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Fully Fuzzy Linear Systems Solving Using MOLP

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Abstract: As can be seen from the definition of extended operations on fuzzy numbers, subtraction and division of fuzzy numbers are not the inverse operations to addition and multiplication, respectively. Hence for solving equations or system of equations, we must use methods without using inverse operators. In this paper, we propose a novel method to find the nonzero solutions of fully fuzzy linear systems (shown as **FFLS**). We suppose that all elements of matrix of coefficients are positive. We employ embedding method to transform $n\times n$(**FFLS**) to $2n\times 2n$ parametric form linear system. We decompose solutions to non negatives and non positives using solutions of one MOLP in the initial step of this method. Finally, numerical examples are used to illustrate this approach.

Key words: Fuzzy numbers . fully fuzzy linear systems . systems of fuzzy linear equations . non-zero solutions . decomposition method

INTRODUCTION

System of equations is the simplest and the most useful mathematical model for a lot of problems considered by applied mathematics. In practice, unfortunately, the exact values of coefficients of these systems are not a rule known. This uncertainty may have either probabilistic or non probabilistic nature. Accordingly, different approaches to the problem and different mathematical tools are needed.

In this article, system of linear equations whose coefficients and right hand sides are fuzzy numbers is considered. The system of linear equations $\mathbf{AX} = \mathbf{b}$ where the elements, $\tilde{a}_{ij}$, of the matrix $\mathbf{A}$ and the elements, $\tilde{b}_i$, of $\mathbf{b}$ are fuzzy numbers, is called Fully Fuzzy Linear System (**FFLS**).

Abramovich and his colleagues in [1], Buckley and Qu in [5-7], Muzzioli and Reynaerts in [14, 15], Dehghan and his colleagues in [8-10], Vroman and her colleagues in [16-18], Allahviranloo and his colleagues in [2-4] suggested different approaches to solving (**FFLS**).

In many applications, that can be solved by solving system of linear equations, system's parameters are positive and hence it is important to propose method to find non-zero solutions of (**FFLS**), where system's parameters are positive.

Allahviranloo *et al*. in [3] suggests a new method for solving (**FFLS**) using decomposition method. In the initial step of their method, they solve 0-cut interval linear system for decompose variables to non positives and non negatives. Base on their work, in this paper, we are going to find non-zero solutions of this (**FFLS**). For this reason, we decompose variables in two groups: non positives and non negatives using solutions of one MOLP in the initial step and transform multiplications of fuzzy numbers to multiplications of functions. We use embedding approach to replace the original $n\times n$ (**FFLS**) by a $2n\times 2n$ parametric linear system and design numerical method for calculating the solutions.

The structure of this paper is organized as follows:

In section 2, we discuss some basic definitions, results on fuzzy numbers and (**FFLS**). In section 3, we discuss our numerical procedure for finding non-zero solutions of (**FFLS**) and the proposed algorithm are illustrated by solving some numerical examples. Conclusion are drawn in section 4.

PRELIMINARIES

The set of all fuzzy numbers is denoted by **E** and defined as follows:

Definition 1: [12, 13] A fuzzy number $\tilde{u}$ is a pair $(\underline{u}(r),\overline{u}(r))$ of functions $\underline{u}(r),\overline{u}(r); 0 \le r \le 1$ which satisfy the following requirements:

- $\underline{u}(r)$ is a bounded monotonic increasing left continuous function;
- $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function;
- $\underline{u}(r) \le \overline{u}(r), 0 \le r \le 1$.

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A crisp number k is simply represented by $\overline{k}(r)=\underline{k}(r)=k;\ 0\le r\le 1$ and called singleton. The fuzzy number space $\{\underline{u}(r),\overline{u}(r)\}$ becomes a convex cone **E** which is then embedded isomorphically and isometrically into a Banach space.

A fuzzy number $\tilde{a}$ can be represented by its λ-cuts (0<λ≤1):

$$\tilde{a}^{\lambda}=\{x\,|\,x\in R,\ \tilde{a}(x)\ge\lambda\}$$

and

$$\operatorname{supp}\tilde{a}=\tilde{a}^{0}=Cl(\{x|x\in R,\ \tilde{a}(x)>0\})=[\underline{a}(0),\overline{a}(0)]$$

Note that the λ-cuts of a fuzzy number are closed and bounded intervals. The fuzzy arithmetic based on the Zadeh extension principle can also be calculated by interval arithmetic applied to the λ-cuts.

For fuzzy number

$$\tilde{u}=(\underline{u}(r),\overline{u}(r)),\ \ 0\le r\le 1$$

we will write (1) $\tilde{u}>0$ if $\underline{u}(0)>0$, (2) $\tilde{u}\ge 0$ if $\underline{u}(0)\ge 0$, (3) $\tilde{u}<0$ if $\overline{u}(0)<0$, (4) $\tilde{u}\le 0$ if $\overline{u}(0)\le 0$. If $\tilde{u}\le 0$ or $\tilde{u}\ge 0$ this fuzzy number is called non-zero fuzzy number. For arbitrary

$$\tilde{u}=(\underline{u}(r),\overline{u}(r))\ ,\ \tilde{v}=(\underline{v}(r),\overline{v}(r))$$

and k>0 we define addition $(\tilde{u}+\tilde{v})$, subtraction $(\tilde{u}-\tilde{v})$ and multiplication $(\tilde{u}.\tilde{v})$ as:

Addition:

$$(\underline{u+v})(r)=\underline{u}(r)+\underline{v}(r),\quad (\overline{u+v})(r)=\overline{u}(r)+\overline{v}(r) \tag{1}$$

Subtraction:

$$(\underline{u-v})(r)=\underline{u}(r)-\overline{v}(r),\quad (\overline{u-v})(r)=\overline{u}(r)-\underline{v}(r) \tag{2}$$

Multiplication:

$$\begin{aligned}(\underline{uv})(r)&=\min\{\overline{u}(r)\overline{v}(r),\overline{u}(r)\underline{v}(r),\underline{u}(r)\overline{v}(r),\underline{u}(r)\underline{v}(r)\},\\(\overline{uv})(r)&=\max\{\overline{u}(r)\overline{v}(r),\overline{u}(r)\underline{v}(r),\underline{u}(r)\overline{v}(r),\underline{u}(r)\underline{v}(r)\}\end{aligned} \tag{3}$$

The important cases where we want to calculate multiplication of two fuzzy numbers are as follows:

Case 1: u≥0 and v≥0

$$\underline{uv}(r)=\underline{u}(r)\underline{v}(r),\quad \overline{uv}(r)=\overline{u}(r)\overline{v}(r)$$

Case 2: u≥0 and v≤0

$$\underline{uv}(r)=\overline{u}(r)\underline{v}(r),\quad \overline{uv}(r)=\underline{u}(r)\overline{v}(r)$$

Definition 2: The n×n linear system of equations

$$\begin{cases}\tilde{a}_{11}\tilde{x}_1+\tilde{a}_{12}\tilde{x}_2+\cdots+\tilde{a}_{1n}\tilde{x}_n=\tilde{b}_1\\ \tilde{a}_{21}\tilde{x}_1+\tilde{a}_{22}\tilde{x}_2+\cdots+\tilde{a}_{2n}\tilde{x}_n=\tilde{b}_2\\ \qquad\vdots\\ \tilde{a}_{n1}\tilde{x}_1+\tilde{a}_{n2}\tilde{x}_2+\cdots+\tilde{a}_{nn}\tilde{x}_n=\tilde{b}_n\end{cases} \tag{4}$$

where the elements, $\tilde{a}_{ij}$, of the coefficient matrix A, 1≤i, j≤n and the elements, $\tilde{b}_i$, of the vector **b** are fuzzy numbers is called a fully fuzzy linear system of equations (**FFLS**).

Definition 3: A fuzzy number vector $(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)^t$ given by $\tilde{x}_i=(\underline{x}_i(r),\overline{x}_i(r))\ \ 1\le i\le n,\ 0\le r\le 1$ is called a solution of (**FFLS**), if

$$\begin{cases}\underline{\sum_{j=1}^{n}a_{ij}x_j}(r)=\sum_{j=1}^{n}\underline{a_{ij}x_j}(r)=\underline{b}_i(r),\\ \overline{\sum_{j=1}^{n}a_{ij}x_j}(r)=\sum_{j=1}^{n}\overline{a_{ij}x_j}(r)=\overline{b}_i(r),\end{cases}\quad i=1,\cdots,n \tag{5}$$

Now, we define non-zero fuzzy number solution of (**FFLS**) as follows:

Definition 4: A fuzzy number solution vector $(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)^t$ of (**FFLS**) is called non-zero fuzzy number solution if for all i, (i=1,2,...,n) $\tilde{x}_i$ are non-zero fuzzy numbers.

Necessary and sufficient condition for the existence of a Non-zero fuzzy number solution of (**FFLS**) is:

Theorem: [3] If (**FFLS**) **AX** = **b** has a fuzzy number solution, then (**FFLS**) **AX** = **b** has non-zero fuzzy number solution if and only if 0-cut system of linear system represented by $\mathbf{A^0X^0}=\mathbf{b^0}$ has non-zero solution.

THE MODEL

As can be seen from the definition of extended operation on fuzzy numbers, subtraction and division of fuzzy numbers are not the inverse operations to addition and multiplication, respectively. Hence for solving equations or system of equations, we must use methods without using inverse operators.

In this section, we are going to replace an original $n\times n$ (**FFLS**) with a $\mathbf{2n\times 2n}$ parametric system and then discuss our algorithm to find this solution.

Let **AX** = **b** is (**FFLS**). Consider i-th equation of this system:

$$\sum_{j=1}^{n}\tilde{a}_{ij}\tilde{x}_j=\tilde{b}_i \quad ,i=1,\ldots,n \tag{6}$$

Now, let **AX** = **b** has a non-zero fuzzy number solution, we define

$$J=\{j\,|\,1\le j\le n\ ,\tilde{x}_j\ge 0\} \tag{7}$$

Hence we can decompose eq. (6) and rewrite it as:

$$\sum_{j=1}^{n}\tilde{a}_{ij}\tilde{x}_j=\sum_{j\in J}\tilde{a}_{ij}\tilde{x}_j+\sum_{j\notin J}\tilde{a}_{ij}\tilde{x}_j=\tilde{b}_i,\ i=1,\ldots,n \tag{8}$$

We define two n-vector $\tilde{W}=(\tilde{w}_1,\tilde{w}_2,\ldots,\tilde{w}_n)^t$ and $\tilde{V}=(\tilde{v}_1,\tilde{v}_2,\ldots,\tilde{v}_n)^t$ where

$$\tilde{w}_j=\begin{cases}\tilde{x}_j & \text{if } j\in J\\ 0 & \text{if } j\notin J\end{cases}$$

$$\tilde{v}_j=\begin{cases}\tilde{x}_j & \text{if } j\notin J\\ 0 & \text{if } j\in J\end{cases} \tag{9}$$

This is obvious that

$$\tilde{w}_j+\tilde{v}_j=\tilde{x}_j,\ \ 1\le j\le n \tag{10}$$

If we replace $\tilde{x}_j$ in (8) with $\tilde{w}_j+\tilde{v}_j$ it can be rewritten as follows:

$$\begin{aligned}\sum_{j=1}^{n}\tilde{a}_{ij}\tilde{x}_j&=\sum_{j\in J}\tilde{a}_{ij}\tilde{x}_j+\sum_{j\notin J}\tilde{a}_{ij}\tilde{x}_j\\&=\sum_{j\in J}\tilde{a}_{ij}\tilde{w}_j+\sum_{j\notin J}\tilde{a}_{ij}\tilde{v}_j\\&=\sum_{j=1}^{n}\tilde{a}_{ij}\tilde{w}_j+\sum_{j=1}^{n}\tilde{a}_{ij}\tilde{v}_j\\&=\tilde{b}_i\ ,i=1,\ldots,n,\end{aligned} \tag{11}$$

If we use definitions 3 and 4 and if $\tilde{x}=(\tilde{x}_1,\cdots,\tilde{x}_n)^t$ is fuzzy number solution of **AX** = **b**, eq (11) can be rewritten as:

$$\underline{b}_i(r)=\sum_{j=1}^{n}\underline{a_{ij}w_j}(r)+\sum_{j=1}^{n}\underline{a_{ij}v_j}(r)\quad i=1,\ldots,n \tag{12}$$

and

$$\overline{b}_i(r)=\sum_{j=1}^{n}\overline{a_{ij}w_j}(r)+\sum_{j=1}^{n}\overline{a_{ij}v_j}(r)\quad i=1,\ldots,n \tag{13}$$

Since $\tilde{w}_j\ge 0,\ \tilde{v}_j\le 0,(1\le j\le n)$ and by applying (3) we can write:

$$\begin{aligned}&\underline{a_{ij}w_j}(r)=\underline{a_{ij}}(r).\underline{w_j}(r)\quad \overline{a_{ij}w_j}(r)=\overline{a_{ij}}(r).\overline{w_j}(r)\\&\underline{a_{ij}v_j}(r)=\overline{a_{ij}}(r).\underline{v_j}(r)\quad \overline{a_{ij}v_j}(r)=\underline{a_{ij}}(r).\overline{v_j}(r)\end{aligned} \tag{14}$$

Now, if we replace above expressions in (12) and (13), they can be rewritten as:

$$\sum_{j=1}^{n}\underline{a_{ij}x_j}(r)=\sum_{j=1}^{n}\underline{a_{ij}}(r).\underline{w_j}(r)+\sum_{j=1}^{n}\overline{a_{ij}}(r)\underline{v_j}(r)=\underline{b_i}(r),\quad i=1,\ldots,n, \tag{15}$$

and

$$\sum_{j=1}^{n}\overline{a_{ij}x_j}(r)=\sum_{j=1}^{n}\overline{a_{ij}}(r)\overline{w_j}(r)+\sum_{j=1}^{n}\underline{a_{ij}}(r).\overline{v_j}(r)=\overline{b_i}(r),\quad i=1,\ldots,n. \tag{16}$$

Note that before solving (**FFLS**) **AX** = **b** since we decompose matrix of coefficients, we must have information about its solution such as: 1-Does this system have a nonzero solution? and 2-Is $\tilde{x}_j$ positive fuzzy number? In [3] Allahviranloo *et al*. find their solutions by solving 0-cut system of **AX** = **b** to find (**FFLS**) **AX** = **b** solution supports. After solving this interval system they find our questions'answers.

Now, we propose another method for find their solutions without solving 0-cut system of **AX** = **b** to find (**FFLS**) **AX** = **b** solution supports. We define a crisp number y_j, $1\le j\le n$ as follows:

$$y_j=\min\{x\in \mathbb{R}\,|\,x\ge 0,\ x+\tilde{v}_j\ge 0\} \tag{17}$$

And define $\tilde{z}_j\ge 0$

$$\tilde{z}_j=\tilde{v}_j+y_j\ge 0,\ \ 1\le j\le n \tag{18}$$

Hence

$$\tilde{v}_j=\tilde{z}_j-y_j,\ 1\le j\le n \tag{19}$$

This is obvious that if $y_j = 0$ then $\tilde{x}_j \geq 0$ and hence $\tilde{v}_j = 0$ and $\tilde{w}_j = \tilde{x}_j$. but if $y_j > 0$ we can not infer $\tilde{x}_j$ is negative fuzzy number and we only can infer $\tilde{x}_j$ is non positive.

By applying (1)-(4) and (19) and replacing in (15) and (16) we can rewrite they as follows:

$$\sum_{j=1}^{n} \underline{a_{ij}x_j}(r) = \sum_{j=1}^{n} \underline{a_{ij}}(r).\underline{w_j}(r) + \sum_{j=1}^{n} \overline{a_{ij}}(r).(\underline{z_j}(r) - y_j) = \underline{b_i}(r) \qquad i=1,\ldots,n \tag{20}$$

and

$$\sum_{j=1}^{n} \overline{a_{ij}x_j}(r) = \sum_{j=1}^{n} \overline{a_{ij}}(r).\overline{w_j}(r) + \sum_{j=1}^{n} \underline{a_{ij}}(r).(\overline{z_j}(r) - y_j) = \overline{b_i}(r), \qquad i=1,\ldots,n \tag{21}$$

And hence,

$$\sum_{j=1}^{n} \underline{a_{ij}}(r).\underline{w_j}(r) + \sum_{j=1}^{n} \overline{a_{ij}}(r).\underline{z_j}(r) = \underline{b_i}(r) + \sum_{j=1}^{n} \overline{a_{ij}}(r)y_j, \; i=1,\ldots,n \tag{22}$$

and

$$\sum_{j=1}^{n} \overline{a_{ij}}(r).\overline{w_j}(r) + \sum_{j=1}^{n} \underline{a_{ij}}(r).\overline{z_j}(r) = \overline{b_i}(r) + \sum_{j=1}^{n} \underline{a_{ij}}(r)y_j, \; i=1,\ldots,n \tag{23}$$

Now we illustrate these equations in matrix forms. If A_1 and A_2 are parametric $n\times n$ matrices by elements

$$(A_1)_{ij} = \underline{a_{ij}}(r), \; (A_2)_{ij} = \overline{a_{ij}}(r), \quad 1 \leq i,j \leq n \tag{24}$$

and if W_1, W_2, Z_1, Z_2, B_1 and B_2 are parametric n-vectors by elements

$$(W_1)_j = \underline{w_j}(r), \quad (W_2)_j = \overline{w_j}(r),$$
$$(Z_1)_j = \underline{Z_j}(r), \quad (Z_2)_j = \overline{Z_j}(r), \quad 1 \leq i,j \leq n \tag{25}$$
$$(B_1)_j = \underline{b_j}(r), \quad (B_2)_j = \overline{b_j}(r),$$

And $Y = (y_1,\ldots, y_n)$, $n\times n$ matrix representation of (**FFLS**) **AX** = **b** transform to

$$\begin{pmatrix} A_1 & 0 & A_2 & 0 \\ 0 & A_2 & 0 & A_1 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} B_1 + A_2Y \\ B_2 + A_1Y \end{pmatrix} \tag{26}$$

where this coefficients matrix represents in $2n\times 4n$. But in fact, by definition of Z_1, Z_2, W_1 and W_2, 2n elements of variable matrix are zero and hence 2n columns of coefficient matrix are omitted and hence we replace $n\times n$(**FFLS**) by an $2n\times 2n$ system of linear parametric equations. We do not know any information about x_j and hence we can not define w_j, z_j and y_j, ($j = 1,\ldots,n$). By definition of positive fuzzy number and definition of y_j, for finding y_j and hence finding w_j and z_j ($j = 1,\ldots,n$) we solve 0-cut of fully fuzzy linear system where transform to (26) and satisfy in (19). Hence we only require to solve the following (MOLP):

$$\begin{aligned}
&\text{Min } y_1 \\
&\text{Min } y_2 \\
&\quad\vdots \\
&\text{Min } y_n \\
&\text{s.t.} \\
&\sum_{j=1}^{n}(\underline{a_{ij}}(0).\underline{w_j}(0) + \sum_{j=1}^{n} \overline{a_{ij}}(0).(\underline{z_j}(0) - y_j) = \underline{b_i}(0), \; i=1,\ldots,n, \\
&\sum_{j=1}^{n} \overline{a_{ij}}(0).\overline{w_j}(0) + \sum_{j=1}^{n} \underline{a_{ij}}(0).(\overline{z_j}(0) - y_j) = \overline{b_i}(0), \; i=1,\ldots,n, \\
&\underline{z_j}(0) \leq \overline{z_j}(0) \quad ,j=1,\ldots,n, \\
&\underline{w_j}(0) \leq \overline{w_j}(0) \quad ,j=1,\ldots,n, \\
&\underline{w_j}(0) \geq 0, \quad ,j=1,\ldots,n, \\
&\overline{z_j}(0) \geq 0, \quad ,j=1,\ldots,n, \\
&y_j(0) \geq 0, \quad ,j=1,\ldots,n,
\end{aligned} \tag{27}$$

If this MOLP does not have any feasible solution, 0-cut of fully fuzzy linear system would not have any solution and hence this (**FFLS**) is unsolvable. If above MOLP has a feasible solution, we can find z_j ($j = 1,\ldots,n$) by solving (26) $2n\times 4n$ parametric linear system. Maybe, this MOLP have alternative solutions, but we only require a solution which satisfy the following condition: if y_j (0) = 00 then $v_j = 0$ ($j = 1,\ldots,n$) (because this MOLP's solution satisfy in(15),(16) and (19).)

After solve above MOLP and find required solution and replace it in (26) and solve this system, if its solutions define fuzzy number, (**FFLS**) **AX** = **b** has fuzzy number solution where

$$x_j = w_j + v_j (j=1,\dots,n)$$

Example [8] Consider the system of equations

$$\begin{cases} (4+r, 6-r)x_1 + (5+r, 8-2r)x_2 = (40+10r, 67-17r) \\ (6+r, 7)x_1 + (4, 5-r)x_2 = (43+5r, 55-7r) \end{cases} \quad (28)$$

Dehghan *et al*. in [8] solved this system. Their solution is

$$x_1 = \left(\frac{43}{11} + \frac{1}{11}r, 4\right)$$

and

$$x_2 = \left(\frac{54}{11} + \frac{1}{11}r, \frac{21}{4} - \frac{1}{4}r\right)$$

Where these are approximated solutions.
We solve this system by our algorithm as follow's :
First, we solve the MOLP.

The solution of this system is $y_1 = y_2 = 0$. Hence $v_1 = v_2 = 0$ and we can replace 2×2 coefficient matrix by 4×4 parametric coefficient matrix;
The solution vector-exact solution of this (**FFLS**)-is:

$$x_1 = \left(\frac{-5r^2 - 28r - 55}{-r^2 - 7r - 14}, \frac{3r^2 + 14r - 105}{r^2 + 3r - 26}\right)$$

and

$$x_2 = \left(\frac{-5r^2 - 37r - 68}{-r^2 - 7r - 14}, \frac{7r^2 + 22r - 139}{r^2 + 3r - 26}\right)$$

CONCLUSION

One of the important applications of mathematics is system of linear equations that when all system's parameters are fuzzy is called fully fuzzy linear system.

Allahviranloo *et al*. in [3], design a novel method for solving (**FFLS**) and for finding its non zero solutions using embedding method. In their method, they decompose variables to two groups: non negatives and non positives and if system's solutions is in two groups they solve this system and find its solutions. For this mean, in the initial step, they solve 0-cut system of (**FFLS**) and find its solutions. If its solutions are non zero, they use embedding method and transform $n\times n$ fuzzy system to $2n\times 2n$ parametric system and solve it. Since 0-cut system is interval linear system, for solving it, 2^t, $t \le n(n+1)$ crisp system must be solved.

Based on their work, in this paper, we suggest in the initial step of algorithm, replace solving 0-cut system by solving MOLP (27). This suggest has useful and harmful because, if $y_j = 0$ then $\tilde{x}_j \ge 0$ and hence $\tilde{v}_j = 0$ and $\tilde{w}_j = \tilde{x}_j$. but if $y_j > 0$ we can not infer $\tilde{x}_j$ is negative fuzzy number and we only can infer $\tilde{x}_j$ is non positive.

Maybe **2n**$\times$**2n** parametric system has solution, but its solutions can not define fuzzy numbers and hence (**FFLS**) does not have any fuzzy number solution.

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