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Application of Differential Transformation Method to Study on Motion of a Sphere Rolling Down an Inclined Plane Submerged in Incompressible Newtonian Media

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Abstract: In this paper, the unsteady motion of a spherical particle rolling down an inclined plane in a Newtonian fluid for a range of Reynolds numbers was solved using a simulation method called the Differential Transformation Method (DTM). The concept of differential transformation is briefly introduced and then we employed it to derive solution of nonlinear equation. The obtained results for displacement, velocity and acceleration of the motion from DTM are compared with those from the exact and numerical solution to verify the accuracy of the proposed method. The results reveal that the Differential Transformation Method can achieve suitable results in predicting the solution of such problems.

Key words: Spherical particle · Acceleration motion · Inclined plane · Non-linear equation · Differential Transformation Method (DTM) · Numerical Solution (NS)

INTRODUCTION

The description of the motion of immersed bodies in fluids is present in several manufacturing processes, e.g. sediment transport and deposition in pipe lines, alluvial channels, chemical engineering and powder process [1-6]. Several works could be found in technical literature which investigated the spherical particles in low and high concentration [7-9].

A particle falling or rolling down a plane in a fluid under the influence of gravity will accelerate until the gravitational force is balanced by the resistance forces that include buoyancy and drag. The constant velocity reached at that stage is called the "terminal velocity" or "settling velocity". Knowledge of the terminal velocity of solids falling in liquids is required in many industrial applications. Typical examples include hydraulic transport slurry systems for coal and ore transportation, thickeners, mineral processing, solid–liquid mixing, fluidization equipment, drilling for oil and gas, geothermal drilling.

The resistive drag force depends upon drag coefficient. Drag coefficient and terminal velocities of particles are most important design parameters in engineering applications. There have been several attempts to relate the drag coefficient to the Reynolds number. The most comprehensive equation set for predicting C_D from Re for Newtonian fluids has been published by Clift *et al.* [10], Khan and Richardson [11], Chhabra [12] and Hartman and Yutes [13]. Comparing between most of these relationships for spheres, demonstrates quite low deviations [13].

Most of mentioned applications involve the description of the particle position, velocity and acceleration during time e.g. classification, centrifugal and gravity collection or separation, where it is often necessary to determine the trajectories of particle accelerating in a fluid for proposes of design or improved operation [14].

For some industrial problems such flow in the rolling ball viscometer which entails the measurement of the rolling velocity of a tightly fitting sphere in an inclined tube, transport of solid particles in inclined pipe lines or sedimentation of solid particles in inclined open channels we need information about the motion of particles rolling down an inclined plane. This topic is received less attention in the technical literature. Jan and Chen [15] developed a C_D – Re correlation for a single spherical particle rolling down a smooth plane in an incompressible Newtonian media for range of $0.1 \le \mathrm{Re} \le 10^5$. In their work, inclination angle was varied between $2° \le \theta \le 10°$. They used this correlation and their own experimental works to numerically solve the equation of motion for a sphere rolling down a smooth inclined plane.

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Jan and Chen presented their correlations for three regimes:

$$\begin{cases} C_D = 322/\mathrm{Re} & \text{if } \mathrm{Re} < 10 \\ C_D = 10^{\left[3.02 - 1.89\log\mathrm{Re} + 0.411(\log\mathrm{Re})^{2-0.033}(\log\mathrm{Re})^3\right]} & \text{if } 10 < \mathrm{Re} < 20000 \\ C_D = 0.74 & \text{if } \mathrm{Re} > 20000 \end{cases} \tag{1}$$

Where Reynolds number is defined as follow:

$$\mathrm{Re} = \frac{\rho u D}{\mu} \tag{2}$$

In Eq. (2), ρ, u, D and μ denote the fluid density, particle velocity, particle diameter and fluid viscosity, respectively.

Chhabra and Ferreira [16] used Eq. (1) to generate one correlation for range of $0.1 \le \mathrm{Re} \le 10^5$ in following structure:

$$C_D = \alpha + \frac{\beta}{\mathrm{Re}} \tag{3}$$

Where α and β are constants. They recommended a relationship with 11% average relative deviation:

$$C_D = 0.861 + \frac{321.906}{\mathrm{Re}} \tag{4}$$

Figure (1) demonstrates the variations of C_D versus Re for Eqs. (4) with experimental points from Jan and Chen [15], in a log-log diagram.

Eq. (3) is of the same form as that used by Rumpf [17], Ferreira [18] and Oseen[19] for free fall of spherical particles such:

$$C_D = 0.5 + \frac{24}{\mathrm{Re}} \tag{5}$$

Which was presented by Ferreira [18] for vertically falling sphere. Comparing Eqs. (4) and (5), it could be found that the drag coefficient for a sphere rolling down a smooth plane is much larger than that for vertically free fall.

In this case study, similarity transformation has been used to reduce the governing differential equations into an ordinary non-linear differential equation. In most cases, these problems do not admit analytical solution, so these equations should be solved using special techniques.

The differential transform method is based on Taylor expansion. It constructs an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions.

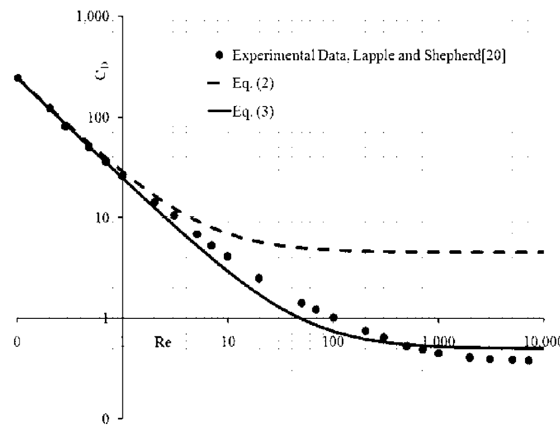

Fig. 1: Drag curve for a sphere rolling down a smooth plane

The Taylor series method is computationally taken long time for large orders. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. Differential transform has the inherent ability to deal with nonlinear problems and consequently Chiou [19] applied the Taylor transform to solve non-linear vibration problems. Furthermore, the method may be employed for the solution of both ordinary and partial differential equations. Jang *et al.* [20] applied the two- dimensional differential transform method to the solution of partial differential equations. Finally, Hassan [21] adopted the Differential Transformation Method to solve some problems. The method was successfully applied to various practical problems [22-24].

The aim of current study is the analytically investigation of acceleration motion of a spherical particle rolling down an inclined boundary with drag coefficient in form of Eq. (3), using the Differential Transformation Method (DTM). Investigation and solution of falling objects' equation is a new application for DTM which was used for some other engineering problems.

Problem Definition: Consider a small, spherical, non-deformable particle of diameter D, mass m and density ρ_s rolling down a smooth plane in an infinite extent of an

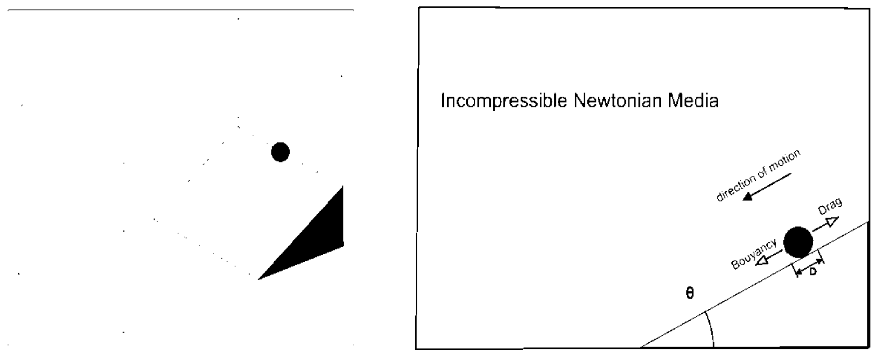

Fig. 2: Schematic picture of a spherical particle rolling down a smooth plane in a Newtonian media

incompressible Newtonian fluid of density ρ and viscosity μ. Let u represent the velocity of the particle at any instant time, t and g the acceleration due to gravity. Figure 2 demonstrates a schematic figure of current problem.

Neglecting lift force and sphere tube friction, the equation of motion is gained as follow [16]:

$$m(1.4+2\frac{\rho}{\rho_s})\frac{d^2w(t)}{dt^2}=mg(1-\frac{\rho}{\rho_s})\sin(\theta)$$
$$-\frac{1}{8}\pi D^2\rho C_D(\frac{dw(t)}{dt})^2 \tag{6}$$

Where C_D represents the drag coefficient. In the right hand side of the Eq. (6), the first term represents the buoyancy affect and the second one corresponds to resistance, drag, force.

The main difficulty in solution of Eq. (6) lies in the non-linear terms which are generated due to non-linearity nature of the drag coefficient, C_D. Substituting Eq. (3) in Eq. (6) and by rearranging parameters, Eq. (6) could be rewritten as follow:

$$a\frac{d^2w(t)}{dt^2}+b\frac{dw(t)}{dt}+c\left(\frac{dw(t)}{dt}\right)^2-d=0,\ w(0)=0,\ \frac{dw(0)}{dt}=0, \tag{7}$$

Where:

$$a=m\left(1.4+2\frac{\rho}{\rho_s}\right) \tag{8}$$

$$b=\frac{\beta}{8}\pi D\mu \tag{9}$$

$$c=\frac{\alpha}{8}\pi D^2\rho \tag{10}$$

$$d=mg\left(1-\frac{\rho}{\rho_s}\right)\sin(\theta) \tag{11}$$

With change of variation as bellow we obtain velocity,

$$u(t)=\frac{dw(t)}{dt} \tag{12}$$

By substituting Eq. (11) into Eq. (6) we will have:

$$a\frac{du(t)}{dt}+bu(t)+c\left(u(t)\right)^2-d=0,\quad u(0)=0 \tag{13}$$

Eqs. (7) and (13) are non-linear ordinary differential equations which could be solved by numerical techniques such Runge-Kutta method. We employed DTM and compared our results with numerical solution of 4th order Runge-Kutta method using the Maple package.

Differential Transformation Method: We suppose $x(\tau)$ to be analytic function in a domain D and $\tau=\tau_i$ represent any point in D. The function $x(\tau)$ is then represented by one power series whose center is located at τ_i. The Taylor series expansion function of $x(\tau)$ is in the form of [23]:

$$x(\tau)=\sum_{k=0}^{\infty}\frac{(\tau-\tau_i)^k}{k!}\left[\frac{d^kx(\tau)}{dt^k}\right]_{\tau=\tau_i}\qquad\forall\tau\in D \tag{14}$$

The particular case of Eq. (13) when τ_i is referred to as the Maclaurin series of $x(\tau)$ and is expressed as:

$$x(\tau)=\sum_{k=0}^{\infty}\frac{\tau^k}{k!}\left[\frac{d^kx(\tau)}{d\tau^k}\right]_{\tau=0}\qquad\forall\tau\in D \tag{15}$$

As explained in [25-31] the differential transformation of the function $x(\tau)$ is defined as follows:

$$X(k)=\sum_{k=0}^{\infty}\frac{H^k}{k!}\left[\frac{d^kx(\tau)}{d\tau^k}\right]_{\tau=0} \tag{16}$$

Where, $x(\tau)$ is the original function and $X(k)$ is the transformed function. The differential spectrum of $X(k)$ is confined within the interval $\tau\in[0,H]$, where H is a constant. The differential inverse transform of $X(k)$ iS defined as follows:

$$x(\tau)=\sum_{k=0}^{\infty}\left(\frac{\tau}{H}\right)^{k}X(k) \tag{17}$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument k are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. The more discrete available, the more precise it is possible to restore the unknown function. The function $x(\tau)$ consists of the T-function $X(k)$ and its value is given by the sum of the T-function with $(\tau/H)^k$ as its coefficient. In real applications, at the right choice of constant H, the larger values of argument k the discrete of spectrum reduce rapidly. The function $x(\tau)$ is expressed by a finite series and Eq. (16) can be written as:

$$x(\tau)=\sum_{k=0}^{n}\left(\frac{\tau}{H}\right)^{k}X(k) \tag{18}$$

Eq. (18) implies that the value $k = n + 1 \to \infty$ is negligible.

If $u(t)$ and $v(t)$ are two uncorrelated functions with time t where $U(k)$ and $V(k)$ are the transformed functions corresponding to $u(t)$ and $v(t)$ then we can easily proof the fundamental mathematics operations executed by differential transformation. The fundamental mathematical operations performed by differential transformation method are listed in Table 1 [25-30].

Table 1: The fundamental operations of differential transformation method

Original function	Transformed function
$x(t) = \alpha f(x) \pm \beta g(t)$	$X(k) = \alpha F(k) \pm \beta g(k)$
$x(t)=\frac{df(t)}{dt}$	$X(k) = (k + 1)F(k +1)$
$x(t)=\frac{d^2f(t)}{dt^2}$	$X(k) = (k + 1)(k + 2)F(k + 2)$
$x(t) = f(t)g(t)$	$X(k)=\sum_{l=0}^{k}F(l)G(k-1)$
$x(t) = t^m$	$X(k)=\delta(k-m)=\begin{cases}1 & k=m\\ 0 & k\neq m\end{cases}$

Application of Differential Transformation Method: Now we apply Differential Transformation Method into Eq. (7) for find $w(t)$ as displacement. Taking the differential transform of Eq. (16) with respect to t according Table 1 gives:

$$\begin{aligned}&a\big((k+2)(k+1)\mathrm{W}_{k+2}\big)+b\big((k+1)\mathrm{W}_{k+1}\big)\\&+c\left(\sum_{j=0}^{k}(k-j+1)\mathrm{W}_{k-j+1}(j+1)\mathrm{W}_{j+1}\right)\\&-d\left(\begin{cases}1 & \mathrm{k}=0\\ 0 & \text{otherwise}\end{cases}\right)=0\end{aligned} \tag{19}$$

By suppose W_0 and W_1 are apparent from boundary conditions by solving Eq. (19) respect W_{k+2}, we will have:

$$W_2=-\frac{1}{2}\frac{\left(bW_1+cW_1^{\,2}-d\right)}{a} \tag{20}$$

$$W_3=-\frac{1}{3}\frac{\left(W_2\left(b+2cW_1\right)\right)}{a} \tag{21}$$

$$W_4=\frac{-1}{12}\frac{\left(3bW_3+6cW_3W_1+4cW_2^2\right)}{a} \tag{22}$$

$$W_5=-\frac{1}{5}\frac{\left(bW_4+2cW_4W_1+3cW_3W_2\right)}{a} \tag{23}$$

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The above process is continuous. Substituting Eq. (20-23) into the main equation based on DTM, Eq. (18), it can be obtained the closed form of the solutions,

$$\begin{aligned}w(t)=W_0+tW_1&-\frac{t^2}{2}\frac{\left(bW_1+cW_1^{\,2}-d\right)}{a}\\&-\frac{t^3}{3}\frac{\left(W_2\left(b+2cW_1\right)\right)}{a}\\&-\frac{t^4}{12}\frac{\left(3bW_3+6cW_3W_1+4cW_2^2\right)}{a}\\&-\frac{t^5}{5}\frac{\left(bW_4+2cW_4W_1+3cW_3W_2\right)}{a}+\cdots\end{aligned} \tag{24}$$

Substituting Eq. (20-23) into the main equation based on DTM, it can be obtained the closed form of the solutions. In this stage for achieve higher accuracy we use sub-domain technique, i.e. the domain of t should be divided into some adequate intervals and the values at the end of each interval will be the initial values of next one. For example for first sub-domain assume that distance of each interval is 0.005. For first interval, $0 \to 0.005$ boundary conditions are From boundary conditions in Eq. (7) at point $t = 0$. By exerting transformation, we will have:

$$W_0 = 0 \tag{25}$$

The other boundary conditions are considered as follow:

$$W_1 = 0 \tag{26}$$

As mentioned above for next interval, 0.005→001, new boundary conditions are:

$$W_0 = w \tag{27}$$

The next boundary condition is considered as follow:

$$W_1 = \frac{dw}{dt}(0.2) \tag{28}$$

For this interval function $w(t)$ is represented by power series whose center is located at 0.005, by means that in this power series t convert to $(t - 0.005)$.

As we can see bellow in similar case for achieves the solution for $u(t)$ as velocity we should apply DTM on Eq. (13) to find transformed function.

$$a\left((k+1)\mathrm{U}_{k+1}\right) + b\mathrm{U}_k + c\left(\sum_{j=0}^{k}\mathrm{U}_{k-j}\mathrm{U}_j\right) - d\left(\begin{cases}1 & \mathrm{k}=0\\ 0 & \text{otherwise}\end{cases}\right) = 0 \tag{29}$$

By assuming that U_0 is apparent from boundary condition by solving Eq. (29) respect U_{k+1}, we will have:

$$U_1 = -\frac{\left(bU_0 + cU_0{}^2 - d\right)}{a} \tag{30}$$

$$U_2 = -\frac{1}{2}\frac{\left(U_1\left(b + 2cU_0\right)\right)}{a} \tag{31}$$

$$U_3 = -\frac{1}{3}\frac{\left(bU_2 + 2cU_2U_0 + cU_1^2\right)}{a} \tag{32}$$

$$U_4 = -\frac{1}{4}\frac{\left(bU_3 + 2cU_3U_0 + 2cU_2U_1\right)}{a} \tag{33}$$

$$U_5 = -\frac{1}{5}\frac{\left(bU_4 + 2cU_4U_0 + 2cU_3U_1 + cU_2^2\right)}{a} \tag{34}$$

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As mentioned above this process is continuous. By substituting Eq. (30-34) into Eq. (18), closed form of the solutions is,

$$\begin{aligned} u(t) = U_0 - t\frac{\left(bU_0 + cU_0{}^2 - d\right)}{a} \\ -\frac{t^2}{2}\frac{\left(U_1\left(b+2cU_0\right)\right)}{a} \\ -\frac{t^3}{3}\frac{\left(bU_2 + 2cU_2U_0 + cU_1^2\right)}{a} \\ -\frac{t^4}{12}\frac{\left(bU_3 + 2cU_3U_0 + 2cU_2U_1\right)}{a} \\ -\frac{t^5}{5}\frac{\left(bU_4 + 2cU_4U_0 + 2cU_3U_1 + cU_2^2\right)}{a} + \cdots \end{aligned} \tag{35}$$

And for achieve higher accuracy we use sub-domain technique as described above.

By substituting Eqs. (8-11) into Eq. (24) and Eq. (35), an exact solution for $w(t)$ and $u(t)$ can be obtained which is only related to the particle and the fluid properties.

Runge-Kutta Method: The Runge-Kutta methods are an important iterative method for the approximation solutions of ordinary differential equations. These methods were developed by the German mathematician Runge and Kutta around 1900. For simplicity, we explain one of the important methods of Runge-Kutta methods, named forth-order Runge-Kutta method.

Consider an initial value problem be specified as follows:

$$y' = f(t, y), \qquad y(t_0) = y_0. \tag{36}$$

Then RK4 method is given for this problem as below:

$$y_{n+1} = y_n + \frac{1}{6}h\left(k_1 + 2k_2 + 2k_3 + k_4\right), \tag{37}$$

$$t_{n+1} = t_n + h. \tag{38}$$

Where y_{n+1} is the RK4 approximation of $y(t_{n+1})$ and

$$k_1 = f\left(t_n, y_n\right), \tag{39}$$

$$k_2 = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1\right), \tag{40}$$

$$k_3 = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_3\right), \tag{41}$$

$$k_4 = f\left(t_n + h, y_n + hk_3\right). \tag{42}$$

Real Combination of Sphere-Fluid: Mentioned method was applied for real combination of solid-fluid. A single Aluminum spherical particle of 3 mm diameter was

Table 1: Physical properties of materials

Material	Density [kg/m^3]	Viscosity [Kg/m.s]
Olive oil	913.0	0.0840
75% Glycerin	1178.2	0.0182
Water	998.0	0.0010
Aluminum	2702.0	-

Table 2: Selected coefficient of Eq. (6)

Solid	Fluid	a	b	c	d/sin (θ)
Aluminum	Olive oil	0.00007929254195	0.03185587204	0.002778281994	0.0002321749754
	75% Glycerin	0.00008679089531	0.006902105608	0.003585292272	0.0001977575336
	Water	0.00008169586033	0.0003792365719	0.003036939135	0.0002211437441

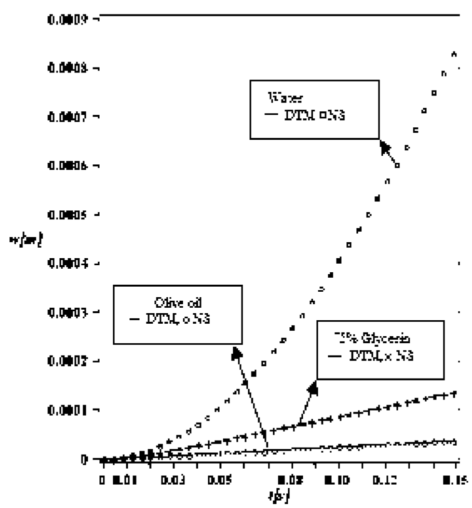

Fig. 3: Displacement variation for different fluid, ($\theta = 2°$)

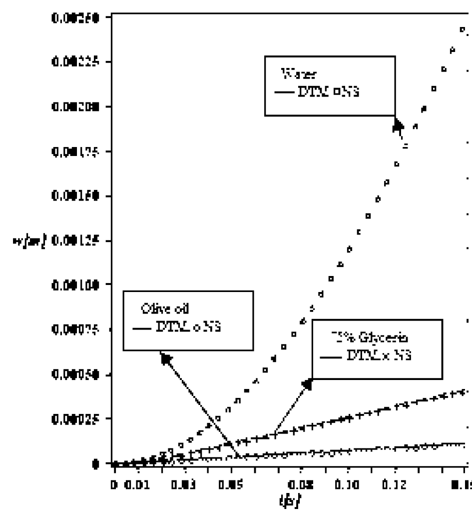

Fig. 4: Displacement variation for different fluid, ($\theta = 6°$)

assumed to roll down a smooth inclined plane in an infinity medium of olive oil, 75% glycerin solution and water. Required physical properties of selected materials are given in Table 1. [31, 32].

Inserting above properties into Eqs. (8) to (11) and using Eq. (4), different combinations are gained which are classified in Table 2.

By substituting above coefficients in Eq. (7) and for four different inclination angles, twelve different nonlinear equations are achieved. Inclination angles were selected to be 2°, 6°, 20° and 60°. Differential Transformation Method was applied to gained equations and results were compared with numerical method. Figures 3 to 6 depict the variation of rolling displacement of the particle versus time for different inclination angles and fluids. These figures clearly illustrate that how inclination angle affects the displacement of particles while other conditions are equivalent. Variable displacement and velocity for sphere which its fluid is water, results of the present analysis for $\theta = 60°$ are tabulated and comprised with the numerical solution obtained by fourth-order Runge–Kutta method in Table 3 and 4 In this case, a very interesting agreement between the results is observed which confirms the excellent validity of the DTM.

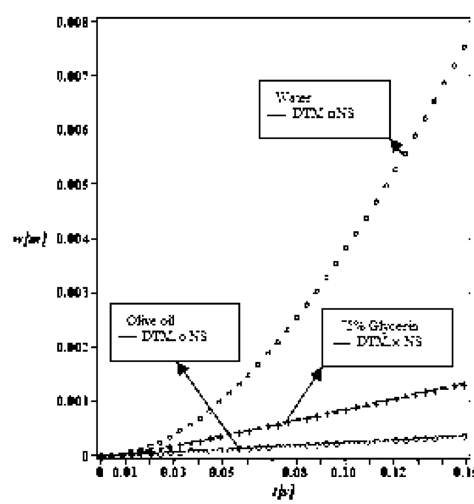

Fig. 5: Displacement variation for different fluid, ($\theta = 20°$)

Table 3: The $w(t)$ obtained from DTM and NS for water ($\theta = 60°$)

Time (s)	NS (m)	DTM (m)	Absolute Error (m)
0.01	0.0001152513	0.0001152596	0.0000000083
0.03	0.0009955233	0.0009955240	0.0000000007
0.05	0.0026320686	0.0026320732	0.0000000046
0.07	0.0048805356	0.0048805405	0.0000000049
0.09	0.0076020439	0.0076020476	0.0000000037
0.11	0.0106764688	0.0106764741	0.0000000053
0.13	0.0140072644	0.0140072704	0.0000000060
0.15	0.0175207012	0.0175207065	0.0000000053

Table 4: The $u(t)$ obtained from DTM and NS for water ($\theta = 60°$)

Time (s)	NS (m/s)	DTM (m/s)	Absolute Error (m/s)
0.01	0.0228428071	0.0228426387	0.0000001684
0.03	0.0640969838	0.0640970297	0.0000000459
0.05	0.0983357245	0.0983357493	0.0000000249
0.07	0.1253427527	0.1253427351	0.0000000176
0.09	0.1458033787	0.1458033546	0.0000000241
0.11	0.1608357457	0.1608357047	0.0000000410
0.13	0.1716326481	0.1716325880	0.0000000601
0.15	0.1792619071	0.1792618477	0.0000000594

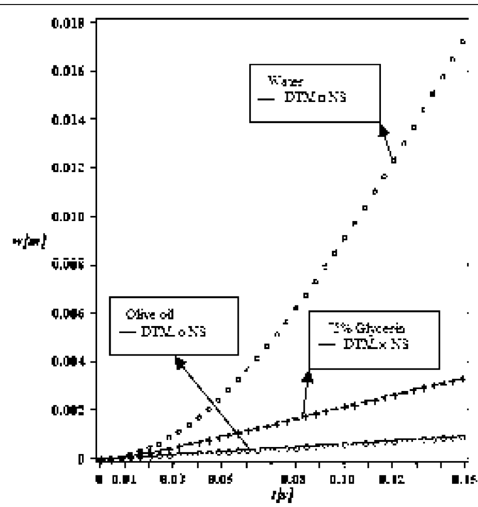

Fig. 6: Displacement variation for different fluid, ($\theta = 60°$)

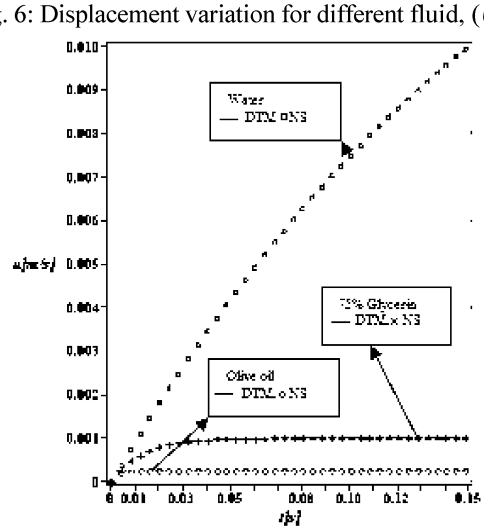

Fig. 7: Velocity variation for different fluids ($\theta = 2°$)

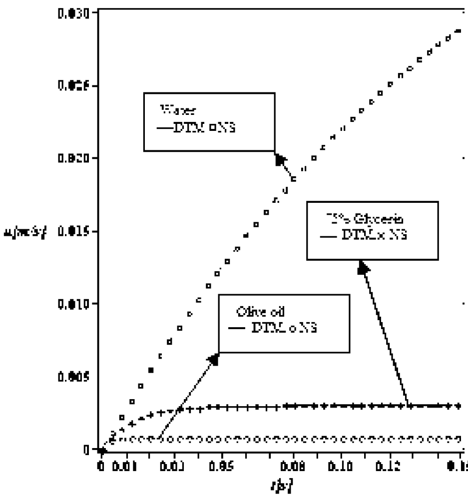

Fig. 8: Velocity variation for different fluids ($\theta = 6°$)

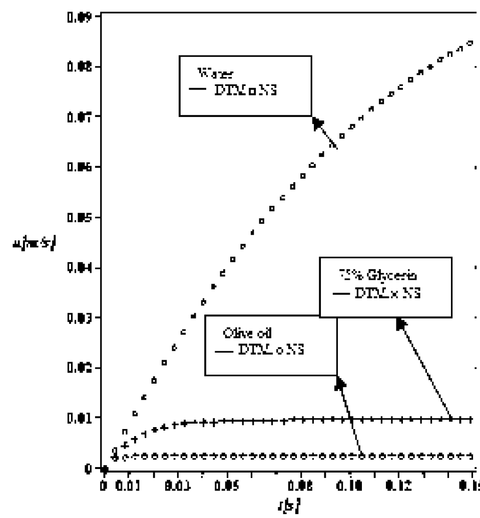

Fig. 9: Velocity variation for different fluids ($\theta = 20°$)

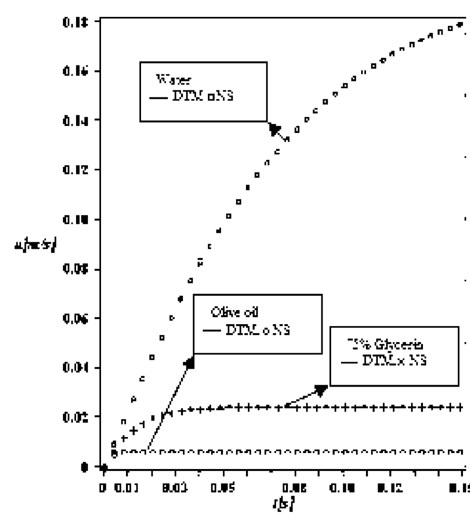

Fig. 10: Velocity variation for different fluids ($\theta = 60°$)

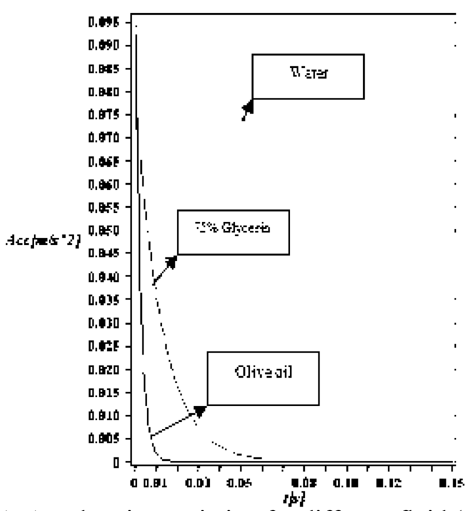

Fig. 11: Acceleration variation for different fluid ($\theta = 2°$)

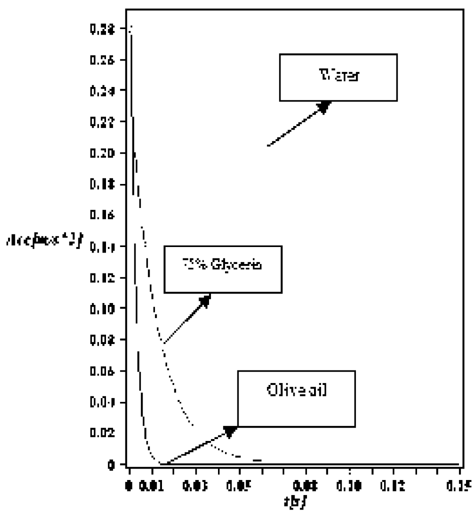

Fig. 12: Acceleration variation for different fluid ($\theta = 6°$)

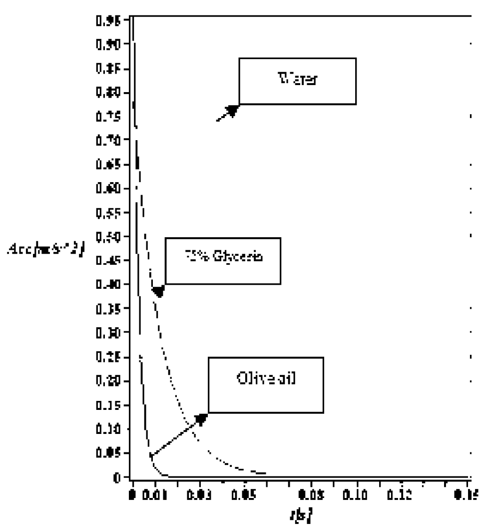

Fig. 13: Acceleration variation for different fluid ($\theta = 20°$)

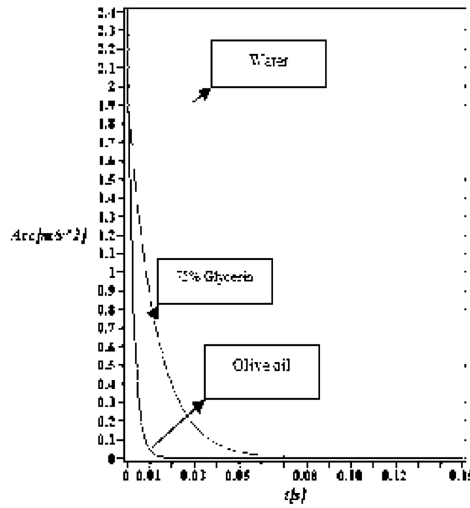

Fig. 14: Acceleration variation for different fluid ($\theta = 60°$)

The variation of rolling velocity of the particle versus time for different inclination angles and fluids are shown in Figs. (7-10). Presented results demonstrate an excellent agreement between DTM and numerical solution. For a given inclination angle, by increasing the fluid viscosity, terminal velocity and acceleration duration are decreased. Results show that increasing of inclination angle increases the terminal velocity as well as acceleration duration and displacement. Also by augmentation of viscosity, the dependence of terminal time on inclination angle is decreased. Employing DTM, the acceleration of the particles, *Acc*, was achieved and presented in Figures 11 to 14 for different fluids. Outcomes illustrated that higher acceleration is obtained for larger inclination angle. Acceleration of particles tends to zero after a while due to constant value of terminal velocity. To show the effect of inclination angle the displacement of particle rolling down in water was obtained for instant time during rolling procedure. Variable displacement and velocity for sphere which its fluid is water, results of the present analysis from Eq. (4) are tabulated and comprised with the numerical solution obtained by fourth-order Runge-Kutta method in Table 3 and 4. In this case, a very interesting agreement between the results is observed which confirms the excellent validity of the DTM.

CONCLUSIONS

In this paper, Differential Transformation Method (DTM) is applied to obtain the solution of rolling particle nonlinear equation with drag coefficient in form of Eq. (4).

Equation was solved generally and for some real combinations of solid-liquid. Instantaneous velocity, acceleration and position were obtained as results and outcomes were compared with Runge–Kutta method solution. Very good agreement has been seen between numerical and current analytical method. Results show that for a given condition of particle and fluid, an increase in inclination angle, θ, results in an increase in terminal distance and a decrease in terminal duration. Current work approved the simplicity and capability of Differential Transformation Method. Solution of equation of motion for an object rolling down an inclined boundary is a new application of DTM and could be used in wide area of scientific problems, especially hydraulic and sedimentation engineering.

Nomenclature

a,b,c,d	Constants	Re	Reynolds number
w	Displacement [m]	α,β	constants
u	Velocity	μ	Dynamic viscosity [*kg/ms*]
t	Time [s]	ρ	Fluid density [kg/m^3]
C_D	Drag coefficient	ρ_s	particle density [kg/m^3]
D	Particle diameter [m]	*m*	Particle mass [*kg*]
g	Acceleration due to gravity [m/s^2]		

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