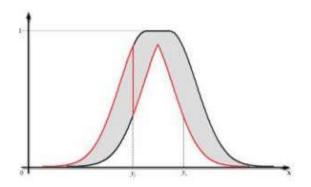


Fig. 5: Computing right and left centroids for an IT2FS

$$\xi_1^k(\underline{x}) = f_1^k(\underline{x}) / \sum_{k=1}^M f_1^k(\underline{x})$$

and

$$\xi_r^k(\underline{x}) = f_r^k(\underline{x}) / \sum_{k=1}^M f_r^k(\underline{x})$$

then we can rewrite (9) and (10) as:

$$y_l(\underline{x}) = \sum_{k=l}^M y_l^k f_l^k(\underline{x}) / \sum_{k=l}^M f_l^k(\underline{x}) = \sum_{k=l}^M y_l^k \xi_l^k(\underline{x}) = \theta^T \xi_l(\underline{x}) \quad (11)$$

$$y_r(\underline{x}) = \sum_{k=1}^M y_r^k f_r^k(\underline{x}) / \sum_{k=1}^M f_r^k(\underline{x}) = \sum_{k=1}^M y_r^k \xi_r^k(\underline{x}) = \theta_r^T \xi_r(\underline{x}) \quad (12)$$

Where

$$\boldsymbol{\xi}_1(\underline{\boldsymbol{x}}) = \left[ \boldsymbol{\xi}_l^1(\underline{\boldsymbol{x}}) \; \boldsymbol{\xi}_l^2(\underline{\boldsymbol{x}}) \cdots \boldsymbol{\xi}_l^M(\underline{\boldsymbol{x}}) \right]$$

and

$$\xi_{r}(\underline{x}) = \left[\xi_{r}^{1}(\underline{x}) \xi_{r}^{2}(\underline{x}) \cdots \xi_{r}^{M}(\underline{x})\right]$$

are the fuzzy basis functions and

$$\boldsymbol{\theta}_{l}\left(\underline{\boldsymbol{x}}\right) = \left[y_{l}^{l}\!\left(\underline{\boldsymbol{x}}\right)y_{l}^{2}\!\left(\underline{\boldsymbol{x}}\right)\cdots y_{l}^{M}\!\left(\underline{\boldsymbol{x}}\right)\right]$$

and

$$\theta_{r}(\underline{\mathbf{x}}) = \left[ y_{r}^{1}(\underline{\mathbf{x}}) y_{r}^{2}(\underline{\mathbf{x}}) \cdots y_{r}^{M}(\underline{\mathbf{x}}) \right]$$

are the adjustable parameters.

Finally, the crisp value is obtained by the defuzzificatin procedure as follows:

$$y(\underline{\mathbf{x}}) = \frac{1}{2} \left( y_{\mathbf{f}}(\underline{\mathbf{x}}) + y_{\mathbf{f}}(\mathbf{x}) \right) = \frac{1}{2} \left( \theta_{1}^{\mathsf{T}} \xi_{1}(\underline{\mathbf{x}}) + \theta_{r}^{\mathsf{T}} \xi_{r}(\underline{\mathbf{x}}) \right) = \frac{1}{2} \theta^{\mathsf{T}} \xi(\underline{\mathbf{x}}) \quad (13)$$

where 
$$\theta = \left[\theta_l^T \theta_r^T\right]^T$$
 and  $\xi = \left[\xi_l^T \xi_l^T\right]^T$ .

#### CONTROLLER DESIGN

By using results of Section 0, the type-1 fuzzy method introduced in Subsection 2.b will be developed in order to propose a more reliable and high performance controller.

First to design an Interval Type-2 Fuzzy Controller (IT2FC), for quarter car suspension system (4), three crisp inputs (i.e. body acceleration  $(\ddot{z}_b)$ , velocity of car body  $(\dot{z}_b)$ , suspension deflection velocity  $(\dot{z}_b - \dot{z}_w)$  and one fuzzy output (i.e. actuator force  $(f_a)$ ) will be introduced.

The working procedure of proposed IT2FC is as following:

- In the fuzzification stage the crisp inputs are transformed to type-2 fuzzy sets.
- These sets are processed by inference machine in relation with type-2 rule base. Result of this procedure is a group of type-2 fuzzy output sets.
- Difuzzifacation stage integrated with type reducer block convert type-2 fuzzy output sets to crisp output values that are fed into actuators.

This procedure is illustrated in block diagram of Fig. 4.

As mentioned before in active suspension system we have four variables, Membership Functions (MFs) of these variables are chosen as interval type-2 Gaussian functions. These MFs are shown in Fig. 6.

In above pictures all abbreviations are based on Table 1. It should be mentioned that the interval type-2

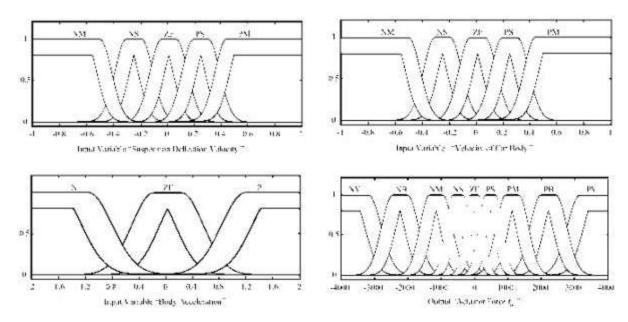


Fig. 6: Input/Output membership functions

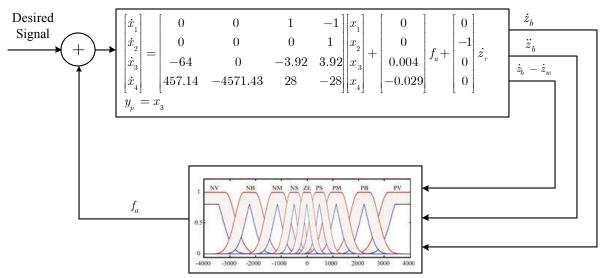


Fig. 7: IT2FC block diagram

fuzzy sets are designed in order to minimize the effect of rule and data uncertainties. In addition, the rule base which is used in IT2FC design is similar to the rule base proposed in Table 2 [15]. The table has two parts; in the left part, body acceleration is zero so the main goal of the control problem is to minimize the relative and the absolute body velocities. However, in the right part, as the body acceleration is nonzero, minimizing the body acceleration is also included in the goal.

Our suspension system needs a crisp control value so defuzzification stage is necessary; we use "center of gravity method" to defuzzify our type-2 fuzzy output sets, by using this method the output  $f_a$  is computed using equations (11), (12) and (13).

The overall block diagram, of the proposed IT2FC method is shown in Fig. 7.

# SIMULATION RESULTS

In this section, we evaluate the performance of our IT2FC scheme by applying the method for regulation of an automobile suspension system. In the first experiment, we give the simulation results of our proposed IT2FC in a regulation problem and the superiority of the proposed controller is shown in comparison with the type-1 fuzzy controller of [15]. In the second experiment, the performance of the schemes is compared with the method in [32],

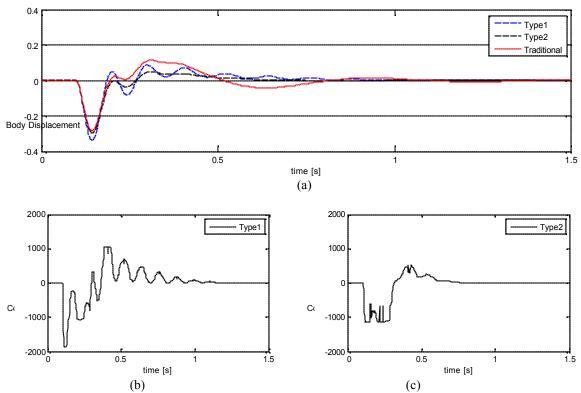


Fig. 8: Simulation results for suspension system control (a) Body displacement (b) Type-1 control signal (c) Type-2 control signal

which is designed based on interval type-2 fuzzy logic systems.

All the simulation procedures are implemented within MATLAB with the step size of 0.001.

In the following experiments, the model used for representing suspension system is the mathematical model represented in (4). All parameter values are chosen based on the values in Table 3

By substituting the aforementioned values in the model represented in (4) the numerical values of model matrices are calculated and the model will be

$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \\ \dot{x}_{3} \\ \dot{x}_{4} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -64 & 0 & -3.92 & 3.92 \\ 457.14 & -4571.43 & 28 & -28 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0.004 \\ -0.029 \end{bmatrix} x_{5} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \dot{z}_{r}$$

$$y_{p} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \end{bmatrix}$$

$$(14)$$

Table 3: Suspension system parameter specifaction

m <sub>b</sub>	Sprung mass (body mass)	250kg
$m_{\rm w}$	Unsprung mass	35kg
$\mathbf{k}_1$	Spring stiffness coefficient	16 000 N/m
$\mathbf{k}_{2}$	Tire stiffness coefficient	160 000 N/m
$c_{\rm s}$	Damping coefficient	980 Ns/m

Moreover, all fuzzy logic systems use singleton fuzzification, center average defuzzification, Mamdani implication in the rule base and product inference engine to achieve the output of the fuzzy system [34]. In addition the type-2 reduction is applied to the interval type-2 fuzzy sets based on method proposed in [33].

Comparison of proposed type-2 controller with the type-1 controller [15]: Consider the active suspension model as in (14). To give a solution for the fuzzy control of the car suspension system, measured data from body acceleration  $(\ddot{z}_b)$ , velocity of car body  $(\dot{z}_b)$  and suspension deflection velocity  $(\dot{z}_b - \dot{z}_w)$  are chosen as the fuzzy system inputs. All the interval type-2 membership functions used for inputs and the output (actuator force  $(f_a)$ ) are chosen as illustrated in Fig. 6.

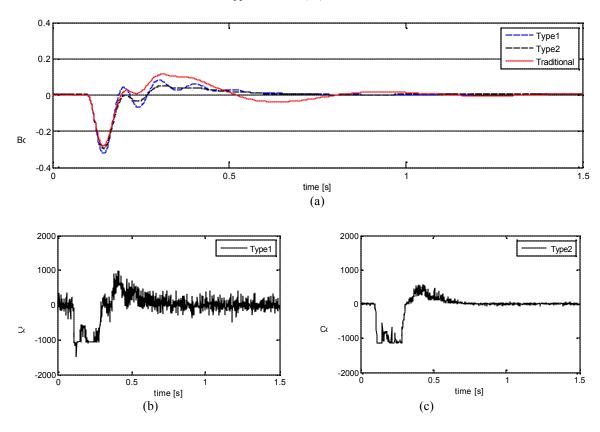


Fig. 9: Simulation results for suspension system control in presence of 25db noise (a) Body displacement (b) Type-1 control signal (c) Type-2 control signal

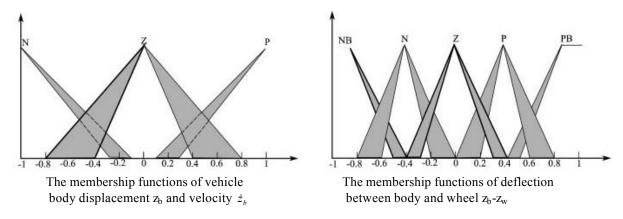


Fig. 10: The membership functions of three inputs variables of [32]

Results for proposed control scheme are illustrated in Fig. 8. The same figure shows the comparisons with the type-1 fuzzy controller [15]. Figure 8(a) shows that the proposed type-2 controller provides much smoother body displacements which can help to achieve more passenger comfort. In addition Fig. 8(b-c) shows that the proposed controller produces control signal with less oscillations, which can conclude to mechanical longevity of suspension system.

Comparison of proposed type-2 controller with the type-1 controller (in presence of noise): In this experiment the robustness of proposed controller in presence of measurement noise will be shown. Therefore a white Gaussian noise with SNR=25db is applied to all the measured data. All of the model characteristics and controller designs are the same as previous example. Figure 9 shows the disturbance rejection behavior of suspension control system. Figure 9(a) shows that the body displacement is very

Table 4: The rule-base of type-2 fuzzy controller [32]

Z <sub>b</sub>	$\dot{z}_{_b}$	$\Delta z$	f <sub>a</sub>	Z <sub>b</sub>	ż	Δz	f <sub>a</sub>	Z <sub>b</sub>	$\dot{z}_{_b}$	Δz	f <sub>a</sub>
NB	NB	N	PV	P	NB	-	ZE	NB	Z	P	PS
NB	N	N	PB	P	N	-	NS	NB	P	Z	ZE
NB	Z	N	PM	P	Z	-	NM	NB	P	P	NS
NB	P	N	PS	P	P	-	NB	NB	PB	Z	NS
NB	PB	N	ZE	P	PB	-	NV	NB	PB	P	NM
N	NB	-	PV	PB	NB	N	PS	PB	NB	Z	ZE
N	N	-	PM	PB	N	N	ZE	PB	NB	P	ZE
N	Z	-	PM	PB	Z	N	NS	PB	N	Z	NS
N	P	-	PS	PB	P	N	NM	PB	N	P	NS
N	PB	-	NS	PB	PB	N	NB	PB	Z	Z	NB
Z	NB	-	PM	NB	NB	Z	PB	PB	Z	P	NB
Z	N	-	PS	NB	NB	P	PM	PB	P	Z	NV
Z	Z	-	ZE	NB	N	Z	PM	PB	P	P	NV
Z	P	-	NS	NB	N	P	PM	PB	PB	Z	NV
Z	PB	-	NM	NB	Z	Z	PS	PB	PB	P	NV

Table 5: Quantitative comparisons of different methods

1	Type-1 fuzzy	controller [15]	Type-2 propose	ed controller	Type-2 controller in [32]			
	No noise	25db noise	No noise	25db noise	No noise			
IAE	0.0313	0.0316	0.0265	0.0269	0.0277			
IAU	372.7300	400.0000	277.7000	294.1200	337.7600			

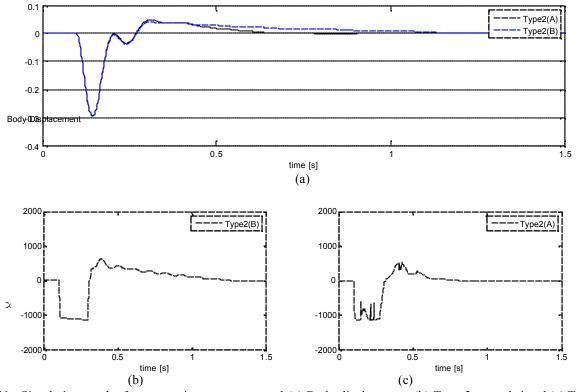


Fig. 11: Simulation results for suspension system control (a) Body displacement (b) Type-2 control signal (c) Type-2 control signal [32]

similar to the one illustrated in last experiment, while Fig. 9 (b-c) concludes that proposed controller is much robust to measurement noises and less noise effect can be seen in its control signal.

Comparison of proposed type-2 controller with the type-2 controller in [32] (Without noise): In this experiment, the proposed IT2FC is compared with controller proposed in [32]. The Interval type-2 method proposed in [32], uses three crisp inputs (i.e. displacement of the car  $(z_b)$  body, vertical velocity of car body  $\ddot{z}_b$ , suspension deflection ( $\Delta z = z_b - z_w$ ) and one fuzzy output (i.e. actuator force  $f_a$ ). Membership Functions (MFs) for variables in [32] are chosen as precise triangles (Fig. 10).

The rule-base of type-2 fuzzy controller proposed in [32] is shown in Table 4.

In this table all abbreviations are the same as Table 1. The rest of simulation conditions are exactly chosen based on previous experiments and the results are shown in Fig. 11. Less settling time of proposed method is shown in Sub Fig. 11(a) and Sub Fig. 11(b-c) illustrates that both control signals are very similar. Next subsection present quantitative comparisons for control efforts and error integrals.

Quantitative comparisons of different methods: In order to have a quantitative comparison between different methods Integral of Absolute Error (IAE) and Integral of Absolute U (IAU), where U is the control signal; are selected as the performance criteria. All quantitative results are shown in (Table 5).

Based on Subsections 5.a and 5.b; I's indicated that although IAE and IAU in no-noise conditions are similar, in presence of noise with SNR = 25db, the proposed IT2FC method controls the suspension system more effectively. In addition, based on results of Subsection 5.c, it is demonstrated that the control effort of proposed controller is less than control effort of control design proposed in [32].

## **CONCLUSION**

In this paper, the regulation problem for a quarter active mechanical suspension system was investigated in presence of system unmodeling, external disturbance and noise. An Interval type-2 fuzzy logic system was developed to improve noise reduction and reliability of suspension system. All the theoretical results were verified by numerical simulations to demonstrate the effectiveness of the proposed control scheme.

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