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Solving Fuzzy Linear Regression by Huan's Algorithm

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Abstract: In this paper, the estimation of Fuzzy Linear Regression (FLR) is computed by using least-square approach. For using this approach, a solution of Fuzzy Linear System (FLS) is required. This solution is computed by applying an iterative method (Huang's Algorithm) which this alternative method is powerful to compute the solutions without using the inverse of coefficient matrix. Experimental results are then presented which indicate the performance of this algorithm.

Key words: Fuzzy linear system . iterative methods . linear regression . least square . triangular fuzzy number

INTRODUCTION

In last years, the applications of linear regression in different fields such as economic, engineering and social science have been studied. There are many regression techniques and formulations based on regression parameters in which some of them have simple structures and some complicated models can be found. Unfortunately, in most of these patterns, unrealistic approaches may be occurred because of the design of model, estimation of parameters and some other factors such that the usual regression methods are not fitting with the natural sources of problems. Thus the concept of fuzzy regression analysis was introduced by Tanaka *et al*. [20] in 1982, where an LP-based method with symmetric triangular fuzzy parameters was proposed. Later on, several fuzzy regression approaches have been proposed, including the mathematical programming based methods [16, 17, 20], least-squares based algorithms [2, 4] and other methods [3, 11, 12, 14, 15].

The basic concept of fuzzy theory of fuzzy regression is that the residuals between estimators and observations are not produced by measurement errors, but rather by the parameter uncertainty in the model and the possibility distribution is used to deal with real observations. This method provides the means by which the goodness of a relationship between two variables, y and x, may be evaluated on the basis of a small sample size. In this approach, the regression coefficients are assumed to be fuzzy number [18].

In this paper, the estimation of Fuzzy Linear Regression (FLR) based on least-square approach is computed in which the unrealistic approaches are considered. This method has some more benefits that will be mentioned later. This method explicitly does not compute the inverse of any square matrix. Thus, this method is suggested to apply for computing a meaningful estimation of a fuzzy linear regression. In chapter 2, the basic notations and definitions have been discussed. The general structure of fuzzy linear regression has been argued in the next chapter and later on, the Huang's algorithm for solving fuzzy linear system of equations is explained in chapter 4. Numerical tests and conclusion are two last chapters of this paper.

BASIC CONCEPTS AND DEFINITIONS

Here, some primary definitions and notes, which are needed in this study, have been indicated.

Definition 2.1: The r-level set of a fuzzy set $\tilde{u}$ is defined as an ordinary set $[\tilde{u}]_r$ of which the degree of membership function exceeds the level r, i.e.

$$\tilde{u}_r = [\tilde{u}]_r = \left\{ x \in R \,\middle|\, \tilde{u}(x) \geq r \quad , r \in [0,1] \right\} \tag{2.1}$$

Definition 2.2: A fuzzy set $\tilde{u}$, defined on the universal set of real number R, is said to be a fuzzy number if its membership function has the following characteristics:

- $\tilde{u}$ is convex i.e.

$$\mu_{\tilde{u}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{u}}(x_1), \mu_{\tilde{u}}(x_2)) \, \forall x_1, x_2 \in R, \forall \lambda \in [0,1] \tag{2.2}$$

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- $\tilde{u}$ is normal i.e. $\exists x_0 \in R$ such that $\mu_{\tilde{u}}(x_0)=1$.
- $\mu_{\tilde{u}}$ is piecewise continuous.

Definition 2.3: A fuzzy number $\tilde{u}$ in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$, $\overline{u}(r)$, $0 \le r \le 1$, that satisfies the following requirement:

- $\underline{u}(r)$ is a bounded monotonically increasing left continuous function;
- $\overline{u}(r)$ is a bounded monotonically decreasing left continuous function;
- $\underline{u}(r) \le \overline{u}(r), 0 \le r \le 1$

Definition 2.4: The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows [9].

For arbitrary $\tilde{u}=(\underline{u}, \overline{u})$, $\tilde{v}=(\underline{v}, \overline{v})$ and $k \in R$, the addition and the scalar multiplication are defined as follows:

- $\tilde{u}=\tilde{v}$ iff $\underline{u}(r)=\underline{v}(r)$ and $\overline{u}(r)=\overline{v}(r)$,
- $(\tilde{u} \pm \tilde{v})(r) = \left(\left(\underline{u}(r) \pm \underline{v}(r)\right), \left(\overline{u}(r) \pm \overline{v}(r)\right)\right)$,
- $k \otimes \tilde{u} = \begin{cases} (k\underline{u}, k\overline{u})(r), & k \ge 0 \\ (k\overline{u}, k\underline{u})(r), & k < 0 \end{cases}$

Remark 2.5: A crisp number α is simply represented by

$$\underline{u}(r)=\overline{u}(r)=\alpha, \ 0 \le r \le 1$$

Definition 2.6: The triangular fuzzy number

$$\tilde{u}=(u_1, u_2, u_3)$$

is a fuzzy set where the membership function is as

$$\tilde{u}(x)=\begin{cases} \dfrac{x-u_1}{u_2-u_1}, & u_1 \le x \le u_2 \\ \dfrac{u_3-x}{u_3-u_2}, & u_2 \le x \le u_3 \\ 0, & \text{Otherwise} \end{cases} \quad (2.3)$$

and its parametric form is

$$\tilde{u}=\left(\underline{u}(r), \overline{u}(r)\right)$$

$$\underline{u}(r)=(u_2-u_1)r+u_1$$

$$\overline{u}(r)=u_3-(u_3-u_2)r \quad (2.4)$$

Definition 2.7: A fuzzy number $\tilde{u}$ is said to be non-negative fuzzy number if and only if $\tilde{u}(x)=0$, $\forall x<0$.

Definition 2.8: A matrix $\tilde{A}=(\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number. $\tilde{A}$ is positive (negative) and denoted by $\tilde{A}>0$ ($\tilde{A}<0$) if each element of $\tilde{A}$ is positive (negative). Similarly, nonnegative and nonpositive fuzzy matrices can be defined [6].

Proposition 2.9: Let the crisp matrix A is nonnegative and $\tilde{x}$ is a nonnegative fuzzy vector then according to Principle extension, it is concluded that

$$A \otimes \tilde{x} = \tilde{b} \quad \Leftrightarrow \quad [A \otimes \tilde{x}]_r = [\tilde{b}]_r \quad (2.5)$$

FUZZY LINEAR REGRESSION (FLR)

A general form of FLR is represented by

$$\tilde{y}_i = X_{i0} \otimes \tilde{a}_0 + X_{i1} \otimes \tilde{a}_1 + \cdots + X_{in} \otimes \tilde{a}_n \quad (3.1)$$

or in matrix form as

$$\tilde{y} = X \otimes \tilde{a} \quad (3.2)$$

where the vector $\tilde{y}$ and matrix $\tilde{X}$ are fuzzy outputs and crisp inputs observations, respectively in which

$$\tilde{a}=[\tilde{a}_0, \tilde{a}_1, \ldots, \tilde{a}_n]^T$$

and

$$X=[X_0, X_1, \ldots, X_n]^T$$

so that

$$X_i=[X_{1i}, X_{2i}, \ldots, X_{ni}]^T$$

is the ith column of crisp coefficients matrix X and $\tilde{a}_i$ is the ith triangular fuzzy variable of $\tilde{a}$ for i = 0,1,2,…,n.

The fuzzy regression analysis is a powerful tool for investigating and predicting data sets by measuring a vague concept that contains a degree of ambiguity, uncertainty or fuzziness [1, 10]. The main purpose of fuzzy regression models is to find the best model with the least error.

In the present study the least-square model, is employed which minimizes the sum of squared errors in the estimated value, based on their specifications

[12, 14, 16]. This approach is indeed a fuzzy extension of the ordinary least-squares, which obtains the best fitting to the data, based on the distance measure under fuzzy consideration, applying information included in the input-output data set.

To compute a meaningful estimation for this FLR problem, the following theorem gives a good strategy.

Theorem 3.1: Let X be a nonnegative crisp n×n matrix and

$$\tilde{y}=\left(\underline{y}(r),\overline{y}(r)\right)$$

is a fuzzy vector. Then a fuzzy vector

$$\tilde{a}=\left(\tilde{a}_0,\tilde{a}_1,\ldots,\tilde{a}_n\right)^T$$

while

$$\tilde{a}_j=\left(\underline{a}_j(r),\overline{a}_j(r)\right)$$

$1\le j\le n$, $0\le r\le 1$, is a solution of $X\otimes\tilde{a}=\tilde{y}$ iff $\tilde{a}$ be the solution of

$$X^TX\otimes\tilde{a}=X^T\otimes\tilde{y}\,.$$

Proof: It is known that a fuzzy vector

$$\tilde{a}=\left(\underline{a}(r),\overline{a}(r)\right)$$

is the solution of

$$X\otimes\tilde{a}=\tilde{y}$$

for any $r\in[0,1]$ iff $\underline{a}(r)$ and $\overline{a}(r)$ satisfy in the equations

$$X\underline{a}(r)=\underline{y}(r)$$

and

$$X\overline{a}(r)=\overline{y}(r)$$

respectively. For simplicity, the parameter r is omitted (for example $\tilde{a}$ is used instead of $\tilde{a}(r)$). Similarly, $\tilde{a}$ is the solution of

$$X^TX\otimes\tilde{a}=X^T\otimes\tilde{y}$$

iff $\underline{a}$ and $\tilde{a}$ are the answers of

$$X^TX\,a=X^T\underline{y}$$

and

$$X^TX\,a=X^T\overline{y}$$

respectively.

To complete the proof, two below propositions should be concluded.

$$X\underline{a}=\underline{y}\ \text{ iff }\ X^TX\,\underline{a}=X^T\underline{y}$$

$$X\overline{a}=\overline{y}\ \text{ iff }\ X^TX\,\overline{a}=X^T\overline{y}$$

As the proofs of (i) and (ii) are similar, the proposition (i) is proved. Now residual vector

$$k=\underline{y}-X\underline{a}$$

is defined and it is denoted as

$$k(\underline{a})=\underline{y}-X\underline{a}$$

to emphasize that given X and $\underline{y}$, k is a function of $\underline{a}$. Let $\underline{w}$ be an n-vector. Then

$$k(\underline{w})=\underline{y}-X\underline{w}=k(\underline{a})+X\underline{a}-X\underline{w}=k(\underline{a})+X(\underline{a}-\underline{w})$$

So

$$\left\|k(\underline{w})\right\|_2^2=\left\|k(\underline{a})\right\|_2^2+2(\underline{a}-\underline{w})^TX^T\left\|k(\underline{a})\right\|+\left\|X(\underline{a}-\underline{w})\right\|_2^2$$

First assume that $\underline{a}$ satisfies in

$$X^TX\underline{a}(r)=X^T\underline{y}(r)$$

so, $X^Tk(\underline{a})=0$.

From the preceding, we have

$$\left\|k(\underline{w})\right\|_2^2=\left\|k(\underline{a})\right\|_2^2+\left\|X(\underline{a}-\underline{w})\right\|_2^2\ge\left\|k(\underline{a})\right\|^2$$

which implying that $\underline{a}$ is a least-square solution.

Next assume $X^Tk(\underline{a})\neq 0$ and set $X^Tk(\underline{a})=v$. Define a vector $\underline{w}$ such that $\underline{w}$ = $\underline{a}$+cv with an arbitrary crisp number c (c>0). Then

$$k(\underline{w})=k(\underline{a})+X(\underline{a}-\underline{w})=k(\underline{a})-cXv$$

$$\left\|k(\underline{w})\right\|_2^2=\left\|k(\underline{a})\right\|_2^2+c^2\left\|Xv\right\|_2^2-2cv^TX^Tk(\underline{a})$$

$$\left\|k(\underline{a})\right\|_2^2+c^2\left\|Xv\right\|_2^2-2c\left\|v\right\|_2^2<\left\|k(\underline{a})\right\|_2^2$$

for sufficiently small value of c. This implies that $\underline{a}$ is not a least-square solution.

From the above theorem, an ordinary method for computing the solution of (3.2) in matrix form is obtained as follows

$$\tilde{a} = B^{-1} \otimes \tilde{z} \text{ or } \tilde{a} = \left(X^T X\right)^{-1} \otimes \left(X^T \otimes \tilde{y}\right) \qquad (3.3)$$

where $B = X^T X$ and $\tilde{z} = X^T \otimes \tilde{y}$.

The matrix $X^T X$ is called the information matrix, because it measures the information contained in the experiment and the matrix $(X^T X)^{-1}$ is known as the variance-covariance matrix [5]. It can be shown that the matrix $X^T X$ is positive definite and hence non-singular, where the columns of X are linearly independent. At first, the case that X has full rank is discussed such that a normal equation instead of linear regression is considered which, in our formulation, have the form

$$X^T X \otimes \tilde{a} = X^T \otimes \tilde{y}$$

Nevertheless, the matrix $X^T X$ is ill-posed and for moderate values of n, the solution $\tilde{a}$ will not be accurate. Hence, an alternative method is used. By using this iterative method, the solution of

$$A^T A \otimes \tilde{x} = A^T \otimes \tilde{b}$$

can be found. In usual, any iterative method can not find the optimum solution of this kind of ill-posed problems. For this reason, the algorithm of Huang, which is described in the next section, is selected that is powerful to solve ill-posed fuzzy linear system of equations.

Remark 2.6: There always exists a solution to the linear least-squares problem $(\tilde{z} = B \otimes \tilde{a})$. This solution is unique iff B has full rank.

SOLVING FUZZY LINEAR SYSTEMS (FLS)

Friedman *et al.* [9] proposed a general model for solving an n×n FLS problem, whose coefficient matrix A is crisp and right-hand side column $\tilde{b}$ is a triangular fuzzy vector, by the embedding approach. So, for solving the following FLS equations

$$A \otimes \tilde{x} = \tilde{b} \qquad (4.2)$$

at first, the system (4.2) is converted to the following 2n×2n crisp function linear system

$$SX = Y \qquad (4.3)$$

described in below. Assume an 2n×2n matrix $S = (s_{ij})$ is obtained as:

$$\begin{cases} a_{ij} \ge 0 \Rightarrow & s_{ij} = a_{ij}, \quad s_{i+m,j+n} = a_{ij} \\ a_{ij} < 0 \Rightarrow & s_{i,j+n} = -a_{ij}, \quad s_{i+m,j} = -a_{ij} \end{cases} \qquad (4.4)$$

and each s_{ij} which is not determined by (4.4) is zero. An approximate solution of (4.2) (that is often used), is the least square solution of (4.3), defined as a vector X which minimizes the Euclidean norm of (Y-SX).
Now, by setting

$$S^T = [s_1, s_2, \ldots, s_{2n}]$$

the procedures of Huang's method [13] for computing the fuzzy solution is described in below.

Huang's Algorithm

1. Let $X_1 \in R^{2n}$ is arbitrary vector and set $H_1 = I \in R^{2n\times 2n}$ and i = 1.
2. Compute the direction vector p_i as $p_i = H_i s_i$.
3. Update the approximation iterate by $X_{i+1} = X_i - \alpha_i p_i$ where α_i is given by $\alpha_i = \left(s_i^T X_i - y_i\right)/ s_i^T p_i$.
4. If i = 2n stop (X_{i+1} is the solution); otherwise Update the matrix H_i by $H_{i+1} = H_i - \left(p_i p_i^T\right)/s_i^T p_i$ set i = i+1 and go to 2.

Definition 4.1: Let

$$x = \left\{\left(\underline{x}_j(r), \overline{x}_j(r)\right), 1 \le j \le n\right\}$$

denotes the unique solution of equation (4.3). The triangular fuzzy vector

$$\tilde{U} = \left\{\left(\underline{u}_j(r), \overline{u}_j(r)\right), 1 \le j \le n\right\}$$

defined by

$$\begin{aligned} \underline{u}_j(r) &= \min\left\{\underline{x}_j(r), \overline{x}_j(r), \underline{x}_j(1)\right\} \\ \overline{u}_j(r) &= \max\left\{\underline{x}_j(r), \overline{x}_j(r), \overline{x}_j(1)\right\} \end{aligned} \qquad (4.5)$$

is called the fuzzy solution of equation (4.3).

Theorem 4.2: Consider the Huang's algorithm with the following choice of X_1,

$$X_1 = \sum_{j=1}^{k} \beta_j s_j$$

with k<2n and some scalars β_j, then for $i \ge k$, X_{i+1} is a vector with minimal Euclidean norm.

Table 1: Numerical results of example 1.Here, the fuzzy iterates $\tilde{x}_i = (\tilde{x}_{i1}, \tilde{x}_{i2})$ are showed in triangular forms separately

Iterate	$\tilde{x}_1^{(i)} = (a_1^i, b_1^i, c_1^i)$			$\tilde{x}_2^{(i)} = (a_2^i, b_2^i, c_2^i)$		
1	1	1	0	0	0	1
2	0	0.50	0	0	0	0
3	0.21	0.74	0	1.263	1.421	-0.263
4	0.70	1.10	3.100	1.100	1.200	0.700
5	1.375	2	3.875	0.875	1	1.375

Proof: Refer to [7].

Theorem 4.3: The vector X_2 generated by the Huang's algorithm is the minimal Euclidean (vector) norm iff $X_1 = \beta s_1$ for some arbitrary scalar β.

Proof: Refer to [7].

NUMERICAL TESTS

Here, we want to test the mentioned algorithms by some popular examples. At first, one well-known FLS equations is solved by Huang's algorithm and later on, one FLR problem which is obtained from a geology study is tested. The computed results and graphs of triangular fuzzy numbers have been showed at the end of each test.

Example 1: Consider the 2×2 FLS problem

$$\begin{aligned} \tilde{x}_1 - \tilde{x}_2 &= (r, 2-r) \\ \tilde{x}_1 + 3\tilde{x}_2 &= (4+r, 7-2r) \end{aligned} \tag{5.1}$$

To solve the above problem by Huang algorithm we have

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}, \tilde{b} = \begin{pmatrix} (r, 2-r) \\ (4+r, 7-2r) \end{pmatrix}$$

So

$$S = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}, Y = \begin{pmatrix} r \\ 4+r \\ r-2 \\ 2r-7 \end{pmatrix} \tag{5.2}$$

This algorithm has been started with the supposition $X_1 = s_1$. The exact solution of this linear system has been computed after 5 iterations that the numerical results are showed in below Table 1.

The computed answers of this problem is indicated in below

$$\tilde{x}_1 = (1.375\ ,\ 2\ ,\ 3.875),\ \tilde{x}_2 = (0.875\ ,\ 1\ ,\ 1.375)$$

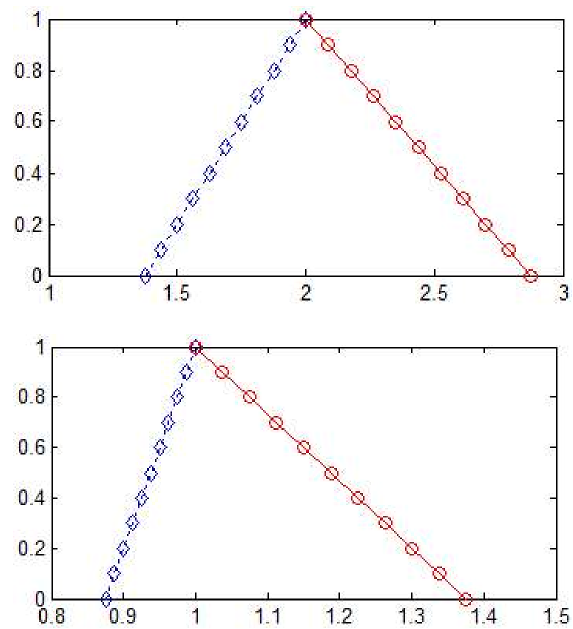

Fig. 1: The graphs of triangular fuzzy solutions of (5.1)

The optimum triangular fuzzy solutions of (5.1) have been illustrated in two below figures.

The authors also tested this problem to be solved by some other methods, for example the proposed methods of [8, 9]. The computed results showed that the new method is more powerful to solve such problems.

Example 2: The following geology data is a study about the effect x:SAR Sodium absorption rate) on y:PSE (percent of Sodium exchange) in Silakhor region of Lorestan province of Iran in 2001 [19]. Due to unconcluded with sufficient accuracy in measurements of PSE, the observations related to variable y are ambiguous. It is reasonable to find an optimum model for relation of PSE parameters based on SAR data. For this aim, an FLR problem has been considered.

To examine the applicable of mentioned fuzzy regression process and to compute the best linear regression with fuzzy coefficients, at first, the matrix $X = (1, x_{i1})$ is set such that the n-vector 1 is as $1 = [1,1,\ldots 1]^T$.

Table 2: Inputs of example 2 which x_{i1} is crisp and $\tilde{y}_i$ is triangular fuzzy number

i	x_{i1}	$\tilde{y}_i = (y_i, z_i, w_i)$	i	x_{i1}	$\tilde{y}_i = (y_i, z_i, w_i)$
1	0.87	3.08 0.31 0.31	13	0.71	5.23 0.52 0.52
2	0.64	2.86 0.29 0.29	14	0.50	5.16 0.52 0.52
3	0.62	6.25 0.63 0.63	15	0.77	11.10 1.11 1.11
4	0.49	4.11 0.41 0.41	16	0.99	4.47 0.45 0.45
5	1.10	1.04 0.10 0.10	17	3.56	28.84 2.88 2.88
6	0.61	2.71 0.27 0.27	18	0.86	9.43 0.94 0.94
7	0.74	4.45 0.45 0.45	19	0.61	4.50 0.45 0.45
8	1.15	6.92 0.69 0.69	20	0.64	9.30 0.94 0.94
9	1.08	7.41 0.74 0.74	21	0.71	9.48 0.95 0.95
10	0.38	9.08 0.91 0.91	22	0.61	3.65 0.37 0.37
11	0.61	6.56 0.66 0.66	23	0.63	10.14 1.01 1.01
12	0.98	5.05 0.51 0.51	24	1.13	3.00 0.30 0.30

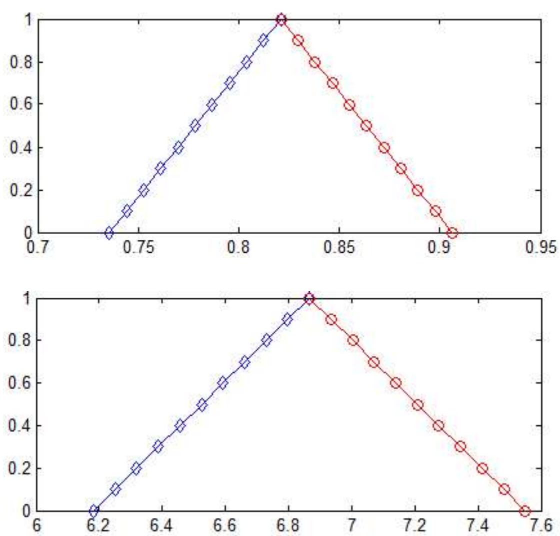

Fig. 2: The graphs of triangular fuzzy solutions of example 2

Huang's algorithm for computing the best estimation for this FLR problem has been applied. By computing B = X^TX and $\tilde{z} = X^T \otimes \tilde{y}$ the system of equations $B \otimes \tilde{a} = \tilde{z}$ is obtained such that the solution $\tilde{a}$ is a triangular fuzzy vector. Thus, this system is solved by Huang's Algorithm to compute the best estimation of fuzzy regression that the numerical results are showed as follows.
Then the computed regression parameters are as

$$\tilde{a}_0 = (0.74356, 0.8209, 0.9062)$$

$$\tilde{a}_1 = (6.1817, 6.8660, 7.5504)$$

and the linear estimation is as follows

$$\tilde{y} = (0.74356, 0.8209, 0.9062) \oplus X_{i1} \otimes (6.1817, 6.8660, 7.5504)$$

The corresponding graphs for computed results have been showed in below.

CONCLUSION

Fuzzy linear regression is a useful technique for fitting the data which many implementations for finding the best estimators of FLR problems have been proposed. In this study, fuzzy linear regression based on least-squares approaches has been considered that a fuzzy linear system, obtained by this approach, is required to be solved. Usually these linear systems are ill-posed which most of alternative solvers can not compute a meaningful estimation but Huang's algorithm is a good iterative method that reaches to the solutions quickly. Numerical results certify that this method is useful. Then Huang's Algorithm is suggested to be used for computing the estimations of FLR and ill-posed FLS problems.

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