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The Modified Variational Iteration Transform Method (MVITM) for Solve Nonlinear Partial Differential Equation (NPDE)

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Abstract: In this paper, we are development the modified variation iteration method for solving nonlinear equation this method used Laplace transformation, the technique solve nonlinear problem without He's polynomial approximation analytical solution of nonlinear equation by the Laplace transform method, the scheme is tested for some examples,it is shown that the solution obtained by the Variational Iterational Methods (VIM) and special cases of Homotopy Analysis Method, comparison that solution by Numerical solution solving by Mathematica Software, result also show that numerical scheme is very effective.

Key words: Variational iteration method . laplace transform method . nonlinear differential equation homotopy analysis method . nonlinear partial differential equation

INTRODUCTION

Many problem and engineering science are modeled by partial differential equation;. Nonlinear equation which one of the basic nonlinear evolution equation

$$u_{tt}(x,t) - cu(x,t)u_x(x,t) + k\left(u(x,t)\right)^2 = f(x,t), x \in R, t > 0 \quad (1.1)$$

where c, k is constant arisen various branched of physics,engineering and applied science with initial condition

$$u(x,0) = f(x), u_t(x,0) = g(x) \quad (1.2)$$

In this work, we will use the modified form of Laplace variational iteration method is called Modified Variational Iteration Transform Methods (MVITM) will be employed in a straight forward manner, The variational iteration method was developed by Ji Huan He [1-3]. This method provides an effective and efficient way of solving a wide range of nonlinear operator equations [4, 5]. HPM for solving differential and integral equation, This method was first proposed by He [5-19, 20] and has been the subject of extensive analytical equation and numerical studies, here we use the Homotopy Perturbation Method coupled with the Laplace transformation for solving the nonlinear eq (1.1), we have more analytical method,These methods include the harmonic balance method (HB) [9, 10], the elliptic Lindstedt-Poincare method (LP), the Krylov Bogoliubov-Mitropolsky Method (KBM) [15, 16] the averaging [17] and Multiple Scales Method (MSM) [50], He [5-17, 20] has proposed a new perturbation technique to eliminate the "small parameter" assumption. This method is called the Homotopy Perturbation Method (HPM) and is applied to various nonlinear oscillator equations; for more details [1, 10-13, 16] and the references therein

BASIC IDEA OF VARIATIONAL ITERATION METHOD (VIM)

We consider the following general differential equation

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = f(x,t) \quad (2.1)$$

With the initial condition

$$u(x,0) = h(x), u_t(x,0) = f(x) \quad (2.2)$$

where L is Linear operator, R is linear operator less than L,N is nonlinear operator, f(x,t) is inhomogeous term. According to variational iteration method [1-4]. We can construct a correction functional as follows:

$$u_{n+1} = u_n + \int_0^t \lambda(\xi)\begin{bmatrix} Lu(x,\xi) + R\tilde{u}(x,\xi) \\ +N\tilde{u}(x,\xi) - f(x,t)\end{bmatrix} d\xi \quad (2.3)$$

where λ is a Lagrange multiplier [17, 18] where L is given λ =-1. The successive approximation, u_{n+1}, $n \geq 0$ of the solution u the solution is given by

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$$u = \lim_{n\to\infty} u_n \tag{2.4}$$

MODIFIED HOMOTOPY PERTURBATION TRANSFORM METHOD (MHPTM)

Now we consider a general nonlinear equation [6, 7] non-homogenous partial differential equation

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = f(x,t) \tag{3.1}$$

with the initial condition and in this paper L is operator ($\partial^2/\partial t^2$)

$$u(x,0) = h(x), u_t(x,0) = f(x) \tag{3.2}$$

Taking the Lapace transforms we obtain:

$$\wp Lu(x,t) + \wp Ru(x,t) + \wp Nu(x,t) = \wp f(x,t) \tag{3.3}$$

With the Laplace transformation

$$\begin{aligned} s^2\wp u(x,t) - su(x,0) - u_t(x,0) = \wp f(x,t) - \wp Ru(x,t) \\ - \wp Nu(x,t) \end{aligned} \tag{3.4}$$

Or

$$\begin{aligned} \wp u(x,t) = \frac{h(x)}{s} + \frac{f(x)}{s^2} + \frac{1}{s^2}\wp f(x,t) \\ - \frac{1}{s^2}\wp Ru(x,t) - \frac{1}{s^2}\wp Nu(x,t) \end{aligned} \tag{3.5}$$

Taking the inverse Laplace

$$\begin{aligned} u(x,t) = h(x) + f(x)t + \wp^{-1}\left(\frac{1}{s^2}\wp f(x,t)\right) \\ - \wp^{-1}\left(\frac{1}{s^2}\wp Ru(x,t)\right) - \wp^{-1}\left(\frac{1}{s^2}\wp Nu(x,t)\right) \end{aligned} \tag{3.6}$$

Let

$$H(x,t) = h(x) + f(x)t + \wp^{-1}\left(\frac{1}{s^2}\wp f(x,t)\right) \tag{3.7}$$

$$\begin{aligned} \wp u(x,t) = H(x,t) - \wp^{-1}\left(\frac{1}{s^2}\wp Ru(x,t)\right) \\ - \wp^{-1}\left(\frac{1}{s^2}\wp Nu(x,t)\right) \end{aligned} \tag{3.8}$$

By Homotopy Perturbation Method

$$u(x,t) = H(x,t) - p\wp^{-1}\left(\frac{1}{s^2}\left(\wp Ru(x,t) + \wp Nu(x,t)\right)\right) \tag{3.9}$$

By He's polynomials the nonlinear operator He's polynomial is given:

$$A_n = \frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{i=0}^{n} p^i u_i\right)\right]_{p=0} \tag{3.10}$$

And apply Construction homotopy Perturbation

$$\left(\sum_{i=0}^{n} p^i u_i\right) = H(x,t) - p\wp^{-1}\left(\frac{1}{s^2}\left(\begin{aligned} \wp R\left(\sum_{i=0}^{n} p^i u_i\right) \\ + \wp N\left(\sum_{i=0}^{n} p^i u_i\right)\end{aligned}\right)\right) \tag{3.11}$$

By He' polynomials eq

$$\left(\sum_{i=0}^{n} p^i u_i\right) = H(x,t) - p\wp^{-1}\left(\frac{1}{s^2}\left(\wp R\left(\sum_{i=0}^{n} p^i u_i\right) + A_n\right)\right) \tag{3.12}$$

Coopering the coefficient of like power of p

$$p^0: u_0(x,t) = H(x,t) \tag{3.13}$$

$$P^1: u_1(x,t) = -\wp^{-1}\left(\frac{1}{s^2}\left(\wp Ru_0(x,t) + A_0\right)\right)$$

$$P^2: u_2(x,t) = -\wp^{-1}\left(\frac{1}{s^2}\left(\wp Ru_1(x,t) + A_1\right)\right) \tag{3.14}$$

$$P^3: u_3(x,t) = -\wp^{-1}\left(\frac{1}{s^2}\left(\wp Ru_2(x,t) + A_2\right)\right) \ldots\ldots$$

$$P^{n+1}: u_{n+1}(x,t) = -\wp^{-1}\left(\frac{1}{s^2}\left(\wp Ru_n(x,t) + A_n\right)\right)$$

MODIFIED VARIATIONAL ITERATION TRANSFORM METHOD (MVTM)

Now we consider a general nonlinear equation non-homogenous partial differential equation

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = f(x,t) \tag{4.1}$$

with the initial condition and in this paper L is operator ($\partial^2/\partial t^2$)

$$u(x,0) = h(x), u_t(x,0) = f(x) \tag{4.2}$$

Taking the Lapace transforms we obtain:

$$\wp Lu(x,t) + \wp Ru(x,t) + \wp Nu(x,t) = \wp f(x,t) \tag{4.3}$$

With the Laplace transformation

$$s^2\wp u(x,t)-su(x,0)-u_t(x,0)=\wp f(x,t)-\wp Ru(x,t)-\wp Nu(x,t) \quad (4.4)$$

Or

$$\wp u(x,t)=\frac{h(x)}{s}+\frac{f(x)}{s^2}+\frac{1}{s^2}\wp f(x,t)-\frac{1}{s^2}\wp Ru(x,t)-\frac{1}{s^2}\wp Nu(x,t) \quad (4.5)$$

Taking the inverse Laplace

$$u(x,t)=h(x)+f(x)t+\wp^{-1}\left(\frac{1}{s^2}\wp f(x,t)\right)-\wp^{-1}\left(\frac{1}{s^2}\wp Ru(x,t)\right)-\wp^{-1}\left(\frac{1}{s^2}\wp Nu(x,t)\right) \quad (4.6)$$

Derivative by $\partial/\partial t$ both side ()

$$u_t(x,t)+\wp^{-1}\left(\frac{1}{s^2}\wp Ru(x,t)\right)+\wp^{-1}\left(\frac{1}{s^2}\wp Nu(x,t)\right)-\wp^{-1}\left(\frac{1}{s^2}\wp f(x,t)\right)+f(x)=0 \quad (4.7)$$

By the correction function of the irrational Method

$$u_{n+1}=u_n-\int_0^t\left[\begin{array}{c} u_\xi(x,t)+\wp^{-1}\left(\frac{1}{s^2}\wp Ru(x,t)\right)\\ +\wp^{-1}\left(\frac{1}{s^2}\wp Nu(x,t)\right)\\ -\wp^{-1}\left(\frac{1}{s^2}\wp f(x,t)\right)+f(x)\end{array}\right]d\xi \quad (4.8)$$

Solution u the solution is given by

$$u=\lim_{n\to\infty}u_n \quad (4.9)$$

ILLUSTRATIVE EXAMPLES

In this section, we solve the linear-nonlinear problem with the last correction Modified Variational Iteration Transform Method (MVITM) and we used the eq (4.8) to solve nonlinear problem

Example 1: We consider the following linear Schrödinger equation

$$u_t+iu_{xx}=0 \quad (5.1)$$

With initial condition

$$u(x,0)=1+2\cosh 2x \quad (5.2)$$

We apply Laplace transform in both side

$$\wp u_t+\wp iu_{xx}=0 \quad (5.3)$$

By Laplace probability

$$s\wp u-u(x,0)+i\wp u_{xx}=0 \quad (5.4)$$

By inverse Laplace and derivative

$$u_t+\frac{\partial}{\partial t}\left(i\wp^{-1}\frac{1}{s}\wp u_{xx}\right)=0 \quad (5.5)$$

Making the correction function is given

$$u_{n+1}=u_n-\int_0^t\left[(u_n)_\xi+\frac{\partial}{\partial\xi}\left(i\wp^{-1}\frac{1}{s}\wp u_{xx}\right)\right]d\xi \quad (5.6)$$

We apply the modified variational iteration transform method and with Mathematica package the solution in series form is given by

$$u_0=1+2\cosh 2x$$

$$u_1=1+2\cosh 2x-8it\cosh 2x$$

$$u_2=1+2\cosh 2x-8it\cosh 2x-16t^2\cosh 2x \quad (5.7)$$

$$u_3=1+2\cosh 2x-8it\cosh 2x-16t^2\cosh 2x+\frac{64}{3}it^3\cosh 2x$$

………

And in closed form by

$$u(x,t)=1+\cosh 2xe^{-4it} \quad (5.8)$$

Example 2: We consider the following linear Schrödinger equation

$$iu_t+u_{xx}-2u=0 \quad (5.9)$$

With initial condition

$$u(x,0)=e^{ix} \quad (5.10)$$

We apply Laplace transform in both sides

$$i\wp u_t+\wp u_{xx}-2\wp u=0 \quad (5.11)$$

By Laplace probability

$$s\wp u-u(x,0)-i\wp u_{xx}+2i\wp u=0 \quad (5.12)$$

By inverse Laplace and derivative

$$u_t - \frac{\partial}{\partial t}\left(i\,\wp^{-1}\frac{1}{s}\wp\, u_{xx} \right) + 2i\frac{\partial}{\partial t}\left(\wp^{-1}\frac{1}{s}\wp u \right) = 0 \qquad (5.13)$$

Making the correction function is given

$$u_{n+1} = u_n - \int_0^t \left[\begin{array}{l} u_\xi - i\frac{\partial}{\partial t}\left(\wp^{-1}\frac{1}{s}\wp\, u_{xx} \right) \\ +2i\frac{\partial}{\partial t}\left(\wp^{-1}\frac{1}{s}\wp\, u \right) \end{array} \right] d\xi \qquad (5.14)$$

We apply the modified variational iteration transform method and with Mathematica package the solution in series form is given by

$$\begin{aligned} u_0 &= e^{ix} \\ u_1 &= e^{ix} - 3ite^{ix} \\ u_2 &= e^{ix} - 3ite^{ix} - \frac{9}{2}t^2e^{ix} \\ u_3 &= e^{ix} - 3ite^{ix} - \frac{9}{2}t^2e^{ix} + \frac{9}{2}i\,t^3e^{ix} \\ &\ldots\ldots\ldots \end{aligned} \qquad (5.15)$$

And in closed form by

$$u(x,t) = e^{i(x-3t)} \qquad (5.16)$$

Example 3: We consider the following nonlinear differential equation

$$u_{xx} - u_x u_{yy} = -x + u \qquad (5.17)$$

With initial condition

$$u(x,0) = \sin y, u_x(x,0) = 1 \qquad (5.18)$$

We apply Laplace transform in both sides

$$\wp u_{xx}(x,y) - \wp u_x u_{yy} - \wp u + \wp x = 0 \qquad (5.19)$$

By Laplace probability

$$\begin{aligned} s^2\wp u(x,y) - su(x,0) - u_x(x,0) \\ -\wp u_x u_{yy} - \wp u + \wp x = 0 \end{aligned} \qquad (5.20)$$

By inverse Laplace and derivative

$$u_x - \frac{\partial}{\partial x}\left(\wp^{-1}\frac{1}{s^2}\wp\, u_x u_{yy} \right) - \frac{\partial}{\partial x}\left(\wp^{-1}\frac{1}{s^2}\wp\, u \right) + \frac{x^2}{2} - 1 = 0 \qquad (5.21)$$

Making the correction function is given

$$u_{n+1} = u_n - \int_0^x \left[\begin{array}{l} u_\xi - \frac{\partial}{\partial \xi}\left(\wp^{-1}\frac{1}{s^2}\wp\, u_\xi u_{yy} \right) \\ -\frac{\partial}{\partial \xi}\left(\wp^{-1}\frac{1}{s^2}\wp\, u \right) + \frac{\xi^2}{2} - 1 \end{array} \right] d\xi \qquad (5.22)$$

We apply the modified variational iteration transform method and with Mathematica package the solution in series form is given by

$$\begin{aligned} u_0 &= x + \sin y \\ u_1 &= x + \sin y \\ u_2 &= x + \sin y \\ u_3 &= x + \sin y \\ &\ldots\ldots\ldots \end{aligned} \qquad (5.23)$$

And in closed form by

$$u(x,t) = x + \sin y \qquad (5.24)$$

CONCLUSION

In this work we carefully proposed a reliable modification of laplace variational iteration method is called Modified Variational Iteration Transform Method, we solve three nonlinear partial differential equation with initial condition.This modified technique has been shown to computationally efficient in these example,that are important to researchers in the field of applied science we applied modified Variational Iteration Transform (MVITM) for solving Schrödinger equation powerful tool to search for solutions of various nonlinear problems. We derived the same results by combining the series, obtained by MHPTM. Comparison the result by Numerical solution

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