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Proper Functions in Chattering Free Adaptive Sliding Mode Control

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Abstract: In this paper, a control strategy with a condition to find proper function in chattering free Adaptive Sliding Mode Control (ASMC) is proposed. To show the feasibility of existence of represented functions, there kind of proper functions evaluate in this article. The effectiveness of our introduced functions and schemes is provided by simulation.

Key words: Adaptive Sliding Mode Control (ASMC) . nonlinear systems.

INTRODUCTION

There is no doubt that chaos is one of the most significant nonlinear phenomenons and has been studied in the recent three decades [1]. In addition, chaos is one of the basic features in nonlinear science and is of fundamental importance in a variety of complex physical, chemical and biological systems [2]. Recently, control of chaotic dynamics increasingly gained researchers interest since the leading work of Ott *et al*. [3]. In recent days, chaos has been found in many engineering systems [4]. Chaos theory and chaotic property have a wide range of useful applications in many engineering fields such as digital communication, secure communication, power electronic devices and power quality, biological systems, chemical reaction analysis and design and information processing [5]

After the publication of two relevant papers by Ott *et al*. [3] and Pecora *et al*. [6] in 1990, control of chaos dynamics have become very important topics on the applications of nonlinear systems and a number of different methods for control of chaos systems have been presented by many researchers. Some of possible application areas are mainly in secure communications, optimization of nonlinear systems performance, modeling brain activity and pattern recognition phenomena [1-8]. It is of great importance to realize that nonlinear dynamics can play very important role in resolving outstanding problems in theoretical physics [9].

It is widely accepted that control of chaotic systems is a difficult problem, mainly on the basis of the extremely sensitive characteristic of chaos to initial conditions. The cascade synchronization method [10] was presented by Pecora and Carroll, in 1990.

Recently, the research of chaos dynamics synchronization has become a very significant research platform in the nonlinear dynamics field and researchers in this field have addressed a number of problems on chaos synchronization, such as the stability conditions for chaos synchronization, the realization for a successful synchronization, the applications of chaos synchronization etc. [11-20].

In recent 15 years, a large number of methods for chaos control and synchronization have been proposed. Such as, periodic parametric perturbation [21], drive-response synchronization [22], adaptive control [23-27], variable structure (or sliding mode) control [28-30], back-stepping control [31, 32], H_8 control [33], fuzzy control [34] and many others.

In [35], Dadras and Momeni introduced an adaptive sliding mode control of chaotic systems. This method successfully reduces the chattering phenomenon and guarantees stability in presence of parameter uncertainties and external disturbance.

The objective of this article is to propose a novel adaptive sliding mode controller for control of any different chaotic system, in the presence of uncertainties and external disturbance.

The organization of this paper is as follows: Section 2 addresses the system description and problem formulation. In section 3, stability analysis is given. In section 4, numerical simulations are used to demonstrate the effectiveness of the proposed method. Finally conclusion is presented.

SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this paper, a class of chaotic n-dimensional systems is studied having the following system description:

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$$\begin{cases} \dot{x}_i = x_{i+1}, & 1 \le i \le n-1 \\ \dot{x}_n = f(x,t) + \Delta f(x,t) + d(t) + u(t) \\ \qquad x = [x_1, x_2, \dots, x_n] \in R^n \end{cases} \tag{1}$$

In essence, the Genesio-Tesibsystem system [36] is one of the chaotic systems with many features of these systems. The equation of this system is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -cx_1 - bx_2 - ax_3 + mx_1^2 + \Delta f(x,t) \\ \qquad + d(t) + u(t) \end{cases} \tag{2}$$

where, x_1, x_2, x_3 are state variables and a, b and c are the positive real constants which should satisfy ab<c. $u \in$ is the control input, f is a given nonlinear function, $\Delta f(x,t)$ is an uncertain term representing the un-modeled dynamics or structural variation of system (2) and $d(t)$ is the disturbance of system (2).

In general, the uncertainty and the disturbance are assumed to be bounded as follows:

$$|\Delta f(y)| \le \alpha \text{ and } |d(t)| \le \beta \tag{3}$$

where α and β are positive unknown constants. The control problem considered in this paper is that for different initial conditions of system (2) the controller u(t) force the dynamic to trace an n-dimensional desired vector X_d. For system (2) n=3 and

$$X_d = [x_{d1}, x_{d2}, x_{d3}]^T = [x_d, \dot{x}_d, \ddot{x}_d]^T \in R^3$$

therefore the tracking error can determine as follows:

$$\begin{aligned} E(t) &= X(t) - X_d(t) \\ &= [x(t) - x_d(t), \dot{x}(t) - \dot{x}_d(t), \ddot{x}(t) - \ddot{x}_d(t)]^T \\ &= [e(t), \dot{e}(t), \ddot{e}(t)]^T \\ &= [e_1(t), e_2(t), e_3(t)]^T \end{aligned} \tag{4}$$

The goal of control method is to have:

$$\lim_{t \to \infty} \|E(t)\| = \lim_{t \to \infty} \|X(t) - X_d(t)\| \longrightarrow 0 \tag{5}$$

Where $\|\cdot\|$ denotes the Euclidian norm of a vector.

In the conventional SMC, a sliding surface S representing the desired system dynamics is chosen as:

$$s = e_n + \sum_{i=1}^{n-1} \lambda_i e_i \tag{6}$$

This equation is comprehensive and for system (2) we have:

$$s = e_3 + \lambda_2 e_2 + \lambda_1 e_1 \tag{7}$$

The switching surface parameters $\{\lambda_i, i=1,\dots, n-1\}$ are chosen based on the following two criteria. First, the values are chosen to stabilize the system during the sliding mode. Routh-Hurwitz criterion [37] is used to determine the range of coefficients c_i that produce stable dynamics. That is, all the roots of the following characteristic polynomial describing the sliding surface have negative real parts with desirable pole placement,

$$P(\lambda) = \lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_2\lambda + c_1 \tag{8}$$

Second, the values are chosen such that the system during sliding mode has fast and smooth response.

Having established an appropriate switching surface, the next step is to design an adaptive sliding mode control scheme to guarantee that the system states are hitting on the sliding surface s = 0 (i.e. to satisfy the reaching condition $s\dot{s} < 0$). In order to ensure the occurrence of the sliding mode, an adaptive sliding mode law is designed. When the closed loop system is in the sliding mode, it satisfies $\dot{s} = 0$. An adaptation law is applied to construct the proposed adaptive sliding mode controller such that the chattering phenomenon, which is inherent in conventional switching-type sliding controllers, is attenuated and the steady error is also alleviated. To ensure the occurrence of the sliding motion, an adaptive control law is proposed as:

$$u = u_{eq} + u_r \tag{9}$$

Where u_{eq} is equivalent control law and obtained by

$$u_{eq} = -\dot{e}_3 - \sum_{i=1}^{2} \lambda_i e_{i+1}$$

In addition, u_r is called reaching control law and it is defined as

$$u_r = -\mu\theta\psi$$

Consequently,

$$u = -\dot{e}_3 - \sum_{i=1}^{n-1} \lambda_i e_{i+1} - \mu\theta\psi$$

And for system (2)

$$u=-\lambda_2 e_3-\lambda_1 e_2+\dot{x}_{d3}(t)-\mu\theta\psi$$

Where μ is a constant as follows

$$\mu>1$$

Also $\psi=\hat{\alpha}+\hat{\beta}$

Where $\hat{\alpha},\hat{\beta}$ are adaptive parameters.

Definition 1: In the adaptive control law, θ is defined as a function of φ and s like $g(\varphi,s)$, where $g(\varphi,s)$ is a function with the following condition

$$-1\le g(\varphi,s)\le 1 \tag{10}$$

Also φ is defined as follows

$$\dot{\varphi}=-\gamma\,\theta h(\varphi,s)$$

Where $h(\varphi,s)$ should apply to the following relation:

$$\frac{dg(\varphi,s)}{d\varphi}h(\varphi,s)=sR(\varphi,s) \tag{11}$$

Whereas $R(\varphi,s)\ge 0$.
So, the adaptive laws are

$$\dot{\hat{\alpha}}=\dot{\hat{\beta}}=|s|,\quad \hat{\alpha}(0)=\hat{\alpha}_0,\quad \hat{\beta}(0)=\hat{\beta}_0 \tag{12}$$

where $\hat{\alpha}_0$ and $\hat{\beta}_0$ are the positive and bounded initial values of $\hat{\alpha}$ and $\hat{\beta}$, respectively and γ is a positive constant.

With the knowledge above we will propose our main results in the following theorem.

STABILITY ANALYSIS

Theorem 1: Consider the system dynamics, if this system is controlled by u(t) in Eq. (9) with adaptation law (11),then the closed loop system is globally asymptotically stable.

Proof: Let

$$\tilde{\alpha}=\hat{\alpha}-\alpha,\tilde{\beta}=\hat{\beta}-\beta \tag{13}$$

α and β are unknown constants. Thus the following expression holds.

$$\dot{\tilde{\alpha}}=\dot{\hat{\alpha}},\dot{\tilde{\beta}}=\dot{\hat{\beta}}$$

Consider the following Lyapunov function candidate

$$V(t)=\frac{1}{2}(s^2+\theta^2+\tilde{\alpha}^2+\tilde{\beta}^2) \tag{14}$$

Then, the derivative of *V* is

$$\dot{V}(t)=s\dot{s}+\theta\dot{\theta}+\tilde{\alpha}\dot{\tilde{\alpha}}+\tilde{\beta}\dot{\tilde{\beta}} \tag{15}$$

where, by direct computation and definition 1, we have:

$$\dot{\theta}=\frac{d\theta}{dt}=\frac{dg(\varphi,s)}{dt}=\frac{dg(\varphi,s)}{d\varphi}\frac{d\varphi}{dt} \tag{16}$$

In the above equation, if $\dot{V}$ is negative for all $s\neq 0$, then the so-called reaching condition [37] is satisfied. That is, the control *u* is designed to guarantee that the states are hitting on the sliding surface s = 0.

$$\begin{aligned}\dot{V}(t)&=s\dot{s}+\theta\dot{\theta}+\tilde{\alpha}\dot{\tilde{\alpha}}+\tilde{\beta}\dot{\tilde{\beta}}\\&=s(\dot{e}_3+\lambda_2\dot{e}_2+\lambda_1\dot{e}_1)+\theta\dot{\theta}+\tilde{\alpha}\dot{\tilde{\alpha}}+\tilde{\beta}\dot{\tilde{\beta}}\\&=s(\dot{e}_3+\lambda_2 e_3+\lambda_1 e_2)+\theta\dot{\theta}+\tilde{\alpha}\dot{\tilde{\alpha}}+\tilde{\beta}\dot{\tilde{\beta}}\\&=s(-cx_1-bx_2-ax_3+mx_1^2+\Delta f(X)+d(t)\\&+u(t)-\dot{x}_{d3}+\lambda_2 e_3+\lambda_1 e_2)+\theta\dot{\theta}+\tilde{\alpha}\dot{\tilde{\alpha}}+\tilde{\beta}\dot{\tilde{\beta}}\end{aligned}$$

As we define θ in Eq(11) we have:

$$-|s|\le|s|g(\varphi,s)\le|s| \tag{19}$$

Also we have

$$\begin{aligned}\theta\dot{\theta}&=g(\varphi,s)\frac{dg(\varphi,s)}{d\varphi}(-\gamma g(\varphi.s)h(\varphi,s))\\&=-s\gamma g^2(\varphi,s)R(\varphi,s)\end{aligned}$$

Furthermore, it is obvious that

$$sg^2(\varphi,s)\le s\le|s|$$

So,

$$\begin{aligned}\dot{V}&\le|c||x_1||s|+|b||x_2||s|+|a||x_3||s|+|m||x_1^2||s|+\alpha|s|\\&+\beta|s|-\mu\psi|s|-\gamma|s|R(\varphi,s)+\tilde{\alpha}\dot{\hat{\alpha}}+\tilde{\beta}\dot{\hat{\beta}}\\&=|c||x_1||s|+|b||x_2||s|+|a||x_3||s|+|m||x_1^2||s|+\alpha|s|\\&+\beta|s|-\psi|s|+\psi|s|-\mu\psi|s|-\gamma|s|R(\varphi,s)\\&+(\hat{\alpha}-|\alpha|)\dot{\hat{\alpha}}+(\hat{\beta}-|\beta|)\dot{\hat{\beta}}\\&=[(1-\mu)\psi-\gamma R(\varphi,s)]|s|\end{aligned}$$

As it was mentioned before

$$R(\varphi,s)\geq 0$$

Since $\mu>1$ has specified in Eq. (9) and γ is a positive constant, we obtain the following inequality

$$\dot{V}\leq -[(\mu-1)\psi+\gamma R(\varphi,s)]|s|<0 \qquad (20)$$

Therefore, the closed-loop system is asymptotically stable. In this regard it is of great importance to realize that the set of functions which can have proper conditions to apply to the Eq. (10) and (11) is not empty. Consequently, many suitable functions can find for this reason and some of them are listed in the following.

Case 1: In the paper [35], one sample for $g(\varphi,s)$ has been introduced as follows:

$$g(\varphi,s)=\frac{(1-e^{-\varphi s})}{(1+e^{\varphi s})} \qquad (21)$$

Also, $h(\varphi,s)=e^{\varphi s}$
Clearly, Eq. (11) is satisfied

$$\frac{dg(\varphi,s)}{d\varphi}h(\varphi,s)=\frac{2se^{-\varphi s}}{(1+e^{-\varphi s})^2}e^{\varphi s}=\frac{2s}{(1+e^{-\varphi s})^2}$$

Therefore, we can define

$$R(\varphi,s)=\frac{2}{(1+e^{-\varphi s})^2}>0$$

And the Eq. (11) is satisfied.

Case 2: Another example which can satisfy the mentioned condition in Eq. (10) is as follows

$$g(\varphi,s)=\frac{1-(\varphi s)^2}{1+(\varphi s)^2} \qquad (22)$$

Consequently,

$$\frac{dg(\varphi,s)}{d\varphi}=\frac{-4(\varphi s)}{(1+(\varphi s)^2)^2}$$

So with definition

$$h(\varphi,s)=\frac{-1}{(\varphi)}$$

the Eq. (11) is satisfied

$$\frac{dg(\varphi,s)}{d\varphi}h(\varphi,s)=\frac{-4\varphi s}{(1+(\varphi s)^2)^2}\left(\frac{-1}{\varphi}\right)=\frac{4s}{(1+(\varphi s)^2)^2}$$

Therefore

$$R(\varphi,s)=\frac{4}{(1+(\varphi s)^2)^2}>0$$

Case 3: The other function that can meet the mentioned conditions is as follows

$$g(\varphi,s)=\frac{2}{\pi}\mathrm{Arctan}(\varphi s) \qquad (23)$$

As a result,

$$\frac{dg(\varphi,s)}{d\varphi}=\frac{2}{\pi}\frac{s}{(1+(\varphi s)^2)}$$

So, with introducing $h(\varphi,s)=1$, the Eq. (11) is satisfied again

$$R(\varphi,s)=\frac{2}{\pi(1+(\varphi s)^2)}$$

NUMERICAL SIMULATIONS

This section of the paper presents illustrative examples to verify and demonstrate the effectiveness of the proposed method. The simulation results are carried out using the MATLAB software. A time step size 0.001 was employed.

In the first place, it seems necessary to mention that the simulation results for function given in case 1 were presented in [35] and it seems unnecessary to show them here.

As a result, only the simulation results of case 2 and 3 have been shown in this section.

The simulation is done with the initial values $[x1\ x2\ x3]^T=[-1\ 1\ 0]^T$, $\hat{\alpha}_0=1$, $\hat{\beta}_0=11$, $\varphi_0=1$, $\gamma=0.9$, $\mu=25$.

The system is perturbed by an uncertainty term

$$\Delta f(x,t)=0.5\sin(\pi x_1)\sin(2\pi x_2)\sin(3\pi x_3)$$

and $d(t)=0.2\cos(t)$, where $\Delta f(x,t)\leq\alpha=0.5$ and $d(t)\leq\beta=0.2$. Also we assume that $X_d=0$.

The system with the parameters a=1.2, b=2.92 and c=6 is chaotic and in [35] has been shown previously. The simulation results for case 2 are shown in Fig. 2 and 3 with the adaptation algorithm and under the given

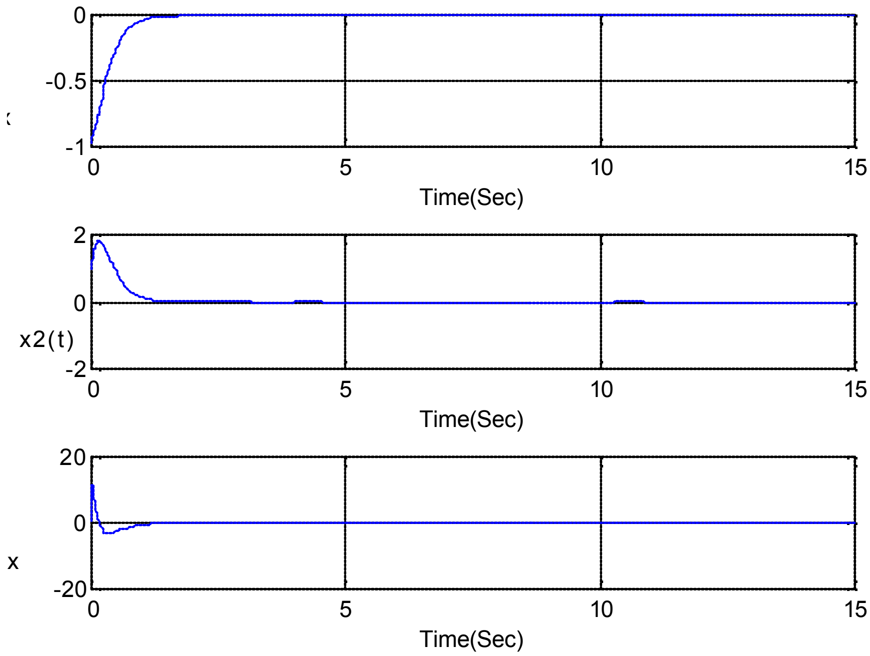

Fig. 1: The time response of system states (case 2)

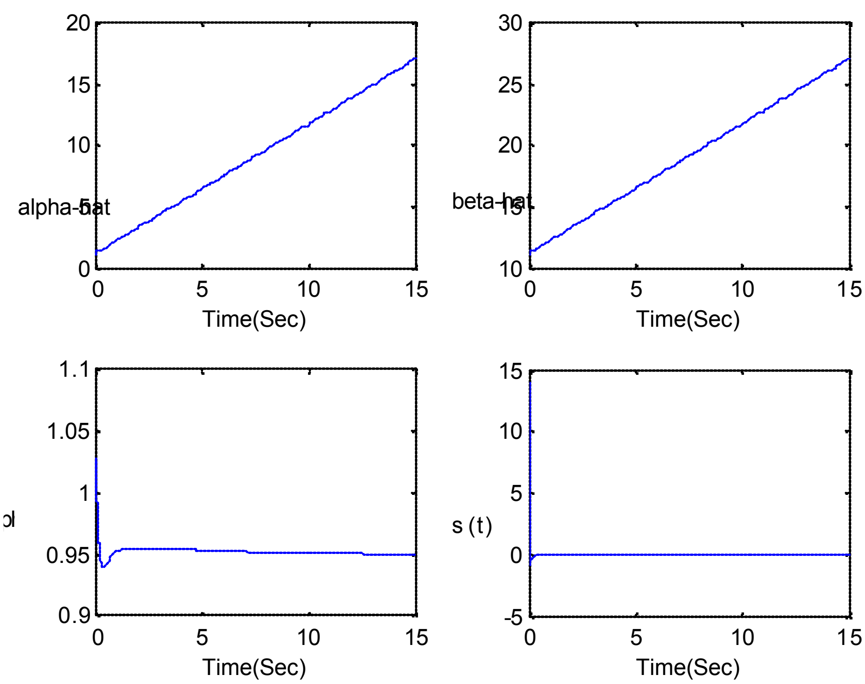

Fig. 2: The response of adaptation parameters and s (t) versus time (case 2)

ASMC. Figure 2 represents respectively the state time responses of system. This figure obviously shows that the dynamic trace the desired state. The adaptation parameters and the sliding surface dynamic (s(t)) are shown in Fig. 2. It can be seen that chattering does not appear, due to continuous control.

Also for the case 3 initial values are

$$[x1\ x2\ x3]^T = [0 -1\ 2]^T$$

The other parameters are initialized as follows

$$\hat{\alpha}_0 = 1,\ \hat{\beta}_0 = 11,\ \varphi_0 = 0.9,\ \gamma = 4,\ \mu = 30$$

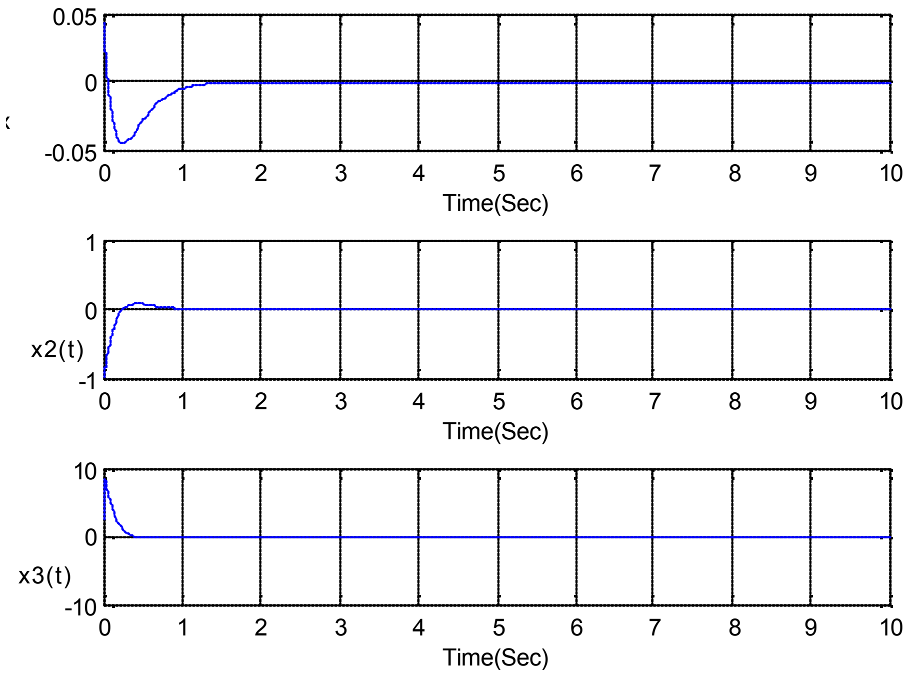

Fig. 3: The time response of system states (case 3)

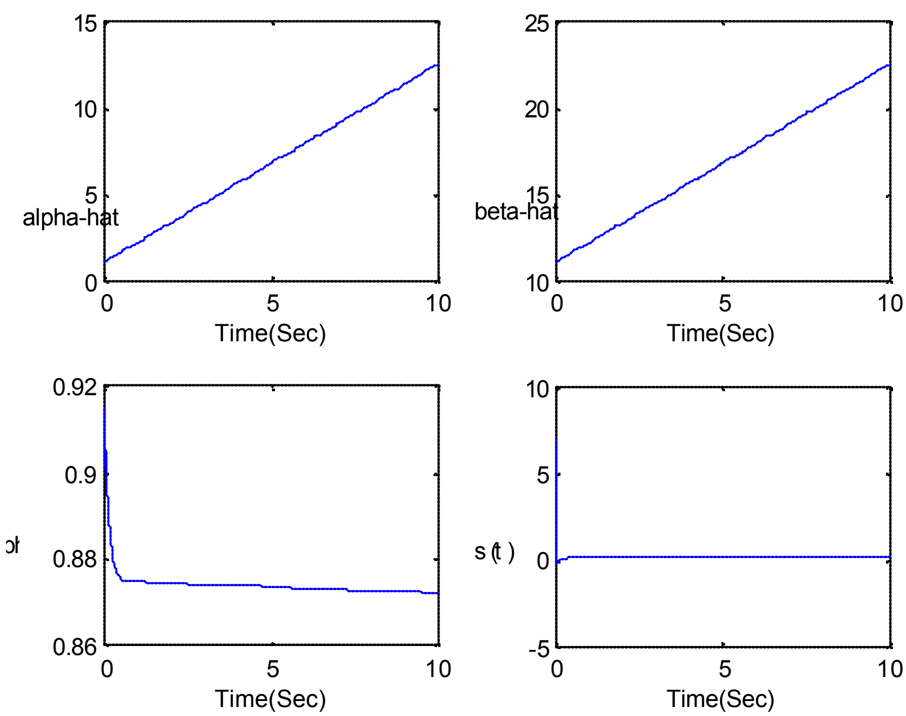

Fig. 4: The response of adaptation parameters and s (t) versus time (case 3)

The simulation results are represented in Fig. 3 and 4. From the simulation results, it is clear that the obtained theoretic results are feasible and efficient for synchronization of two uncertain chaotic dynamical systems.

CONCLUSION

In this paper, the control of chaotic nonlinear systems is studied, using Lyapunov method. The proposed control scheme can be implemented without requiring the bounds of unknown parameters, disturbance and uncertainties to be known in advance and the chattering phenomenon is eliminated. In comparison with the other method presented in the past, our proposed scheme showed better synchronization performance and is more comprehensive.

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