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An Overview of Two-Dimensional Systems

Ahmadreza Argha and Mehdi Roopaei

Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, FARS, Iran

Abstract: This paper intends to make an overview of two-dimensional systems. Recently, 2-D systems play very important role in industry. As a result, it is of great importance to pay more attention to this kind of dynamics. Generally, in the 1-D systems the various quantities are the function of time. On the other hand in many phenomena in the nature, some quantities are function of two independent variables which mainly none of these two variables is time. Consequently, for modeling of these phenomena which have two independent variables, the 2-D signals and systems are used. In this kind of systems the process is done in two independent coordinate axes. In this article, 2-D systems and their specifications are studied briefly.

Key words: Two-dimensional systems . 2-D stability . WAM model

INTRODUCTION

In the theory of 1-D systems, generally various quantities are functions of time. For instance, the electrical potential of an electrical capacitor is only a function of time. On the other hand many phenomena in the nature have quantities which are function of two independent variables and generally none of them is time. As an example, the intensity of an image is function of horizontal and vertical axes of image. In this case, for modeling of this kind of systems 2-D signals are used. In these systems the processing operation is done in two independent axes. The main applications of 2-D systems are such as distributed systems, heat transfer, image processing, biologic systems, earthquake signals processing, sonar etc. To describe 2-D systems in addition to state space equation, transfer functions and difference equations are also used [1]. Similar to 1-D systems, if the system is time variant, state space equation or difference equation is used. For various applications, one of the description methods is used. For example in the case of stability analysis of 2-D systems, generally state space equation and transfer function are used [1, 2]. Also for issues such as state and parameter estimation, state space equation and difference equation are used [3-10]. For optimal control of 2-D systems, mostly state space equation form has been used [11-15]. The organization of this paper is as follows: Section 2 presents the 2-D system various descriptions. In section 3, the WAM model of 2-D systems is studied. In section 4, stability analysis, controllability and observability and minimality are given. Finally conclusion is presented.

D SYSTEMS DESCRIPTIONS

As it was mentioned previously, for describing the 2-D systems there are some methods. In this section these descriptions are introduced.

Transfer function: For finding the transfer function of discrete 2-D systems, similar to 1-D systems the z-transform is used. In this case, it is called 2-D z-transform [1]. The general form of transfer function is as follows

$$P(z,w) = \frac{N(z,w)}{D(z,w)} \quad (1)$$

Which z and w are the shift operator of horizontal and vertical axes respectively. Also N(z, w) and D(z, w) are polynomials based on z and w. For instance, consider the following transfer function

$$P(z,w) = \frac{1+zw-z-w}{2-z-w}$$

The roots of numerator and denominator of transfer function are considered as zeros and poles of 2-D system respectively.

Difference equation: Difference equation for a 2-D system is as follows

$$\begin{aligned} y(m,n) &= ay(m-1,n) + by(m,n-1) + cy(m-1,n-1) + \ldots \\ &+ du(m-1,n) + eu(m,n-1) + \ldots \end{aligned} \quad (2)$$

Corresponding Author: Dr. Mehdi Roopaei, Department of Electrical Engineering, Science and Research Branch-Islamic Azad University, FARS, Iran

where u and y are the input and output of system respectively. Also this system is assumed linear. With taking the z-transform of above equation the transfer function is obtained. In the case that the mentioned 2-D system is stochastic this form is known as ARMA model.

State space equation: One of the most significant points of the subject of 2-D systems is the multiplicity of presented models for state space equation. This fact is due to this point that in this equations local state vector is used instead of state vector because 2-D systems have state vectors with infinite dimension. These models include GR (Givone-Roesser) model [16], FM (Fornasini-Marchesini) model [17], Attasi model [2], MFM (Modified FM) model [18] and FTR (Fundamental Transition Matrix) model [18]. In the model which was introduced in 1984 by Porter and Aravena [18] a 1-D expression for 2-D systems was presented. This model was named WAM (Wave Advanced Model) and is used for consideration of some 2-D problems such as stability problem. In [2] this issue is studied. In the following different models for 2-D systems description are presented.

GR model: This model was introduced in 1972 by Givone and Roesser [16]. This model was used in consideration of recursive 2-D systems. After that GR model is used in other problems such as stability, image processing, control and prediction.

This model has the following formulation

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(i,j)$$

$$y(i,j) = [C_1 \quad C_2] \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \tag{3}$$

where $x^h \in R^n$ and $x^v \in R^m$ are local horizontal state vector and local vertical state vector respectively. Also u and y are respectively the input and output vectors of system. i and j are the indices in horizontal and vertical direction. Matrices A, B and C generally are functions of i and j. As can be seen the local state vector in every point is dependent to the previous state vectors located in a cell before the recent point. This matter is known as First Quadrant Causality. More information about this issue is found in [1].

FM model: This model was introduced by Fornasini and Marchesini in 1972 [17]. The formulation of this model is as follows

$$\begin{aligned} x(i+1,j+1) &= A_1 x(i+1,j) + A_2 x(i,j+1) \\ &\quad + A_0 x(i,j) + Bu(i,j) \\ y(i,j) &= Cx(i,j) \end{aligned} \tag{4}$$

where, $x \in R^n$, $u \in R^p$ and $y \in R^q$ are respectively local state, input and output vectors. This relation is a recursive 2-D equation.

Attasi model: Attasi introduced this model in 1976 [2]. This model is as follows

$$\begin{aligned} x(i+1,j+1) &= A_1 x(i,j+1) + A_2 x(i+1,j) \\ &\quad - A_1 A_2 x(i,j) + Bu(i,j) \\ y(i,j) &= Cx(i,j) \end{aligned} \tag{5}$$

where, $x \in R^n$, $u \in R^p$ and $y \in R^q$ are respectively local state, input and output vectors. This model is a special case of FM model, where the following condition is considered

$$A_0 = -A_1 A_2 = -A_2 A_1$$

MFM model: This model is introduced in [18] for the first time by Porter and Aravena in 1984. This model has the following form

$$\begin{aligned} x(i+1,j+1) &= Jx(i,j+1) + Kx(i+1,j) \\ &\quad + Eu(i,j+1) + Fu(i+1,j) \\ y(i,j) &= Cx(i,j) \end{aligned} \tag{6}$$

The most significant characteristic of this model is that the order of the equation is one.

FTR model: This model is introduced simultaneously in 1987 by Porter, Aravena [18] and Kaczorec, Kurec [19]. The form of this model is as follows

$$\begin{aligned} x(i+1,j+1) &= J_{10} x(i+1,j) + J_{01} x(i,j+1) \\ &\quad + K_{00} x(i,j) + E_{10} u(i+1,j) + E_{01} u(i,j+1) \\ y(i,j) &= Cx(i,j) \end{aligned} \tag{7}$$

This model is second degree and also is First Quadrant Causal.

1-D FORM OF 2-D SYSTEMS

Porter and Aravena introduced this form (1-D form of 2-D systems) called WAM model in 1984 [18]. In this model the 2-D systems are considered as advanced wave. Obviously, because the dimension of state vectors of 2-D systems is infinite, the dimension of defined state vector in this model is huge. Despite this problem because of the possibility of generalization of some characteristics of 1-D systems to 2-D systems this model is of great importance. In this model with the proper classification of local state vectors of a 2-D system a novel form of local state vectors is achieved. In [20], it is presented that the new vector has more capability in the terms of controllability and observability, compared with state vectors in mentioned 2-D models.

The 1-D WAM model of MFM model: If the MFM model is considered as follows

$$\begin{aligned} x(i+1,j+1) &= Jx(i,j+1) + Kx(i+1,j) \\ &\quad + Eu(i+1,j) + Fu(i,j+1) \end{aligned} \tag{8}$$

With introducing the following vectors

$$\phi(n) = \begin{bmatrix} x(n,0) \\ x(n-1,1) \\ \vdots \\ x(0,n) \end{bmatrix} \text{ and } v(n) = \begin{bmatrix} u(n,0) \\ u(n-1,1) \\ \vdots \\ u(0,n) \end{bmatrix}$$

In this case, it is obvious that

$$\phi(n+1) = A(n)\phi(n) + B(n)v(n) \tag{9}$$

Where A(n) and B(n) are as follows

$$A(n) = \begin{bmatrix} J & 0 & 0 & \cdots & 0 & 0 \\ K & J & 0 & \cdots & 0 & 0 \\ 0 & K & J & \cdots & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \cdots & K & J \\ 0 & 0 & 0 & \cdots & 0 & K \end{bmatrix}$$

and

$$B(n) = \begin{bmatrix} E & 0 & 0 & \cdots & 0 & 0 \\ F & E & 0 & \cdots & 0 & 0 \\ 0 & F & E & \cdots & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \cdots & F & E \\ 0 & 0 & 0 & \cdots & 0 & F \end{bmatrix}$$

The dimensions of above matrices are

$$\begin{aligned} &A(n):[(n+1).T].[(n+1).T] \\ &B(n):[(n+1).T].[(m+1).P] \end{aligned}$$

where T and P are respectively the dimensions of x and u in the MFM model. This method is also usable for time varying 2-D MFM model.

Furthermore, it is possible to assume the local state vectors on the horizontal and vertical axes as the boundary conditions. In this case, in $\phi(n)$, x(n,0) and x(0,n) is removed from the beginning and end. If the primary 2-D system was nonlinear, it is possible by defining the new state vector as above to have a 1-D nonlinear model.

According to the mentioned definitions, it is seen that the dimensions of the state and input vectors in this model depends on the index n. It means that by increasing n the matrices' dimensions become greater. On the other hand, matrix A (n) unlike the situation in 1-D system has a rectangular shape and its dimensions increase with increasing n. Hence, despite the 1-D form of equations in the WAM model, many results of 1-D systems are not easy to generalize to this model.

The 1-D WAM model of GR model: The GR model was introduced in (3). Defining new vectors as follows

$$\phi(n) = \begin{bmatrix} x^v(0,n) \\ x^h(1,n-1) \\ x^v(1,n-1) \\ \vdots \\ x^h(n,0) \end{bmatrix}, \ v(n) = \begin{bmatrix} u(0,n) \\ u(1,n-1) \\ \vdots \\ \vdots \\ u(n,0) \end{bmatrix},$$

$$\mu(n) = \begin{bmatrix} y(0,n) \\ y(1,n-1) \\ \vdots \\ y(n,0) \end{bmatrix}, \ f(n) = \begin{bmatrix} x^h(0,n) \\ x^v(n,0) \end{bmatrix}$$

then, it is obtained

$$\begin{aligned} \phi(n+1) &= A(n)\phi(n) + B(n)v(n) + E(n)f(n) \\ \mu(n) &= C(n)\phi(n) + D(n)v(n) + H(n)f(n) \end{aligned} \tag{10}$$

Where

$$A(n) = \begin{bmatrix} A_4 & 0 & 0 & \cdots & 0 & 0 & 0 \\ A_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & A_3 & A_4 & \cdots & 0 & 0 & 0 \\ 0 & A_1 & A_2 & \cdots & 0 & 0 & 0 \\ \vdots & & & & & & \vdots \\ & \cdots & & & A_3 & A_4 & 0 \\ & \cdots & & & A_1 & A_2 & 0 \\ 0 & 0 & 0 & \cdots & & & A_3 \\ 0 & 0 & 0 & \cdots & & & A_1 \end{bmatrix}$$

$$B(n)=\begin{bmatrix} B_2 & 0 & \cdots & 0 \\ B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ 0 & B_1 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & A_3 \\ 0 & 0 & \cdots & A_1 \end{bmatrix}$$

$$C(n)=\begin{bmatrix} C_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & C_1 & C_2 & \cdots & 0 & 0 & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & 0 & \cdots & C_1 & C_2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & C_1 \end{bmatrix}$$

$$E(n)=\begin{bmatrix} A_3 & 0 \\ A_1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & A_3 \\ 0 & A_1 \end{bmatrix},\ H(n)=\begin{bmatrix} C_1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & C_2 \end{bmatrix}$$

The dimensions of above matrices are as follows

$$\begin{aligned} &A(n):\{[(n+1).(h+v)].[(n).(h+v)]\} \\ &B(n):\{[(n+1).(h+v)].[(n+1).(m)]\} \\ &C(n):\{[(n+1).(p)].[(n+1).(h+v)]\} \\ &E(n):\{[(n+1).(h+v)].[(h+v)]\} \\ &H(n):\{[(n+1).(p)].[(n+1).(m)]\} \end{aligned}$$

where h,v,m and p are the length of the horizontal and vertical state vectors, input and output vector of GR model respectively. In this case, obviously the dimensions of matrices increase with increasing n. The significant point in the WAM model of MFM and GR model of 2-D systems is that these models are first order. While, for other models of 2-D systems the WAM forms are second order [2].

The 1-D WAM model of FTR and FM models: In [18], the FTR model for a 2-D system was introduced. This model is as follows

$$\begin{aligned} &x(i+1,j+1)=J_{10}x(i+1,j)+J_{01}x(i,j+1)- \\ &K_{00}x(i,j)+E_{10}u(i+1,j)+E_{01}u(i,j+1)-F_{00}u(i,j) \end{aligned} \quad (11)$$

In the case of time varying systems the matrices J_{10}, J_{01}, K_{00}, E_{10}, E_{01} and F_{00} are functions of the indices i and j. In [18], to determine the WAM model of above 2-D system, the state vector $\phi(k)$ is defined as follows

$$\phi(k)=Col[x(k,0),x(k-1,1),...,x(0,k)] \quad (12)$$

where Col stands for column vector. In this case, the resulting WAM form is as follows

$$\begin{aligned} \phi(n+1)&=J(n)\phi(n)+K^{-}(n-1)\,\phi(n-1) \\ &+E(n)v(n)+F^{-}(n-1)v(n-1) \end{aligned} \quad (13)$$

where

$$v(n)=Col[u(n,0),u(n-1,1),...,u(0,n)] \quad (14)$$

$$E(n)=\begin{bmatrix} E_{01} & 0 & \cdots & 0 & 0 \\ E_{10} & E_{01} & \cdots & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \cdots & E_{10} & E_{01} \\ 0 & 0 & \cdots & 0 & E_{10} \end{bmatrix}$$

$$J(n)=\begin{bmatrix} J_{01} & 0 & \cdots & 0 & 0 \\ J_{10} & J_{01} & & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & & J_{10} & J_{01} \\ 0 & 0 & & 0 & J_{10} \end{bmatrix}$$

$$K^{-}(n-1)=I(n)K(n-1)$$

$$F^{-}(n)=I(n)F(n-1)$$

$$K(n)=-diag[K_{00}]$$

$$F(n)=-diag[F_{00}]$$

and

$$I(n)=\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & I_n & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

According to (13) it is obvious that by defining the following vector the obtained model is converted to a 1-D equation

$$r(n)=K(n-1)\,\phi(n-1)+f(n-1)v(n-1) \quad (15)$$

The obtained state vector equation is as follows

$$\begin{bmatrix} \phi(n+1) \\ r(n+1) \end{bmatrix}=\begin{bmatrix} J(n) & I(n) \\ K(n) & 0 \end{bmatrix}\begin{bmatrix} \phi(n) \\ r(n) \end{bmatrix}+\begin{bmatrix} E(n) \\ F(n) \end{bmatrix}V(n) \quad (16)$$

As can be seen from definition (13), instead the local state vectors in (16) to be in the form of a column vector, in fact we will have the linear combination of these vectors.

In the terms of problems such as optimal control and state vector estimation, having state space

equations with direct access to the state vector is required. To illustration, we consider the issue of optimal state estimation. In the problem of state estimation at each stage of estimation $\phi(n+1)$ and $r(n+1)$ are determined. If the matrix $K_{00}(i,j)$ has inverse in all points, with having r $(n+1)$, $\phi(n)$ also can be estimated. But if $K_{00}(i,j)$ does not have inverse in some points, this action will be impossible. In the following, by changing the definition of state vector a 1-D sate space equation is obtained where direct access to the state vectors $\phi(n)$ and $\phi(n+1)$ is possible.
We define the state vector as follow

$$\phi(n) = \mathrm{Col}[x(n,0), x(n-1,1), \ldots, x(0,n)] \tag{17}$$

Also the input vector is defined similar to the relation (17). With this definition, the 1-D state space equation is as follows

$$\phi(n+1) = A(n)\phi(n) + B(n)v(n) \tag{18}$$

Where

$$A(n) = \begin{bmatrix} J_{01} & 0 & 0 & \cdots & 0 & 0 & 0 \\ I & 0 & 0 & \cdots & 0 & 0 & 0 \\ J_{10} & -K_{00} & J_{01} & \cdots & 0 & 0 & 0 \\ 0 & 0 & I & \cdots & 0 & 0 & 0 \\ \vdots & & & \cdots & & & \vdots \\ 0 & 0 & 0 & \cdots & I & 0 & 0 \\ 0 & 0 & 0 & \cdots & J_{10} & -K_{00} & J_{01} \\ 0 & 0 & 0 & \cdots & 0 & I & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & J_{10} \end{bmatrix}$$

$$B(n) = \begin{bmatrix} E_{01} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ E_{10} & -F_{00} & E_{01} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & E_{10} & -F_{00} & E_{01} & \cdots & 0 & 0 & 0 \\ \vdots & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & E_{10} & -F_{00} & E_{01} \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & E_{10} & -F_{00} \end{bmatrix} \tag{19}$$

To sum up, the positive points of relation (18) in comparison with (16) are as follows

- In relation (18) the state space vector $\phi(n+1)$ includes local state vectors x, whereas in relation (16) there is linear combination of vectors x and disturbance input ν.
- The state matrices in relation (18) are iterative. While, in relation (16) such a situation can not be observed.
- In the issues such as optimal control and optimal state estimation, with using relation (18) there is no need to additional calculations in comparison with using relation (16). If matrix $K_{00}(i,j)$ is singular using relation (16) will be combined with problem. According to the definition of vector ϕ, this vector has the following shape.

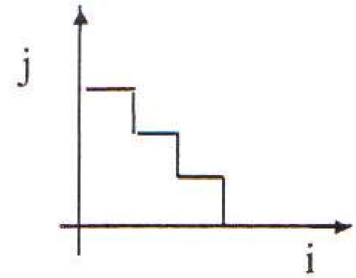

Fig. 1: The shape of vector ϕ

As can be seen, in this definition instead of using local vectors on the line i+j=n+1, local vectors are used as one among on the mentioned line and on the line located one stage before (i+j = n).

- Generally, for WAM description of 2-D systems which are at least second order using this method is useful. For instance, the 2-D ARMA model as the following relation

$$y(m,n) = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} c_{ij} y(m-i, n-j) + \sum_{i=1}^{M_1}\sum_{j=1}^{M_2} d_{ij}\mu(m-i, n-j) \tag{20}$$

w(k) and x(k) are defined as follows

$$x(k) = \mathrm{Col}[\phi(k), \phi(k-1), \ldots, \phi(k-d)] \tag{21}$$

$$w(k) = \mathrm{Col}[v(k), v(k-1), \ldots, v(k-l)] \tag{22}$$

Vectors ϕ and ν are defined as the proper definitions of WAM for u and y. Also, l and d are respectively the length of the first and second above summations. In this case, a relation as (18) can be written again.

STABILITY, CONTROLLABILITY, OBSERVABILITY OF 2-D SYSTEMS

About the stability of 2-D systems because of its importance a wide range of research were done [2, 21-26]. In [2, 21, 22], the stability analysis is done on the basis of WAM model. The technique that used in this method is the use of 'norm'. In [23-26], the presented results are for linear systems with constant coefficients. In this situation, the stability analysis has been done with considering the position of poles of transfer function to the unit circle. To describe the stability issue, in the first place its definition is given.

Definition 1: BIBO stability (Bounded Input Bounded Output): A 2-D system is BIBO stable if for every bounded input u(m,n) the output y(m,n) is bounded.

Like 1-D systems, if the output of system achieves zero, the system will be asymptotically stable.

Theorem 1 [23]: A 2-D system is BIBO stable, if and only if its sequence of impulse response is an 'absolutely bounded sequence'.

In the following, a special property of the 2D systems which plays an important role in the stability issue will be presented.

Definition 2: If H(z,w) is the transfer function of a 2-D system, then the common and non-removable roots of numerator and denominator of the transfer function are called the second type of unessential singularity. For instance, consider the following polynomials as the numerator and denominator of the transfer function

$$N(z,w) = (1-z)(1-w)$$

$$D(z,w) = 2 - z - w$$

It is seen that z = w = 1 is the common points between N and D and at the same time it can not be removed from numerator and denominator.

Theorem 2 [24]: If H(z,w) is the transfer function of a 2-D system, then its poles are located out of unit circle T, as the following. Also, it does not have the second type of unessential singularity located on the boundary of T.

$$T = \{(z,w)\big|\ |z| < 1, |w| < 1\}$$

As a result, one of the most significant problems in determination of the stability of 2-D systems is to determine the location of the poles to the T and to consider the second type of unessential singularity.

Theorem 3 [25, 26]: The 2-D polynomial D(z,w) will not have any roots in the region T, if and only if:

- D(0,w) is non-zero in the region |w|≤1.
- D(z,w) is non-zero in the region |w|=1 and (|z|<1 or |z|=1).

With this method, 1-D techniques such as 'Juri test' can be used to fulfill the first condition. In this regard another theorem is expressed in the following.

Theorem 4 [27]: 2-D polynomial D(z,w) will not have any roots outside the region T, if and only if $D(z,ze^{jr})$ does not have any roots in the 1-D unit circle. This condition should be established for all values of r between zero and 2π.

It is of great importance to realize that for every fixed r polynomial $D(z,ze^{jr})$ is a 1-D polynomial of z. On the other hand, to check the stability of a 2D system, 1-D methods can be used.

Controllability and observability: In the first place, before description the controllability, observability and minimality of 2-D systems it should be noted that in the 2-D systems the local state vector which is a part of the state vector is used.

Definition 3 [16]: A 2-D system with GR form is local controllable, if and only if local state vector x(i,j) with zero initial conditions can be transferred to the coordinate origin. Also, this system is local observable, if and only if the value of state vector in the coordinate origin can be determined from outputs y(i,j) uniquely.

Theorem 5 [16]: A 2-D system with GR form is local controllable, if and only if the following matrix is full row rank

$$\left[B, A^{1,0}B, A^{0,1}B, A^{1,1}B, \ldots, A^{i,j}B, \ldots, A^{m,n}B\right]$$

m and n are respectively the order of local horizontal and vertical state vectors. $A^{i,j}$ is the transfer matrix of GR model.

Also, this system is observable, if and only if the following matrix is full column rank.

$$\begin{bmatrix} C \\ CA^{1,0} \\ CA^{0,1} \\ CA^{1,1} \\ \vdots \\ CA^{i,j} \\ \vdots \\ CA^{m,n} \end{bmatrix}$$

Generally, it can be shown that a 2-D system with controllability and observability conditions is not minimal and the reverse of this issue is not true [28]. In [20], the controllability and observability conditions for WAM form of 2-D systems are presented.

CONCLUSION

In this article, an overview of 2-D systems is presented. In this regard, the 2-D models given in literature are presented and 1-D form of 2-D models called WAM form is expressed. These models are used to consider the stability of 2-D systems. Finally, the specifications such as controllability, observability and minimality are given. It is obviously, crystal clear that 2-D systems and their characteristics are different from 1-D systems.

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