Lattice Boltzmann Simulation for Magnetohydrodynamic Mixed Convective Flow in a Porous Medium

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Abstract: A numerical study of the magnetohydrodynamic (MHD) mixed convective flow in a lid-driven cavity filled with porous medium was presented by Lattice Boltzmann Method (LBM). Mathematical formulations for flow in porous media were constructed on Brinkman-Forchheimer model. The multi-distribution-function (MDF) model was used for studying the magnetic field effect. The top horizontal wall is moving in its own plane at a constant speed, while the other walls are fixed. A uniform horizontal magnetic field is applied on the cavity. Numerical results are obtained for effect of the Richardson, Hartman and Darcy Numbers over the flow field and heat transfer. It is observed that the fluid circulations within the cavity are reduced by increasing magnetic field strength as well as Darcy number reduction. It is concluded that the heat transfer depends on the Darcy and Hartman numbers. The average Nusselt number increases by decreasing Darcy numbers.

Key words: Lattice Boltzmann Method · Darcy Number · Hartman Number · MHD Flow · Lid-Driven Cavity

INTRODUCTION

The subject of mixed convective flow in a lid-driven cavity has been an important subject for research studies due to its common occurrence in technological applications such as quartz expansion, electronic cooling, oil extraction, etc [1-3]. Prasad and Koseff [4] performed an experimental investigation of mixed convective flow in a lid-driven cavity. Mixed convective flow in two-sided lid-driven differentially heated square cavity was presented by Oztop and Dagtekin [5]. They have found that both Richardson number and direction of moving walls have a great effect on the fluid flow and heat transfer. Effect of heating location on mixed convective flow in lid-driven enclosure was considered by Sivakumar *et al.* [6]. They have found that the heat transfer rate was enhanced by reducing the heating portion. Mixed convective flow in a lid-driven porous cavity has been received large concentration for its applications in engineering and science. Mixed convective flow in an enclosure filled with porous medium with heat generation was numerically investigated by Khanafer and Chamkha [7]. Their results show that heat transfer mechanisms and the flow characteristics inside the cavity depend to the Richardson number. Mixed convective flow in a lid-driven cavity filled with a fluid saturated porous medium was

numerically studied by Kandaswamy *et al.* [8]. They have established that conduction dominates for low Prandtl numbers. Laminar mixed convective flow in a parallel two-sided lid-driven differentially heated square cavity filled with a fluid-saturated porous medium was studied by Vishnuvardhanarao and Kumar Das [9]. Kharicha *et al.* [10] presented a numerical study of a steady laminar MHD flow driven by a disk at the top of cavity. They have found that for fixed values of the Hartmann and Reynolds numbers, the velocity distribution depends on the conductance ratio. MHD mixed convective flow of a heat conducting horizontal circular cylinder in a rectangular lid-driven cavity was investigated by Rahman *et al.* [11]. It is found that the flow field at the cylinder center depends to the Richardson number and the aspect ratio of cavity.

Many researchers studied MHD flow and heat transfer in different porous and non porous geometry [12-17]. Natural convection of an electrically conducting fluid in an enclosure in the presence of a magnetic field was numerically studied by Rudraiah *et al.* [18]. Other experimental studies dealing with MHD flows in porous media were explored by McWhirter *et al.* [19] and Kuzhir *et al.* [20]. Khanafer and Chamkhab [21] numerically considered natural convection in an inclined square enclosure filled by porous medium with heat

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generation. Their results show that the fluid instability within the cavity is reduced by increasing the magnetic field strength. Iliuta *et al.* [22] investigated the MHD of trickle bed reactors with an experimental investigation. Robillard *et al.* [23] numerically and analytically concerned the effect of an electromagnetic field on the free convection in a vertical porous cavity. They have found that under constant heat and mass fluxes the flow is parallel to the core of the cavity. The LBM is an appropriate method for modeling fluid flow and heat transfer in many applications [24-27]. This method has been also separately applied to flow in porous media and MHD flow. A most commonly approach to apply LBM in porous medium is the flow field modeling at the representative elementary volume (REV) [28] scale. This is accomplished by including an additional term to the standard lattice Boltzmann equation for consideration the presence of a porous medium. Spaid and Phelan [29] presented a model based on the Brinkman equation for single-component flow in porous media. Although the Brinkman model has been widely used to describe flows in porous media, some limitations still exist in this model [28]. In this present study, linear and nonlinear matrix drag components are considered as well as the inertial and viscous forces using Brinkman-Forchheimer model [30]. One of the first MHD techniques has been developed by Montgomery and Doolen [31]. In general, the lattice Boltzmann MHD models fall into two categories: the multi-speed (MS) [32] and multi-distribution-function (MDF) [33] approaches. The MS approach has some limitations and thus these restrictions can be ignored by using the MDF approach.

The main purpose of the present numerical work is to investigate the MHD mixed convective flow in a rectangular lid-driven porous cavity using lattice Boltzmann method. The mathematical relations for porous media are based on the Brinkman- Forchheimer equation model [30] and the MDF model is used for investigating the magnetic effect.

MATHEMATICAL MODEL

Mixed convective flow in a rectangular lid-driven cavity has been simulated using LBM (Figure 1). It is assumed that the left and right walls are maintained at a constant but different temperature, θ_h and θ_c ($\theta_h > \theta_c$) and the both top and bottom walls are insulated. Also a uniform magnetic field is applied in the horizontal direction and the top wall has constant velocity U_0. The physical properties are constant except the density variation in the body force term of the momentum equation which is satisfied by Boussinesq's approximation.

Lattice Boltzmann Method for MHD Flow Through Porous Media: The Lattice Boltzmann model for incompressible fluid flow in porous media has been proposed by several researchers [30, 34]. The difference between present model and the noticed models ([30, 34]) is the magnetic effect consideration by modifying the density distribution functions (f_i^{eq}). In LBM, the fluid is modeled by a single-particle distribution function. The distribution functions for porous media are governed by lattice Boltzmann equation as follow [30]:
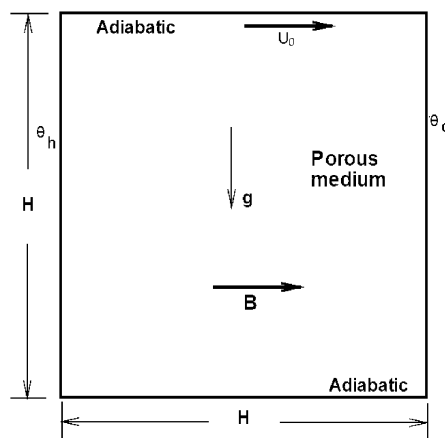

Fig. 1: Geometry of the problem.

$$f_i\left(\vec{x}+\vec{e}_i\delta_t,t+\delta_t\right)=f_i(\vec{x},t)-\frac{f_i(\vec{x},t)-f_i^{eq}(\vec{x},t)}{\tau_v}+\delta_t^{F_i} \tag{1}$$

$$g_i\left(\vec{x}+\vec{e}_i\delta_t,t+\delta_t\right)=g_i(\vec{x},t)-\frac{g_i(\vec{x},t)-g_i^{eq}(\vec{x},t)}{\tau_c} \tag{2}$$

For the D_2Q_9 model, the discrete velocities are defined by:

$$\vec{e}_i = \begin{cases} (0,0) & \text{for} \quad i=0 \\ \left(\cos[(i-1)\frac{\pi}{4}, \sin[(i-1)\frac{\pi}{4}] \right) & \text{for} \quad i=0...4 \\ \sqrt{2}\left(\cos[(i-1)\frac{\pi}{4}, \sin[(i-1)\frac{\pi}{4}] \right) & \text{for} \quad i=5...8 \end{cases} \tag{3}$$

Here δ_t is the lattice time step. The equilibrium functions of density distribution (f_i^{eq}) for D_2Q_9 model in presence of porous media with magnetic field are:

$$f_i^{eq} = \left\{ \omega_i \rho \left[1 + \frac{\vec{e}_i.\vec{u}}{c_s^2} + \frac{(\vec{e}_i.\vec{u})^2}{2\varepsilon c_s^4} - \frac{|\vec{u}|^2}{2\varepsilon c_s^2} \right] \right\} + \left\{ \frac{\omega_i}{2c_s^2} \left[\frac{|\vec{B}|^2 |\vec{e}|^2}{2} - \left(\vec{e}.\vec{B} \right)^2 \right] \right\} \tag{4}$$

Where $\square$ is the porosity of the medium and $\vec{B}$ is the magnetic field and ù$_i$ is weighting factor and c_s is the speed of sound that is defined by $c_S = \frac{c}{\sqrt{3}}$ [30]. The weighting factors are:

$$\omega_i = \left\{ \frac{4}{9} \text{ for } i=0, \frac{1}{9} \text{ for } i=1...4, \frac{1}{36} \text{ for } i=5...8 \right\} \tag{5}$$

Similarly, a magnetic equilibrium functions (h_i^{eq}) are introduced by Dellar [33] for calculating the magnetic field effect:

$$h_{ix}^{eq}x = \lambda_i \left[\vec{B}_x + \frac{1}{c_s^2} \vec{e}_{ix} (\vec{u}_y \vec{B}_x - \vec{u}_x \vec{B}_y) \right]$$
$$h_{iy}^{eq}y = \lambda_i \left[\vec{B}_y + \frac{1}{c_s^2} \vec{e}_{iy} (\vec{u}_x \vec{B}_y - \vec{u}_y \vec{B}_x) \right] \tag{6}$$

Evolutions of the distribution functions for magnetic field consideration are:

$$h_i\left(\vec{x} + \vec{e}_i \delta_t, t + \delta_t \right) = h_i\left(\vec{x}, t \right) - \frac{h_i\left(\vec{x}, t \right) - h_i^{eq}\left(\vec{x}, t \right)}{\tau_m} \tag{7}$$

Where λ_i is the weighting factor of magnetic field:

$$\lambda_i = \begin{cases} \dfrac{1}{3} & \text{for} \quad i=0 \\ \dfrac{1}{6} & \text{for} \quad i=1...4 \end{cases} \tag{8}$$

Thermal distribution functions are [35]:

$$g_i^{eq} = \omega_i T \left(1 + \frac{1}{c_s^2} \vec{e}_i.\vec{u} \right) \tag{9}$$

And the Brinkman-Forchheimer equation is [30]:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)(\frac{\vec{u}}{\varepsilon}) = -\frac{1}{\rho_f} \nabla (\varepsilon P) + \upsilon \nabla^2 u + \vec{F} \tag{10}$$

Here, $p = \dfrac{c^2 \rho}{3\varepsilon}$ and the viscosity $\upsilon = c^2 \left(\tau_\upsilon - 0.5 \right) \dfrac{\delta_t}{3}$. The body force is [30]:

$$\vec{F} = -\frac{\varepsilon \upsilon}{K} \vec{u} - \frac{1.75}{\sqrt{150 \varepsilon K}} |u| \vec{u} + \varepsilon \vec{G} \tag{11}$$

$$K = Da \, H^2, \qquad F_\varepsilon = \frac{1.75}{\sqrt{150 \varepsilon^3}} \tag{12}$$

Where K is the permeability; G is the acceleration due to gravity, Da is the Darcy number and H is the characteristic length. The total body force ($\vec{F}$) encompasses the viscous diffusion and the inertia due to the presence of a porous medium and the external force. The suitable choice for the forcing term, F_i (see Eq. (1)), to obtain the correct equations of hydrodynamics is [30]:

$$F_i = \omega_i \rho \, (1 - \frac{1}{2\tau_\upsilon}) \left[\frac{\vec{e}_i.\vec{F}}{c_s{}^2} + \frac{(\vec{u}.\vec{F}:\vec{e}_i\vec{e}_i)}{\varepsilon c_s{}^4} - \frac{\vec{u}.\vec{F}}{\varepsilon c_s{}^2} \right] \tag{13}$$

The fluid velocity $\vec{u}$ is defined as:

$$\rho \vec{u} = \sum_i \vec{e}_i f_i + \frac{\delta_t}{2} \rho \vec{F} \tag{14}$$

As shown in Eq. (11); $\vec{F}$ contains the velocity $\vec{u}$, therefore Equation (14) is a nonlinear equation for $\vec{u}$. This nonlinearity is ignored by defining a temporal velocity $\vec{v}$:

$$\vec{u} = \frac{\vec{v}}{c_0 + \sqrt{c_0^2 + c_1 |\vec{v}|}} \qquad \vec{v} = \frac{\sum_i \vec{e}_i f_i}{\rho} + \frac{\delta_t}{2} \varepsilon \vec{G} \tag{15}$$

$$c_0 = \frac{1}{2}(1 + \frac{\delta_t}{2} \varepsilon \frac{\upsilon}{K}) \qquad c_1 = \frac{\delta_t}{2} \varepsilon \frac{1.75}{\sqrt{150 \varepsilon^3 K}} \tag{16}$$

The fluid density, temperature and magnetic field are defined as [30 and 33]:

$$\vec{B} = \sum_i h_i, \quad T = \sum_i g_i, \quad \rho = \sum_i f_i \tag{17}$$

Through the Chapman-Enskog procedure, in the limit of small Mach number, equations (1, 2, 6 and 7) recover the continuity, energy and magnetic field equations, respectively [30, 33 and 35].

RESULTS AND DISCUSSIONS

In this study, MHD flow in a rectangular lid-driven cavity saturated with porous medium was performed. The effects of magnetic field and porous parameters on the shear- buoyancy driven cavity are investigated by lattice Boltzmann method. The present computation focused on the parameters having the following ranges: Ri=10^{-3} to 10^{-1}, Da= 10^{-3} to 10^{-1}, Ha=0 t0 50, ε=0.6 and Pr=13.4.

Table 1: Comparison of the present results with Seta *et al* [34] and Nithiarasu *et al* [36]

Da	Ra	ε	Present Study	Seta *et al* [34]	Nithiarasu *et al* [36]
10^{-2}	10^4	0.4	1.41	1.34	1.40
		0.6	1.54	1.55	1.55
		0.9	1.68	1.65	1.64
10^{-4}	10^5	0.4	1.06	1.06	1.06
		0.6	1.07	1.06	1.07
		0.9	1.07	1.07	1.07

To validate the numerical simulation, natural convection in rectangular cavity filled with porous medium in the absence of a magnetic field were simulated and compared with the previous studies [34, 36] (Table 1).
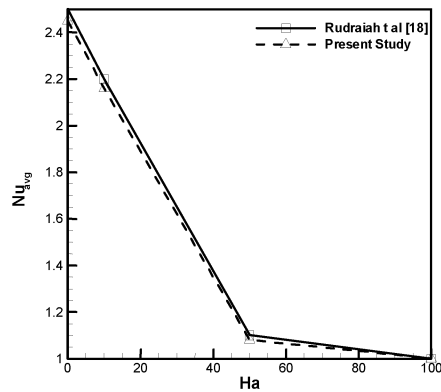
Fig. 2: Average Nusselt number versus Hartman number at $Gr = 2 \times 10^4$.
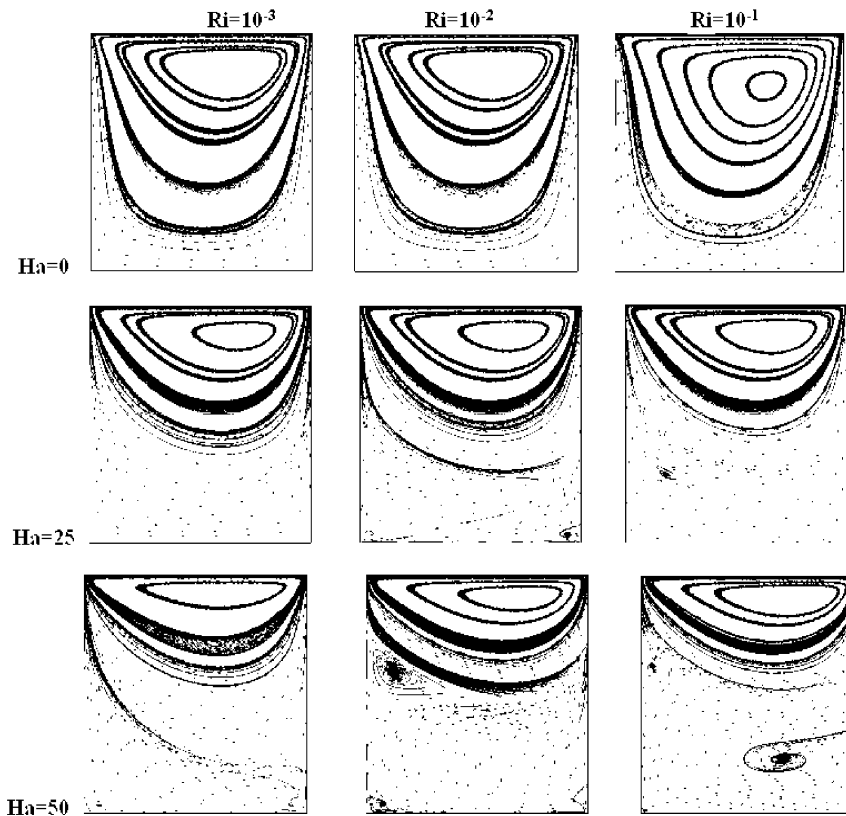

Fig. 3: Streamlines with variable Ha and Ri numbers for Da $= 10^{-2}$ and $\varepsilon = 0.6$.

Also, natural convection in a rectangular enclosure in non-porous medium was modeled and the results were compared with earlier research [18] (Figure 2). The results are provided good agreement between the present and previous numerical solutions. Effect of magnetic field on flow patterns is shown in Figure 3 at different Richardson number. In the absence of magnetic field (Ha=0) the intensity of circulation is increased by increasing the bouncy force. The flow patterns are changed by adding the magnetic field (Ha=25 and 50) and some vortexes are

observed. The Lorenz force (magnetic force) is against the bouncy force so the vortexes are conducted in the cavity by increasing the Hartman number. Figure 4 shows the effect of magnetic field on temperature contours at different value of Richardson number. By increasing the Richardson number and consequently raising the bouncy force, convection heat transfer becomes dominate. Therefore it can be concluded that for higher value of Richardson number, the flow mixing is better than lower value of this parameter (Figure 4).
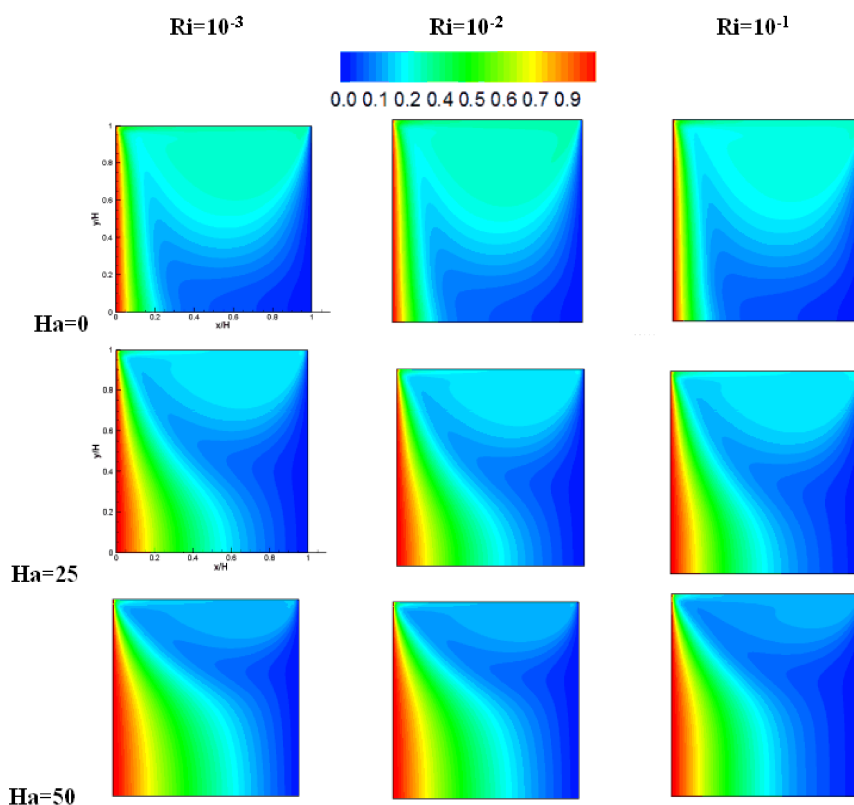
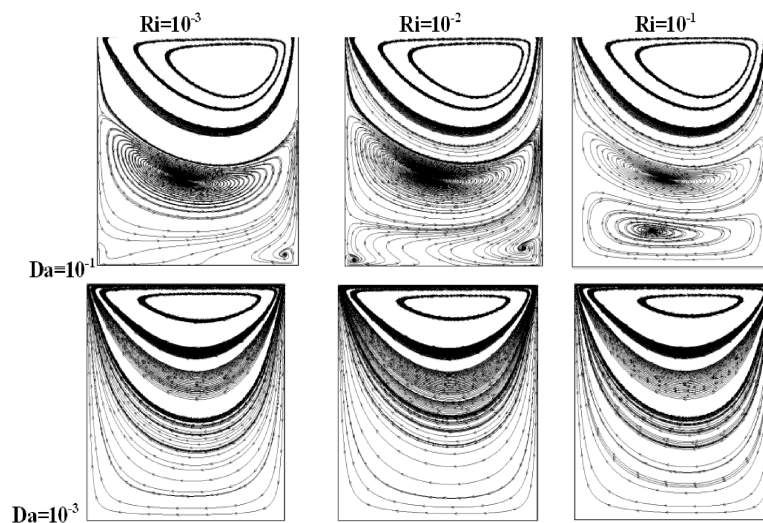Fig. 4: Temperature contours at different values of Ha and Ri numbers for $Da = 10G^2$ and $g = 0.6$.

Fig. 5: Streamlines at different values of Da and Ri numbers for Ha = 25 and $g = 0.6$.

Figure 5 displays the effect of the Darcy number on flow fields at different Richardson numbers. It is observed that the low Darcy number reduces the flow speed in the cavity and the flow field is stabilized at different Ri numbers. In other worlds, the effects of increasing the Hartman number and decreasing the Darcy number are similar to each other but in different mechanisms. Figure 6 shows the temperature contours at different Da and Ri number. It is observed that the buoyancy effect decreases by decreasing the Darcy number. The Richardson number does not have any sensitive effect at low Da number. For high Darcy value (Da $=10G^1$) mixed convective flow is more dominating in the entire cavity.
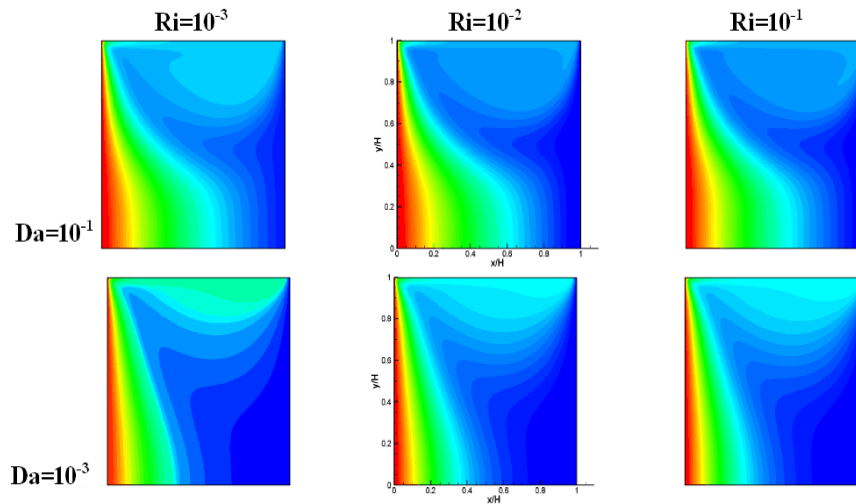
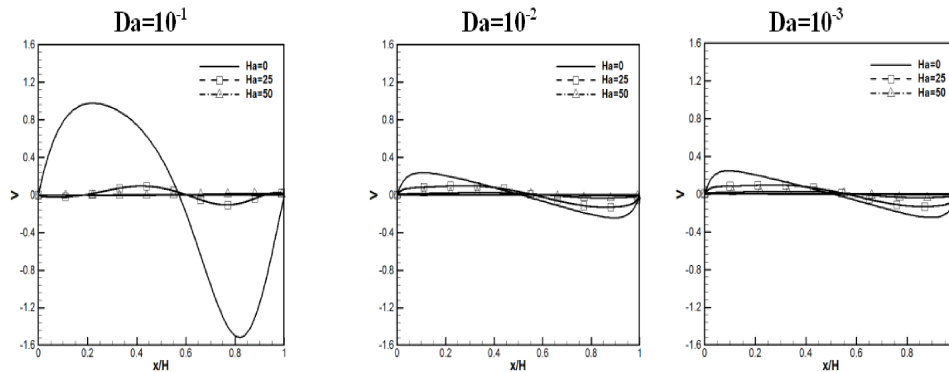Fig. 6: Temperature contours at different values of Da and Ri numbers for Ha = 25 and g = 0.6.

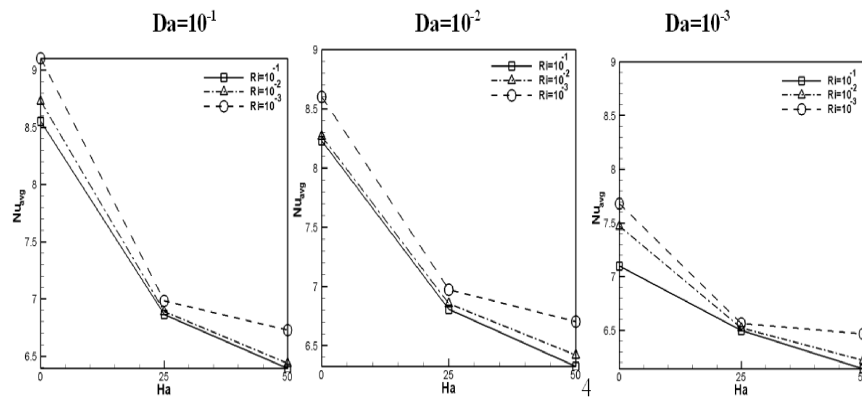Fig. 7: Vertical velocity profile at mid-plane of the cavity at different values of Da for Ri=$10G^2$ and g = 0.6.

Fig. 8: Average Nusselt number for different Ri number at Ra=10^5 and g=0.6.

The velocity profile is plotted at mid-sections of the cavity to illustrate the effect of Hartman and Darcy numbers (Figure 7). Magnetic field creates a force (Lorentz force), opposite to the flow direction, which tends to oppose the flow. It can be observed that the flow instability is damped by decreasing the Darcy number. The average Nusselt number on the hot wall of the cavity is shown in figure 8 for different value of Hartmann numbers. For a constant Richardson number, the average Nusselt number decreases by increasing the Hartman number which is due to dominate the conduction heat transfer. It is observed that the lower Richardson number has a greater heat transfer due to the stranger convective heat transfer which is dominant mechanism of heat transfer.

CONCLUSION

In this study, mixed convective flow in a lid-driven porous cavity with the presence of magnetic field has been presented using lattice Boltzmann method. Flow field in porous media has been simulated by Brinkman-Forchheimer model and the MDF approach has been used for considering the magnetic field. The main observation can be listed as follow:

- Lattice Boltzmann method is an applicable approach for simulating mixed convective flow in a porous cavity at the presence of a magnetic field.
- The flow field and heat transfer are strongly depended on the magnetic field strength and the Darcy number.
- The effect of the Darcy number and magnetic field and is found to reduce the fluid circulation inside the enclosure. It causes to decrease the convection heat transfer.
- The average Nusselt number decreases with increasing the Hartmann number at a constant value of Richardson number.
- The average Nusselt number increases by decreasing the Richardson number. It is due to increase the convection heat transfer in total heat transfer.

Nomenclature

B	Magnetic field strength	*Greek symbols*	
c	Speed of sound in lattice		
c_s	Speed of sound in lattice	α	Thermal diffusivity
Da	Darcy number, K/H^2	δ_t	Time step
e_i	Discrete velocity	ε	Porosity
F_i	Forcing term	η	Magnetic resistivity
	Body force	θ	Dimensionless temperature, $(\theta = T-T_c/T_h-T_c)$
f_i	Hydrodynamic distribution function	λ_i	Weighting factor of magnetic field
f_i^{eq}	Equilibrium distribution function	μ	Viscosity
G	Acceleration due to gravity	ρ	Density
Gr	Grashof number, $(g\beta\Delta\theta H^3/\upsilon^2)$	τ_c	Thermal relaxation time
g_i	Thermal distribution function	τ_m	Magnetic relaxation time
g_i^{eq}	Thermal equilibrium distribution function	τ_v	Hydrodynamic relaxation time
H	Channel width		
Ha	Hartmann number		
h_i	Magnetic equilibrium distribution function		
h_i^{eq}	Magnetic equilibrium distribution function	*Subscripts*	
K	Permeability		
k	Thermal conductivity	*avg*	Average
Nu_{avg}	Average Nusselt number	c	Cold
Pr	Prandtl number (υ/α)	h	Hot
Re	(U_0H/υ)	v	Velocity
Ri	Richardson number (Gr/Re^2)	w	Wall
V	Dimensionless velocity in y direction		
x, y	Cartesians coordinate		

REFERENCES

1. Moallemi, M.K. and K.S. Jang, 1992. Prandtl number effects on laminar mixed convection heat transfer in a lid-driven cavity. Int. J. Heat Mass Transfer, 35: 1881-1892.
2. Iwatsu, R., J.M. Hyun and K. Kuwahara, 1993. Mixed convection in a driven cavity with a stable vertical temperature gradient. Int. J. Heat Mass Transfer, 36: 1601-1608.
3. Singh, S. and M. Sharif, 2003. Mixed convective cooling of a rectangular cavity with inlet and exit opening on differently heated side walls. Numer. Heat Transfer, Part A, 44: 233-253.
4. Prasad, A.K. and J.R. Koseff, 1996. Combined forced and natural convection heat transfer in a deep lid-driven cavity flow. Int. J. Heat and Fluid Flow, 17: 460-467.
5. Oztop, H. and I. Dagtekin, 2004. Mixed convection in two-sided lid-driven differentially heated square cavity. Int. J. Heat Mass Transfer, 47(8): 1761-1769.
6. Sivakumar, V., S. Sivasankaran and J. Lee, 2010. Effect of heating location on mixed convection in lid-driven enclosure. Comp. & Math. with Appl., 59(9): 3053-3065.
7. Khanafer, K.M. and A.J. Chamkha, 1998. Hydromagnetic natural convection from an inclined porous square enclosure with heat generation. Numer. Heat Transfer Part A, 33: 891-910.
8. Kandaswamy, P., M. Muthtamilselvan and J. Lee, 2008. Prandtl Number Effects on Mixed Convection in a Lid-Driven Porous Cavity. J. Porous Media, 11(8): 791-801.
9. Vishnuvardhanarao, E. and M. Kumar Das, 2008. Laminar mixed convection in a parallel two-sided lid-driven differentially heated square cavity filled with a fluid-saturated porous medium. Numer. Heat Transfer, Part A, 53(1): 88-110.
10. Kharicha, A., A. Alemany and D. Bornas, 2004. Steady laminar magnetohydrodynamic flow driven by a disk at the top of the cavity. In. J. Heat and Mass Transfer, 47: 1997-2014.
11. Rahman, M.M., M.A. Alim and M.K. Chowdhury, 2009. Magnetohydrodynamics mixed convection around a heat conducting horizontal circular cylinder in a rectangular lid-driven cavity. J. Scientific Res., 1(3): 345-353.
12. Oreper, G. M and J. Szekely, 1983. The effect of an externally imposed magnetic field on buoyancy driven flow in a rectangular cavity. J. Crystal Growth, 64: 505-515.

13. Vajravelu, K. and A. Hadjinicolaou, 1998. Nonlinear hydromagnetic convection at a moving surface. Nonlinear Analysis, Theory, Meth. and Appl., 31: 867-882.

14. Al-Nimr, M.A. and M.A. Hader, 1999. Mhd free convection flow in open ended vertical porous channels. Chemical Eng. Sci., 54: 1883-1889.

15. Chamkha, A.J., 2002. Hydromagnetic combined convection flow in a vertical lid-driven cavity with internal heat generation of absorption. Numer. Heat Transfer, Part A, 4(1): 529-546.

16. Borjini, M.N., H.B. Aissia, K. Halouani and B. Zeghmati, 2006. Effect of optical properties on oscillatory hydromagnetic double-diffusive convection within semi transparent fluid. Int. J. Heat Mass Transfer, 49: 3984-3996.

17. Muthtamilselvan, M., P. Kandaswamy and J. Lee, 2009. Hydromagnetic Mixed Convection in a Lid-Driven Cavity Field with a Fluid-Saturated Porous Medium. Int. J. Appl. Math. and Mech., 5(7): 28-44.

18. Rudraiah, N., R.M. Barron, M. Venkatachalappa and C.K. Subbaraya, 1995. Effect of a magnetic field on free convection in a rectangular enclosure. Int. J. Engng. Sci., 33: 1075-1084.

19. McWhirter, J.D., M.E.M.E. Crawford and D.E. Klein, 1998. Magnetohydrodynamic flow porous media ii: Experimental results. Fusion Sci. and Tech., 34: 187-197.

20. Kuzhir, P., G. Bossis, V. Bashtovoi and O. Volkova, 2003. Flow o f magnetorheological fluid through porous media. Europ. J. of Mech. B., 22: 331-343.

21. Khanafer, K.M. and A.J. Chamkha, 1998. Hydromagnetic natural convection from an inclined porous square enclosure with heat generation. Numer. Heat Transfer, Part A, 33: 891-910.

22. Iliuta, I. and F. Larachi, 2003. Magnetohydrodynamics of trickle bed reactors: Mechanistic model, experimental validation and simulations. Che. Eng. Sci., 58: 297-307.

23. Robillard, L., A. Bahloul and P. Vasseur, 2006. Hydromagnetic natural convection of a binary fluid in a vertical porous enclosure. Chem. Eng. Comm, 193: 1431-1444.

24. Seta, T. and K. Kono, 2004. Thermal Lattice Boltzmann Method for Liquid-Gas Two-Phase Flows in Two Dimension. JSME Int. J. B, 47(3): 572-583.

25. Delavar, M.A., M. Farhadi and K. Sedighi, 2009. Effect of the Heater Location on Heat Transfer and Entropy Generation in the Cavity Using the Lattice Boltzmann Method. Heat Transfer Res., 40(6): 521-536.

26. Delavar, M.A., M. Farhadi and K. Sedighi, 2010. Numerical Simulation of Direct Methanol Fuel Cells Using Lattice Boltzmann Method. Int. J. Hydrogen Energy, 35: 9306-9317.

27. Nemati, H., M. Farhadi, K. Sedighi, E. Fattahi and A. A. R. Darzi, 2010. Lattice Boltzmann simulation of Nano-fluid in lid-driven cavity, Int. Comm. in Heat and Mass Transfer, Article in Press.

28. Nield, D.A. and A. Bejan, 2006. Convection in Porous Media, Springer, New York.

29. Spaid, M.A.A and F.R. Phelan, 1997. Lattice Boltzmann methods for modeling micro scale flow in fibrous porous media. Phys. Fluids, 9: 2468-2474.

30. Guo, Z. and T.S. Zhao, 2002. Lattice Boltzmann model for incompressible flows through porous media. Phys. Rev. E., 66: 036304-1-036304-9.

31. Montgomery, D. and G.D. Doolen, 1987. Magnetohydrodynamic cellular automata. Phys. Lett. A, 120: 229-241.

32. Chen, H. And W.H. Matthaeus, 1988. An analytical theory and formulation of a local magnetohydrodynamic lattice gas model. Phys. Fluids, 31: 1439-1455.

33. Dellar, P.J., 2002. Lattice kinetic schemes for magnetohydrodynamics. J. Comp. Phys., 179: 95-126.

34. Seta, T., E. Takegoshi, K. Kitano and K. Okui, 2006. Thermal Lattice Boltzmann Model for Incompressible Flows through Porous Media. J. Thermal Sci. and Tech., 1(2): 90-101.

35. Mohammad, A.A., 2007. Applied lattice Boltzmann method for transport phenomena, momentum, heat and mass transfer, Sure printing, Calgary.

36. Nithiarasu, P., K.N. Seetharamu and T. Sundararajan, 1997. Natural convective heat transfer in a fluid saturated variable porosity medium, Int. J. Heat Mass Transfer, 40: 3955-3967.