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Metrics on Row-finite Directed Graph and Relationship with Graph C*-algebras

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Abstract: In this paper, the authors are trying directed graph row-finite $E = (E^0, E^1, r, s)$. Two metric d and ρ Order on the set $E^* = \{v \in E^0 \mid v \in s(\dot{E})\}$ and the edges set E^1 Are defined, the metric defined some properties E-Are defined, the metric defined some properties $\{S_e, P_v\}$. The graph produced C^* Algebra $C^*(E)$ Involved are identified, such that for every two distinct edges e, f, $\|S_e+S_f\| \geq 1$. For both custom and head $v, w \in E^*$, $\|P_v+P_w\| \geq 1$.

Key words: Directed graph . Algebra

INTROUDUCTION

In 1980, Cuntz and Krieger considered generalized version of the Cuntz algebra. Rather than having a C^*-Algebra for each positive integer n, then C^*-Algebra were instead associated to certain with entries in {0,1}. If A is an n*n matrix in {0,1}, then the Cuntz-Kriger algebra Q_A is defined to be the C^* -Algebra generated by partial isometric S_1 ,S_2 ,...S_n orthogonal ranges that satisfy

$$S_i^* S_i = \sum_{j=1}^{n} A(i,j) S_j S_j^*$$

And for which $1 \leq i \leq n$ Where A = A(i,j) Find the matrix Member {0,1} C^{*-}Algebra O_A where C^* Algebra Cuntz and Krieger called generalized O_n and therefore Category Algebra Cuntz and Krieger including Algebra Cuntz and Krieger. But in 1982 Watatani category of Cuntz Algebra and Krieger- considers that the graphs Loaded depend, in fact, Graf-Algebra or the Cuntz Algebra and Krieger dependent graphs Loaded Following closed algebra B(H) Is, where H is a Hilbert space [1,3,4].

Since the square matrices with the row or column Member that they are not zero, so directed with corresponding graphs to obtain better results and easier to read Cuntz Algebra and Krieger, their corresponding directed graphs we consider and so requires that they determine Let the elements-a graph C^*-algebra depends what features should be directed. We define two metric head and edges, a necessary condition but not sufficient for the Projection operator P from $B(H)$ member of depends on the graph C^{*-}Algebra is directed to determine.

Terms:

Definition 1.1: A directed graph $E = (E^0, E^1, r, s)$ consists of E^0 a set you like the outline of the edges as you like E^1 and r,s: $E^1 \to E^0$ set two records which are called respectively the range and origin and s early r end of the edge determines.[4]

Definition 2.1: A row finite directed graph $E = (E^0, E^1, r, s)$ call when the maximum number of each vertex is finite edge edge, in other words for each $v \in E^0$ vertex, $s^{-1}(v)$ such as collection, $\{e \in E^1 | s(e) = v\} \subseteq E^1$ ie a set is finite.[1]

Definition 3.1: let A-is an C^*-algebra $p \in A$ if is Projection: $p = p^* = p^2$. [6]

Definition 4.1: the element $s \in A$ in which a C^*-algebra is a partial call whenever Isometric; ss^* is an projection.[6]

Theorem 5.1: let A C^*-algebra and $u \in A$, then the following equivalent conditions: [2]

1) u is partial isometric
2) u^*u is an projection
3) $u = uu^*u$.

Definition 6.1: let $E = (E^0, E^1, r, s)$ a line directed graphs-is finite and $\{S_e, P_v\}$ (non zero elements) a Cuntz

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and Krieger family this graph, the vertices in which each and every edge $e \in E^1$, is Both Perpendicular projections (if $v \neq w$: $P_v P_w = 0$ and is Isometric (or the same part Isometric) are attributed to the relationship that ties Cuntz and Krieger-Kraygr are valid [1,5]:

(1) $S_e^* S_e = p_{r(e)} \qquad \forall e \in E^1$

(2) $p_v = \sum_{\{e:s(e)=v\}} S_e^* S_e \quad \forall v \in s(E^1)$

Definition 7.1: We assume $E = (E^0, E^1, r, s)$ Directed graph and $\{S_e, P_v\}$ is a finite line E- Cuntz and Krieger family, then the $-C^*$ algebra generated by $\{S_e, P_v\}$ with $E = (E^0, E^1, r, s)$ graph defined **$-C^*$**Algebra. Present with **C*(*E*)** [3, 5].

Lemma 8.1: Assume $\{S_e,P_v\}$ a family of operators Non zero Cuntz and Krieger and then projections $\{S_e S_e^*\}$ orthogonal family [1].

Definition 9.1: Assume $E = (E^0,E^1,r,s)$ is a directed graph of row-finite and Isometric (Isometric or partial) are in relationships that Cuntz and Krieger are true, we define the mapping

$$d : E^1 \times E^1 \to [0,\infty)$$
$$d(e,f) = \|S_e - S_f\|$$

(in which the $\|\cdot\|$ norm is operator in B(H))

Theorem 10.1: Assume $E = (E^0,E^1,r,s)$ a directed graph line-A metric is a finite directed graph E defines a set Edges .

Proof:

(i) $\forall e,f \in E^1 : 0 \le d(e,f) = \|S_e - S_f\| < \infty$

(ii) $\forall e,f \in E^1 : d(e,f) = \|S_e - S_f\| = 0 \Leftrightarrow S_e = S_f$

It is evident that if $e = f$, then $S_e = S_f$ and therefore

$$d(e,f) = \|S_e - S_f\| = 0$$

reverse the order $S_e = S_f$, according $e \neq f$(e, $f \in E^1$) to Lemma 8.1 we have:

$$S_e^* S_e = S_e^* S_f \Rightarrow P_{r(e)} = 0$$

And because members E-non Zero family, $\{S_e,P_v\}$ this is a contradiction does not necessarily result, $e = f$ in other words

$$d(e,f) = 0 \Leftrightarrow e = f$$

$$\forall e,f,g \in E^1: d(e,f) = \|S_e - S_f\|$$

(iii) $= \|S_e - S_g + S_g - S_f\| \le \|S_e - S_g\| + \|S_g - S_f\|$
$= d(e,g) + d(g,f)$

So there (E^1, d). So this is a metric space.

Definition 11.1: Assume $E = (E^0,E^1,r,s)$ is a directed graph row-finite both these projections are in relationships that Cuntz and Krieger plus true put

$$E^* = \{v \in E^0 \mid v \in s(E^1)\}$$

on E^* the map ρ to define

$$\rho : E^* \times E^* \to [0,\infty)$$
$$\rho(v,w) = \|P_v - P_w\|$$

Theorem 12.1: Assume $E = (E^0,E^1,r,s)$ a directed graph line-is finite, then the set defines E^* the mapping of a metric.

Proof:

(i) $\forall v,w \in E^*; 0 \le \rho(v,w) = \|P_v - P_w\| < \infty$

In fact, according to the line-a finite directed graph $E = (E^0,E^1,r,s)$ is a $\{e:s(e) = v\}$ finite set $C^*(E)$ and because one C^*-and so there is determinism $\|S_e S_e^*\| = \|S_e\|^2$:

(i) $$\forall v,w \in E^*; 0 \le \rho(v,w) = \|P_v - P_w\| = \left\| \sum_{\{e:s(e)=v\}} S_e S_e^* - \sum_{\{f:s(f)=w\}} S_f S_f^* \right\|$$
$$\le \left\| \sum_{\{e:s(e)=v\}} S_e S_e^* \right\| + \left\| \sum_{\{f:s(f)=w\}} S_f S_f^* \right\| \le \sum_{\{e:s(e)=v\}} \|S_e S_e^*\| + \sum_{\{e:s(e)=v\}} \|S_f S_f^*\| = \sum_{\{e:s(e)=v\}} \|S_e\|^2 + \sum_{\{e:s(e)=v\}} \|S_f\|^2 < \infty \Rightarrow 0 \le \rho(v,w) < \infty.$$

(ii) $\rho(v,w) = 0 \Leftrightarrow v = w$

If ν = w, P_ν = p_w and $\rho(v,w) = \|P_v - P_w\| = 0$ as a result of the software, $\rho(\nu,w) = 0$ but $\nu \neq w$ conversely to assume, therefore, we have:

$$\rho(v,w) = 0 \Rightarrow \|P_v - P_w\| = 0 \Rightarrow P_v = P_w \Rightarrow P_v P_v = P_v P_w \Rightarrow P_v{}^2 = 0 \Rightarrow P_v = 0$$

And this is not possible, thus necessarily ν=w.

(iii) $\rho(v,w) = \|P_v - P_w\| = \|P_w - P_v\| = \rho(w,v) \Rightarrow \rho(v,w) = \rho(w,v)$

(iv) $\forall u,v,w \in E^*;\rho(v,w) = \|P_v - P_w\| = \|P_v - P_u + P_u - P_w\| = \|P_v - P_u\| + \|P_u - P_w\| = \rho(v,u) + \rho(u,w) \Rightarrow \rho(v,w) \le \rho(v,u) + \rho(u,w)$

The result (E^*,ρ) is a metric space.

Theorem 13.1: Assume E = (E^0,E^1,r,s) a directed graph row-finite and $\{S_e,P_v\}$ a-Cuntz and Krieger E family-Kraygr is the graph, then:

$$\forall e,f \in E^1, r(e) = v, r(f) = w; \rho(v,w) \le d(e,f)\|S_e + S_f\|$$

Proof: there;

$$\begin{aligned}
\rho(v,w) &= \|P_v - P_w\| \\
&= \|P_{r(e)} - P_{r(f)}\| = \|S_e^* S_e - S_f^* S_f\| \\
&= \|S_e^* S_e S_e^* S_e - S_f^* S_f S_f^* S_f\| = \|(S_e - S_f)(S_e{}^* + S_f{}^*)\| \\
&= \|(S_e - S_f)(S_e + S_f)^*\| \text{£} \|S_e - S_f\| \|(S_e + S_f)^*\| \\
&= \|S_e - S_f\| \|S_e + S_f\| = d(e,f)\|S_e + S_f\| \\
&\text{Þ}\rho(v,w)\text{£}d(e,f)\|S_e + S_f\|
\end{aligned}$$

Theorem 14.1: Assume a directed graph row-finite and a-Cuntz and Krieger family-Kraygr for this graph is then

$$\forall e,f \in E^1, (e \neq f) : \|S_e + S_f\| \ge 1$$

Proof: The metric d we have defined:

$$\begin{aligned}
d(e,f) &= \|S_e - S_f\| \\
&= \|S_e S_e^* S_e - S_f S_f^* S_f\| = \|(S_e S_e^* - S_f S_f^*)(S_e + S_f)\| \\
&\le \|S_e S_e^* - S_f S_f^*\| \|S_e + S_f\| = \|(S_e - S_f)(S_e + S_f)^*\| \|S_e + S_f\| \\
&\le \|S_e - S_f\| \|(S_e + S_f)^*\| \|S_e + S_f\| \\
&= \|S_e - S_f\| \|S_e + S_f\| \|S_e + S_f\| = \|S_e - S_f\| \|S_e + S_f\|^2 \\
&= d(e,f)\|S_e + S_f\|^2 \Rightarrow d(e,f) \le d(e,f)\|S_e + S_f\|^2
\end{aligned}$$

As a result,... $\|S_e + S_f\|^2 \ge 1$ so $\|S_e + S_f\| \ge 1$.

Theorem 15.1: Assume E = (E^0,E^1,r,s) a directed graph row-finite and $\{S_e,P_v\}$ a-Cuntz and Krieger E family-Kraygr for this graph is supposed to

$$r(e) = v, r(f) = w$$

$$(e \neq f) e,f \in E^1$$

have $\|p_v + p_w\| \ge 1$
Proof:

$$\begin{aligned}
d(e,f) &= \|S_e - S_f\| = \|S_e S_e^* S_e - S_f S_f^* S_f\| \\
&= \|S_e P_{r(e)} - S_f P_{r(f)}\| \\
&= \|(S_e - S_f)(P_{r(e)} + P_{r(f)})\| \\
&= \|(S_e - S_f)(P_v + P_w)\| \\
&\le \|S_e - S_f\| \|P_v + P_w\| \\
&\Rightarrow d(e,f) \le d(e,f)\|P_v + P_w\|
\end{aligned}$$

As a result $e \neq f$, we have $\|p_v + p_w\| \ge 1$: and that is complete.

CONCLUSION

If the assumptions ν=w of Theorem 1.15 we will then $\|p_v\| \ge 1/2$. With the assumptions necessary nor sufficient for being a B(H) member-C^* of files-to force a row finite directed graph that is $\|p_v\| \ge 1/2$.

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