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Estimation of Failure Probability in Water Pipes Network Using Statistical Model

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Abstract: In this paper, a statistical model is presented for decision making in repairing water pipes network. The water distribution system has been considered as a "repairable" system which is under repeating failure modes. From this, a practical model for anticipating the failure of the water pipes in repairable systems has been presented using the trend renewal process concept. In this process, the statistical Power law has been used for projecting the failure rate to account for the effects of repairs and for different failure modes in estimation of failure intensity. After finding the failures as a function of time, the reliability of the system efficiency is then estimated using survival analysis. At the end, a sample pipes network has been modeled using presented statistical model and the values of failure intensities with respect to time and the curve for reliability function has been found.

Key words: Reliability function · Power law · Repairable system · Instantaneous failure intensity · MLE

INTRODUCTION

Various techniques regarding pipe reliability assessment have been developed through the years. Normally, in a pipe network the amount of failure is estimated by a statistical model with respect to time. Historically, the most often used models in the previous works are general statistical models such as Renewal Process (RP), power model of Weibull, Homogenous Poison Process (HPP) and recently, Non-homogenous Poison Process (NHPP). Another model that has been used by many references is modeling the failed pipes with Shamir and Howard method, [1]. In this method, the optimum time for pipe replacement is found. In Shamir and Howard model, the old replaced pipes are considered together with the replaced pipes. Walski *et al.* have presented a model similar to the model of Shamir and Howard except that the history of the failure has been also entered the model, [2, 3]. In several references the statistical model of Cox's Semi-Parametric has been used for estimating the failure of the pipe. In this method the Proportional Hazard Model (PHM) has been adopted for calculating the risk rate in terms of time, [4]. Besides the last mentioned method, another method has been presented for estimating failure which adopts PHM model as well, with a set of extra random variables. These variables consist of several factors such as geometric characteristic and unavailable information, [5]. In another work, the Markov model has been extended for stating the water distribution systems cases. In all the above mentioned works the time in which the system or portion of the system is under repair is ignored, [6]. Wengstrom has presented a method for predicting the response of a water network using the risk function of Additive Hazard Model (AHM), [7]. In his model, the covariates have been related to the time between the fails with regression and then the time for repairing the pipes has been computed. In another work, nonlinear regression has been used. The coefficients for the nonlinear equation have been found first and then the amount of failure average has been computed in terms of time and place. Non-homogenous Poison Process (NHPP) has been used for failure probability distribution, [8]. A new statistical method called Herz distribution has been presented for estimation of useful age of the pipes in which the statistical Cohort Survival Model has been developed. In the recent model

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the pipes are categorized based upon time of consuming start, geometric characteristic and material type and then the failure rate and the time of replacement after ending their useful age have been anticipated, [9]. Rostum has modeled the water network assuming it as repairable system and then he has compared the existing methods for anticipating the failure time of the pipes, [10]. He has assumed that the system has become "good-as-new" after any repair stage. It is indicated in Pulcini's studies that if a system is repaired repeatedly it may result in producing a finite bound for increasing the failure intensity rate. So, for such systems the failure function is introduced together with a Bounded Intensity Process (BIP) interval, [11]. In many of the above mentioned references it is assumed that the water network is repairable and has a form of Non-homogenous Poison Process (NHPP).

In all methods of previous references, it has been assumed that the type of repair does not affect the improving process of the system and the system after any round of repair becomes "good-as-new", [13]. In other words, the system after any repair process becomes the same as before failure. Such repairs are usually called Minimal Repair. But the reality is that the system may become better than its state before failure. In these models, it has been assumed that after repair, there has been no variation with respect to the base line, instead the function moves vertically along the intensity axis. Guo has presented a model for estimating the failure intensity in repairable systems using power rule. In this model the effects of failure for a failure mode has been presented, [14]. Recently a new statistical method called Trend Renewal Process (TRP) using a failure intensity function, $\lambda(t)$, similar to that of NHPP method has been presented by Lindqvist, [15, 16]. The difference is that in TRP method the effect of the type of failure has been considered in repairing the system. Also, in this model, the failure intensity has a uniform process. However, although this model as a package is able to consider the effects of the type of failure on the system, its application in engineering problems is complex and difficult.

In this paper a practical model for estimating the failure intensity in water pipes network is presented. In this method the type of failure is considered in addition to the effects of repair on the model. Repairing the system decreases the number of failures in the future and including this fact in the method results in more realistic model. For this aim a combination of parameters is defined which is able to account for effect of repairing on the type of failure. After this stage, the full likelihood function is formed and the model parameters are calculated by Maximum Likelihood Estimation (MLE) method. Moreover, the Instantaneous Intensity Failure (IIF) and also, the number of cumulative failure are estimated. Therefore, the reliability of the pipes network can be anticipated. The goals for each network are to develop a model that can correctly classify network pipes to successes or failures; define the pipe characteristics to be blamed for the pipes behavior; and predict whether a pipe will fail or not. At the end of this paper, a sample water pipes network is modeled by the model proposed in this work and by the obtained results the effects of repairing and the failure modes are indicated in Intensity Failure Function (IFF). It is also shown that the model has enough simplicity and capability.

Table 1:Types of failures of the pipes

Damage and Degradation Mechanisms	
ID	Description
CF	Corrosion-fatigue
COR	Corrosion attack/MIC/Pitting
DandC	Design and construction flaws
E/C	Erosion/flow-accelerated corrosion (FAC)
E-C	Erosion–cavitation
FP	Frozen pipe
HE	Human error
OVP	Overload
SCC	Stress corrosion cracking
TF	Thermal fatigue
UNR	Unreported
VF	Vibration-fatigue
WH	Water hammer

Types of Failure Mode in Water Pipes: Failure in water pipes network is a complex event because the pipes are subjected to different environmental factors and so, the reasons for their failure are different. Generally, the factors of pipes failure are categorized in two main groups: the first group includes definite factors such as loads applied through the vehicle tires, internal pressure, etc. and the second group includes probabilistic factors such as corrosion, ice loads, etc. It must be noted that material, diameter and length of the pipe have also serious role on its failure. Hence, all of the failure modes are detected and categorized in different references. The most important failure modes are stated in Table 1. On the other hand, the studies have shown that the failure rate and the breakdown frequency depend on the pipe size, pipe material and the method of pipe production. In this paper, it is assumed that the conditional probability of the pipes failure depends on the type of the pipe from the view point of material and size and also its failures mechanisms.

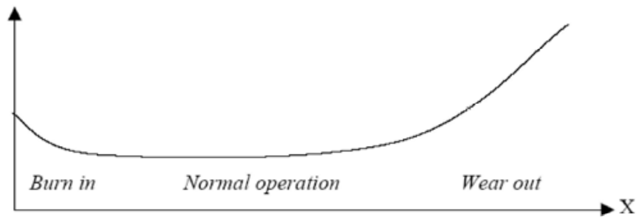

Fig. 1: The Bathtub Curve of the risk rate of statistical systems

The Model of Relative Failure for Pipes Network: Studies have shown that the risk of a statistical system may vary during its performance. Generally, the risk function is in bathtub curve form for many systems, [8]. Such curve has three parts as shown in Fig. 1. Part (1) is the phase of initials defects just after starting the system. Such defects are due to the defects of the materials consumed or bad installation and so, bad starting the system ($\beta < 1$). Part (2) is the normal performance phase of the system. In this phase sudden stresses may cause failure modes in the system. Part (3) is the "out of service" phase. In this phase the system may fail at an increasing rate due to deteriorating stresses such as fatigue stresses in the members of the system as the pipe material deteriorates. In statistical studies, the systems may consider as repairable systems or as non-repairable systems. The first types may fail several times and be repaired after each failure and the second ones have only one performance round and are abended and/or replacement.

In most of previous studies, the effects of repairs and failure modes have been ignored in anticipating the failure of the water networks' pipes. But in the present work, a simple model is considered for stating failure intensity accounting for effects of repairs in terms of failure mode.

Repairable systems like water networks' pipes start from $t = 0$ and each failure mode occurs in times t_1, t_2..... respectively. On the other hand, types of failure modes such as cracking due to overloading, different corrosion types, wall seepage, etc. are indicated by j_1, j_2.... , Therefore, the process can be defined as $(t_1,j_1),(t_2,j_2)$,......, The times between the failures which are the service times are shown as X_1, X_2,...., namely $X_i = t_i - t_{i-1}$ for the moments i = 1,2,...

Using counting process, the failure number, $N_j(t)$, from mode j can be stated in a time interval (0,1], so the total number of failures in the system is $N(t) = \sum N_j(t)$, [9]. Furthermore, the type of failure mode j may be function of the covariates vector $\{z(s)\ ;\ \ 0 \le s \le t\}$ In this type, the covariates history $z(s)$ should be added to the history F_{t-} for each $t > 0$. Generally, the failure intensity corresponding to the type of failure mode j is defined as,

$$\lambda_j(t) = \ell im_{\Delta t \to 0} \frac{\Pr(Failure\ of\ Type\ j\ in\ [t, t+\Delta t) \mid F_{t-})}{\Delta t} \tag{1}$$

Which in this work is summarized as follows,

$$\lambda_j(t) = \lambda_j^o g(z(t)) \tag{2}$$

Where $\lambda_j(t)$ is the failure intensity of failure mode j. Also, the parameter λ_j^o is depending only on the event history $\{z(s)\ ;\ \ 0 \le s \le t\}$. Thus, $\lambda_j(t)\Delta t$ isthe conditional probability of failure mode j in time interval $[t, t+\Delta t]$ given the history before time t. In this study, using the power law, the function of failure intensity corresponding to failure mode j is proposed as follows,

$$g(z(t)) = \beta_j t^{\beta_j - 1} \exp\{\gamma_j m_j(t)\} \tag{3}$$

Where $m_j(t) = E[N_j(t)]$ and has the form of $m_j(t) = \int_0^t \lambda_j(t)$.

Substituting Eq. (3) into Eq. (2) results in,

$$\lambda_j(t) = \lambda_j^o \beta_j t^{\beta_j - 1} \exp\{\gamma_j m_j(t)\} \tag{4}$$

Where β_j is the covariate in power law corresponding to failure mode j for the failure not modified by any repair which is called "reference failure".

β_j and λ_j^o are dependent on the event history $\{N_j(s):\ \ 0 \le s \le t\}$. Eq. (4) physically means that each repair modifies the failure intensity. The cumulative repair effect is reflected by the term $\gamma_j N_j$ (t). Obviously, if $\gamma_j > 0$, the repair will have negative effects and the failure intensity will increase. On the other hand, if $\gamma_j < 0$, the repair will have positive effects and repairs make the system better. When $\gamma_j = 0$,, the repair has no effect and the failure intensity does not change. Thus, Eq. (4) covers all the cases that may happen.

Now, using Eq. (4) gives,

$$\frac{dm_j(t)}{dt} = \lambda_j^o \beta_j t^{\beta_j - 1} \exp\{\gamma_j m_j(t)\} \tag{5}$$

By solving the differential Eq. (5) the closed form of $m_j(t)$ can be calculated as,

$$m_j(t) = -\frac{1}{\gamma_j}\ell n(1-\gamma_j \lambda_j^o t^{\beta_j}) \quad (6)$$

Finally, by substituting Eq. (6) into Eq. (4) the failure intensity corresponding to failure mode j by time t can be obtained as follows,

$$\lambda_j(t) = \frac{\lambda_j^o \beta_j t^{\beta_j - 1}}{1-\gamma_j \lambda_j^o t^{\beta_j}} \quad (7)$$

In water pipes network, it is assumed that repairs may restore the system to a better state comparing to the state just before the failure or, the repairs will not affect the system. So, as the negative effect of repair has no meaning, the parameter γ_j must be equal to zero or less than zero at different times. Therefore, it can be seen that the denominator of Eq. (7) should be equal to, or greater than one. In fact, if the number of the repairs of failure corresponding to failure mode j is increased then the failure intensity will be decreased.

In order to study the change of $\lambda_j(t)$ in Eq. (7) in terms of time, the first derivative of $\lambda(t)$ with respect to time t is calculated as,

$$\frac{d\lambda_j(t)}{dt} = \lambda_j^o \beta_j t^{\beta_j - 2}\exp\left\{\gamma_j m_j(t)\right\}\left[(\beta_j - 1) + \gamma_j \lambda_j(t) t\right] \quad (8)$$

The sign of the derivative in Eq. (8) is relating to the term

$$(\beta_j - 1) + \gamma_j \lambda_j(t) t \quad (9)$$

Since all the other terms are positive. Obviously, term (9) can be equal to, less than, or greater than zero at different times. The sign of term (9) is related to the parameter β_j, repair effect, γ_j and the current system state, $\lambda_j(t)t$. If term (9) results in a positive value at time t, $\lambda_j(t)$ will be an increasing function and the system is deteriorating. If term (9) is a negative value, $\lambda_j(t)$ will be a decreasing function and the system is improving. Finally, if term (9) is close to zero at some times, the system has constant failure intensity and the model will be the same as the Homogenous Poison Process (HPP). Moreover, the above analysis clearly indicates the capability of the proposed model.

Estimation of the Model Parameters: In this section, the value of Likelihood is introduced first. Then the Maximal Likelihood Estimation (MLE) method is used to estimate the model parameters. Let $t_1 < t_2, ...t_n$ denote the n failure times corresponding to failure mode j. In order to obtain the MLE estimators of the parameters, consider the following definition of conditional probability:

$$F_j(t_i \mid t_{i-1}) = \Pr(t \le t_i \mid t > t_{i-1}) = \frac{F_j(t_i) - F_j(t_{i-1})}{R_j(t_{i-1})} = 1 - \frac{R_j(t_i)}{R_j(t_{i-1})} \quad (10)$$

Where R_j is the reliability function.

Using the empirical failure intensity in Eq. (4), the conditional reliability just before the i th failure will be:

$$R_j(t_i \mid t_{i-1}) = 1 - F_j(t_i \mid t_{i-1}) \quad (11)$$

$$R_j(t_i \mid t_{i-1}) = \exp\left\{-\int_{t_{i-1}}^{t_i} \lambda_j(t)dt\right\} = \exp\left\{-\exp\left\{(i-1)\gamma_j\right\}\lambda_j^o(t_i^{\beta_j} - t_{i-1}^{\beta_j})\right\} \quad (12)$$

and the conditional probability density function (PDF) is

$$\begin{aligned} f_j(t_i \mid t_{i-1}) &= \lambda_j^o \beta_j t_i^{\beta_j - 1}\exp\{(i-1)\gamma_j\} \\ &\exp\left\{-\exp\{(i-1)\gamma_j\}\lambda_j^o(t_i^{\beta_j} - t_{i-1}^{\beta_j})\right\} \end{aligned} \quad (13)$$

Consider now a point process $N_j(t)$, observed from timet_{i-1} to time t_i , with corresponding failure times $(t_1,j_1),(t_2,j_2),....(t_n,j_n)$. The Likelihood function of the process is then given by,

$$\begin{aligned} L_j(\lambda_j^o, \beta_j, \gamma_j \mid Data) &= f_j(t_n \mid t_{n-1}) \\ &f_j(t_{n-1} \mid t_{n-2}) .... f_j(t_1 \mid t_0) \end{aligned} \quad (14)$$

$$L_j = \left\{\prod_{i=1}^{n} f_j(t_i \mid t_{i-1})\right\}\exp\left\{-\int_{t_{i-1}}^{t_i}\lambda_j(t)dt\right\} \quad (15)$$

Where $t_0 = 0$ shows the beginning time of the system performance. The parameters are easily calculated taking the natural log on both sides of Eq. (15) as follows,

$$\begin{aligned} \ell n(L_j) &= n\ell n\lambda_j^o + n\ell n\beta_j + (\beta_j - 1)\sum_{i=1}^{n}\ell n t_i + \frac{N_j(N_j - 1)}{2}\gamma_j \\ &-\lambda_j^o \sum_{i=1}^{n}\left\{\exp\left\{(i-1)\gamma_j\right\}t_i^{\beta_j}\right\} + \lambda_j^o \sum_{i=1}^{n+1}\left\{\exp\left\{(i-1)\gamma_j\right\}t_i^{\beta_j}\right\} \\ &-\lambda_j^o t_n^{\beta_j}\exp\left\{N_j \gamma_j\right\} \end{aligned} \quad (16)$$

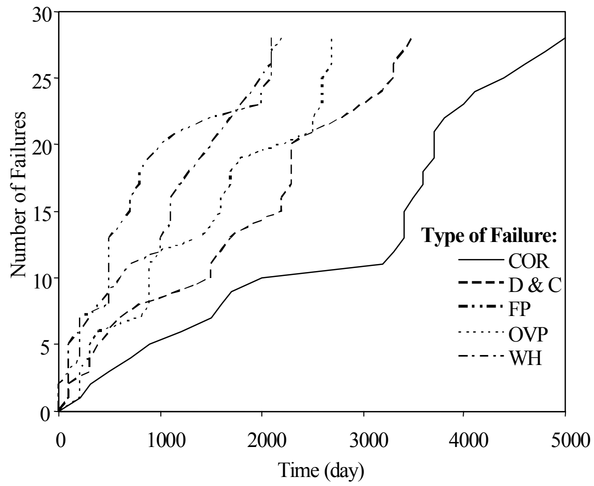

Fig. 2: The number of failures recorded for each type of failure in different days

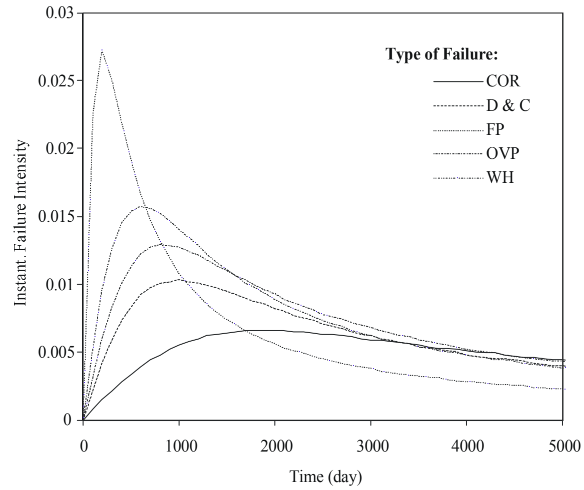

Fig. 4: Instantaneous Failure Intensity function for different type of failure

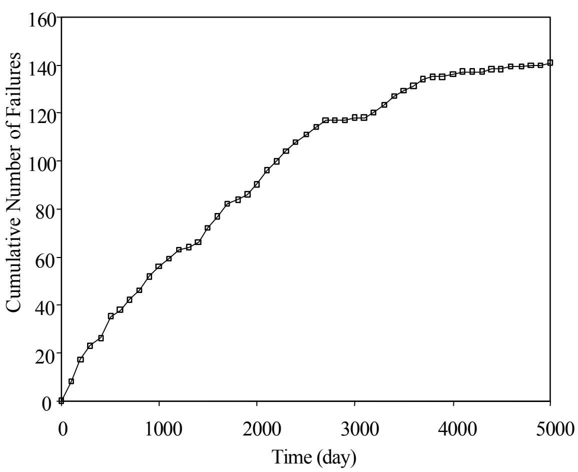

Fig. 3: Cumulative number of failures of different types recorded in each day

It must be noted that the system with failure corresponding to failure mode j has three unknown parameters namely $\lambda_j, \beta_j, \gamma_j$ Consequently, Eq. (16) has $3 \times m$ unidentified parameters. By solving the non-linear system of equations of the first order derivatives with respect to the model parameters of the log Likelihood function, the MLE estimations of $\lambda_j, \beta_j, \gamma_j$ can be obtained. In this study, confidence intervals on $\lambda_j, \beta_j, \gamma_j$ have been obtained using the MATLAB (R2007a) software.

Numerical Study: In order to verify the proposed model, a water distribution network as a sample is considered. The main aim is to predict trends of failures over a time period. Furthermore, the reliability of the system will be calculated. In this example, the total length of pipes in the network is 20 kilometers and the material of pipes is grey cast iron with the mean age of 16 years. For simplifying the data process, it is assumed that only five types of failure modes may be occurred in the network pipes. Network pipes data has been taken from a pilot city and is used in the analysis. This database contains time of each reported failure, type of failure and cumulative number of failures.

Fig. 2 shows the recorded number of failures in the period of 5000 days. The time between each type of failure is the difference between consecutive times. Based upon the recorded data, the number of failures for each type of failure for 5000 days is shown in Fig. 2.

Also, the cumulative number of observed failures of different types recorded in each time is shown in Fig. 3. The expected number of failures is calculated for next 2000 days using proposed model. This calculation is arranged in two steps. In the first step, the natural log on Likelihood for each type of failure is calculated and then the full log Likelihood is obtained. In this example, five modes of failures are observed during the time interval 0 to 5000 days. Therefore, the total unidentified parameters will be 3×5. Now, these parameters have been obtained through MLE process. In this process, by solving the non-linear equations of the first order derivatives with respect to the model parameters of the log Likelihood function, the MLE estimates $\lambda_j, \beta_j, \gamma_j$ for $j = 1,2,..5$ can be obtained. The results of the proposed model are given in Table 2.

In the second step, the instantaneous failure intensity for each type of failure is obtained based upon estimated parameters in the previous step (Fig. 4).

Table 2:Parameter Estimates for the water distribution network

	Type of Failure				
Model Parameter	COR	D &C	FP	OVP	WH
β_j	1.716	1.955	1.884	1.961	1.950
λ_j	5.043e-06	1.426e-05	2.558e-04	2.004e-05	3.558e-05
γ_j	-3.440e-02	-9.450e-02	-1.657e-01	-8.910e-02	-9.960e-02

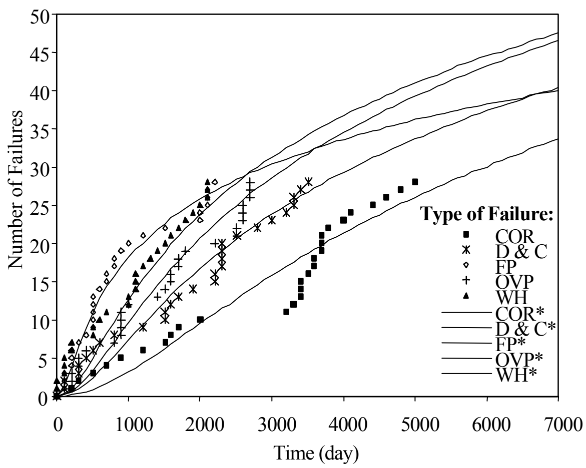

Fig. 5: The predicted number of failures for next 2000 days, (superscript * denote the predicted values by proposed model)

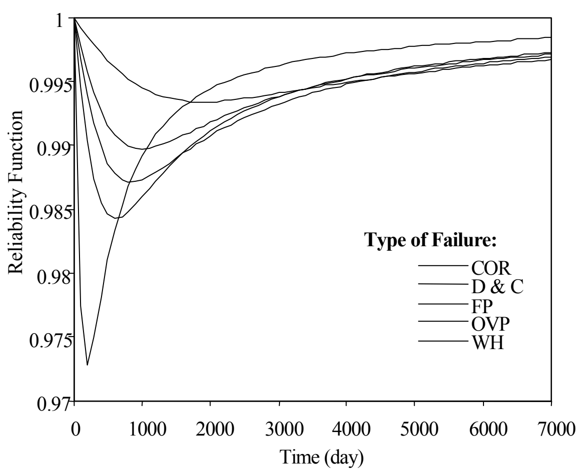

Fig. 6: Reliability function of the pipe for each type of failure

In Fig. 4, the failure intensity increase first and then decreases. The increase could be caused by the fact that more and more functions and components are tested and used. Then the rate of uncovering latent failures increases. The decrease is caused by the repair activities. Then the expected number of failures for next 2000 days is predicted using the proposed model and the results are shown in Fig. 5. From case study, it can be shown that the proposed model can fit repairable system data very well and the repair effects are reflected by introducing one more model parameter.

Furthermore, Fig. 6 has shown the reliability functions for each type of failure mode for successive failures. The reliability functions become steeper as the failures numbers increase. Also, the increase is caused by the repair activities. This situation implies that the average time between failures becomes shorter and shorter.

CONCLUSION

In this paper, a statistical model for predicting failures for each pipe in a water distribution network has been presented. The proposed method is able to model the effects of failure modes and repairs using a modified power law. The model has several parameters which include both the effects of repair and the types of failure. Thus, this model which uses the failure intensity function is capable and convenient for prediction of failures. The model has been used in a numerical example for the urban water network. The pipes in the network have been divided into groups based on different failure characteristics. The case study showed that after each repair, the repair will have positive effects and repairs make system improved. That means, after each repair, the failure intensity has been reduced. In additional, the parameters appear in model can be easily obtained by Maximum Likelihood Estimation (MLE) method. In the present study it is possible to select the most effective modes from total modes. It is important that the proposed method is a feasible method for large water distribution networks in which many failure modes can be considered besides the other probabilities parameters, such as repair effect parameter. The model can be extensively used in management and planning the water networks.

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