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Portfolio Optimization with Fuzzy Returns

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Abstract: Financial problems and particularly portfolio selection have been the issues which a corpus of studies has been conducted on them. The purpose of the present study was to optimize portfolio selection, in order to have concordance with environmental condition and obtain better results compared to previous methods. The problems of portfolio selection along with asset return which were in the form of fuzzy variables were investigated in this study. We provided a fuzzy non linear integer programming model for portfolio selection. A numerical example from Tehran Stock Exchange demonstrates the application of the proposed method.

Key words: Portfolio selection · Fuzzy programming · Linear integer programming ·Hybrid algorithm

INTRODUCTION

A lot of studies have recognized the well-known work of Markowitz as the introduction of mathematics into the financial domain. Additionally, they have claimed that it has turned some concepts such as risk and return into quantitative and have regarded it as the commencement of the modern analysis of portfolio [1]. Prior to work of Markowitz, decision-making about securities was done independently and the relationships among them were not taken into account. As a matter of fact, the Markowitz theory was the juncture of synergy (the elements working together produce results which can not be obtained by any of the components alone) and the field of securities [2].

The study of Markowitz has been the cornerstone of lots of studies that endeavored to expand the modern theory of portfolio [3-6]. However, on the other hand many studies have made effort to develop Markowitz's model itself. Such as Capital Asset Pricing Model (CAPM) [7-10], Semi-Variance Model [11] and Safety-First Model [4].

Inasmuch as in the primary studies, the rate of future return was considered as random variables, while the expected return and variances were regarded as constant, the investors acquired incomplete knowledge from environment, consequently the issue of vagueness factors appeared. Today, many researchers make attempt to create different mathematical ways for development of portfolio selection methods [12]. At the same time probability distributions and fuzzy theory, make vagueness and uncertainty come into contact with new approaches. Some of the researchers have used fuzzy models in order to deal with the issue of possibility. For instance, Watada has undertaken a study on the vague objectives for expected return and risk by using the issue of fuzzy portfolio selection [13].

In the mathematical portfolio modeling, some studies have adopted the probability-based dynamic programming approach [14], some of the studies like those of Kumar and his colleagues [15], Lee and Chesser [16] were on the basis of goal programming approach and some other studies such as those of Abdelaziz and his associates [17, 18], Ziemba and Mulvey have underlain multi-criteria linear objective function approach [19]. Among the studies which were dependent on the fuzzy logic [20-23] Lacagnina and Pecorella have proposed a fuzzy programming model with multi-level random constraints to answer both uncertainty and imprecision and at the same time, solve the portfolio selection problem

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[24]. Huang in one of his articles has proposed the portfolio selection based on two models in which certain outputs were in the form of random variables based on fuzzy inputs [25]. Ammar has solved the portfolio optimization model as a convex quadratic programming problem and has obtained an acceptable answer [26]. Furthermore, Inuiguchi and Ramik [20], Leon *et al* [21], Tanaka and Guo [27], Tanaka *et al* [22] and Watada [13] have considered the vagueness factors as fuzzy sets, in their studies.

Statement of the Problem: In the recent years, many financial problems have been subject of many studies. Portfolio selection is a vital activity in all of the organizations which are related to the complicated processes of decision making in various and sometimes conflicted situations [28]. The debate of portfolio selection relates to creation of a satisfactory portfolio, which due to uncertainty about asset return encounters some difficulty. In the portfolio selection models, the decision-maker should accord all the relevant information that is available or required, to some problems or assumptions [1].

So far, lots of studies have been undertaken on portfolio selection, which majority of them has underlain Markowitz's approach and his proposed mathematical model that was based on mean-variance [2]. This model by maintaining its centrality has been a basis for new portfolio models [29, 3-5].

However it is obvious that all the information related to the investment decision-making cannot be only based on asset return and risk; rather the performance can be improved with inspection of more criteria in respect of these factors. Therefore, the models which have considered more criteria than the Markowitz's model have gained prominence. The inadequacy of mathematical models of Hard or Crisp in covering the uncertain, complicated and vague states or existence of imprecise concepts and values makes the employment of fuzzy principles and methods necessary [4]. Particularly, increasing the development of vagueness and uncertainty sources and the diverse approaches for controlling them reveals the necessity of proposing new optimization models. The information that the decision makers obtain are mainly expressed with verbal account such as high risk, low benefit or high rate of interest [30] which were identified by Zade's Fuzzy theory [31] it was determined that incomplete knowledge around asset return and uncertainty about financial markets' behaviors can be improved to some extent by means of fuzzy values or fuzzy constraints.

On the other side, there is an interaction between investment expenditure and financial potential in portfolio selection. Usually, the less a portfolio's value, the less financial return of it. For a decision maker finding an optimum point is difficult and because of plurality of choices it is usually a time consuming task. Such a decision making which is pertinent to selection or non-selection, are most often formulated as 0-1 functions [28]. Employing integer programming for reduction of risk priority [32] using linear programming with infinite dimensions [6] have been undertaken in recent years, but none of these studies has investigated objective function of nonlinear integer along with constraints of fuzzy asset return.

In the models which were on the basis of linear programming methods or its combination with integer programming, favorable outcomes were achieved. However in the cases that objective function was nonlinear, more perfect combination could be obtained by using nonlinear integer programming and fuzzy programming.

Since traditional evaluation methods often consider uncertain data gathered from environment as linear approximation, in this paper we consider a neural network for simulating of fuzzy nonlinear equations. Neural networks, with their remarkable ability to derive meaning from complicated or imprecise data, can be used to extract patterns and detect trends that are too complex to be noticed by either humans or other computer techniques. A trained neural network can be thought of as an "expert" in the category of information it has been given to analyze.

The solution of the fuzzy nonlinear equations in terms of Neural Network possesses several interesting features such as: 1) Neural Networks learn to solve the fuzzy nonlinear equations analytically 2) Small number of parameters 3) Computational complexity does not increase quickly with number of sampling points 4) Rapid calculation of the solution values 5) Implementation ability on existing specialized hardware (neuroprocessor), a fact that will result in a significant computational speedup. [33]

A Common Criticism of Artificial Neural Networks Is That:

- They require a large diversity of training for real-world operation
- So limited that cannot take them seriously as a general problem-solving tool.
- Its implementation needs much processing and storage resources to be committed
- typical neural network involves hundreds of internal parameters, which can lead to "over-fitting" and poor generalization [34]

Some other criticisms came from believers of hybrid models (combining neural networks and symbolic approaches e.g. genetic algorithm). They believe that hybrid models can better capture the mechanisms of the human mind and is a possible solution for many search and optimization problems [35-38].

Genetic algorithm is a heuristic solution search or an optimization technique, originally motivated by the Darwinian principle of evolution through (genetic) selection. A GA uses a highly abstract version of evolutionary processes to evolve solutions to some given problems [39].

Flow chart for hybrid genetic algorithm is illustrated as figure 1.

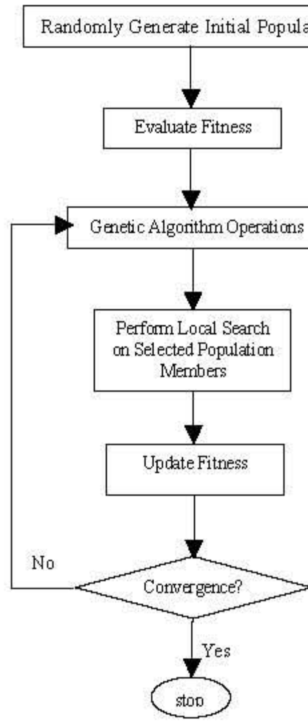
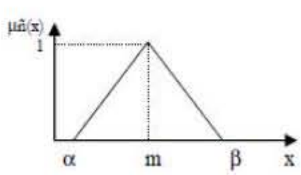


Fig. 1: Flow chart for hybrid genetic algorithm

A Proposed Model: In the mean and variance model of Markowitz [2, 40], the returns have been considered as random variables and it has been supposed that the investors are aiming at striking balance between maximizing the return and minimizing the risk of investment. Thus, the returns with the mean and the risk with the variance portfolio became quantitative.

If we suppose that the return of stock i is a fuzzy variable, its membership function can be displayed in the simple triplet (l, m, u) form, as it is shown in the following:

$$\mu = \begin{cases} \frac{x-l}{m-l} & l \le x \le m \\ \frac{x-u}{m-u} & m \le x \le u \\ 0 & otherwise \end{cases} \quad (1)$$



The reason of using triangular fuzzy numbers for the fuzzy parameters is the augmentation of calculating efficiency [41].

The return rate of each stock is in the form of fuzzy triangular numbers as: $r_i = (m_i - \alpha_i, m_i, m_i + \beta_i)$ in which m_i, is the variable with normal distribution $m_i \sim N(\mu_{m_i}, \sigma_i^2)$, a_i and β_i are also left and right spreads, respectively. Covariance of m_i and m_j has been displayed with $Cov(m_i, m_j) = \sigma_{ij}$ which its matrix equals $V = (Cov(m_i, m_j))_{n \times n}$ that $(a_i, \beta_i) > 0$.

The Objective Function of the Problem: The objective function of the problem can be reflected in the form of defining variables of the future incomes as fuzzy and reducing risk.

$$Max \sum_{i=1}^{n} \tilde{r}_i X_i \quad (2)$$

$$Min \ \ Var(\sum_{i=1}^{n} \tilde{r}_i X_i) = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j \quad (3)$$

Where:

$\tilde{r}_i$: The future return of stock i which is in fuzzy variable form

σ_{ij} : The common covariance

$$X_i = \begin{cases} 0 & \text{if security } i \text{ is purchased} \\ 1 & \text{otherwise} \end{cases}$$

We can revise the above objective function in the following way:

$$Mininize\ \lambda\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j\right]+(1-\lambda)\left\{-\sum_{i=1}^{n}\tilde{r}_i X_i\right\} \quad (4)$$

Where $\lambda \in [0.1]$ is the parameter for risk aversion. So, $\lambda = 0$ represents the state of the maximizing the return and it is the optimal point with the highest return mean. Any value between 0 and 1 is the representative of the exchange between the return mean and variance that will have an answer between the two ends of the problem when $\lambda = 0$ and $\lambda = 1$.

The Constraints of the Problem: The budget constraint: this constraint is related to the prices of stocks. The stocks prices should not be higher than the initial budget of investment, that:

$$\sum_{i=1}^{n} P_i X_i \le B \quad (5)$$

P_i, is the market price for purchasing stock i and
B, is the initial budget of investment

The Minimum Return Constraint: The return rate of any portfolio for fulfilling investors' objectives should be higher than the lower bound of investors' expected return rate for portfolio i in a span of time. The expected rate is taken as fuzzy into account, inasmuch as the vagueness and uncertainty of the rate, in order to make every specified ranges of return, for each portfolio, possible to investigation. That:

$$\sum_{i=1}^{n} \tilde{r}_i X_i \ge \tilde{R}_{\exp} \quad (6)$$

$\tilde{r}_i$ is the future return rate of stock i
$\tilde{R}_{\exp}$ is the future expected rate for stock i

The Volume Constraint: This limitation is the representative of total number of possible stocks type included in portfolio.

$$\sum_{i=1}^{n} X_i = k \quad (7)$$
$$ly \le x_i \le uy$$

K is number of stocks type considered by the stockholders
l,μ are lower and upper number of each stock i, respectively

$$y \in \{0,1\}$$

The Variables Constraint:

$$l \le x_i \le u \ \ ;integer \ \ i = 1,2,...,n \ \ y \in \{0,1\} \quad (8)$$

Case Study: In order to test the model, investment in Tehran Stock Exchange was studied. First, we selected twenty stocks, randomly. Second, the rate of the expected return of each selected stock and the right and left limits have been calculated. Calculation of the right and left sides' values, the vector of the expected return rate and the matrix of covariance has been made by employing historical data and experts' (the traders and stakeholders) ideas. For gathering this data, at first the historical data of each stock during past 12 months in the years 2009-2010 have been collected and used for approximating the future return rate of stock and covariance matrix V= $(Cov(a_i, a_j))_{n\times m}$. Subsequently, the experts and active individuals in this area were asked to state their own estimation of the rate of a_i and β_i. The mean of these estimates were considered as right and left limits and the return rate of fuzzy variable of Stock i is $\mu(r_i) = (\mu(m_i) - ai, \mu(m_i) + \beta_i)$ as illustrated in table 1.

The number of stocks type considered by the stockholders, based on results, is aggregately six ($k = 6$) securities and maximum accepted number of each stock i, is 2000 Unit. Also the initial budget of investment is 100 million (Rials). Minimum expected rate ($R_{\exp}$) considered by the stockholders has obtained 0.47 with 0.14 and 0.32 upper and lower spreads respectively from questioners.

Running the Model: Since it is difficult to find the optimal solution of the proposed model [42] in traditional ways, we use linear and nonlinear (hybrid) approximation methods simultaneously. We can use the technique of simulation [43] and genetic algorithm [44] based on fuzzy non linear integer simulation to help find the optimal solution with hybrid algorithm. When fuzzy simulation is integrated into GA, the algorithm will take a fairly long

Table 1: Future return and expected return

Stock	Return mean, right and left spreads	Future expected return	Stock	Return mean, right and left spreads	Future expected return
x_1	U(0.36 , 0.8 , 0.24)	(0.23,1.03,0.32)	x_{11}	U(0.1, 0.81, 0.65)	(0.24, 1.5, 0.26)
x_2	U(0.39 , 1.1 , 0.52)	(0.14, 0.71, 0.39)	x_{12}	U(0.34, 1.27, 0.65)	(0.32, 1.11, 0.2)
x_3	U(0.15 , 1.3 , 0.15)	(0.44, 2.01, 0.15)	x_{13}	U(0.16, 0.7 , 0.65)	(0.29, 1.22, 0.44)
x_4	U(0.24, 1.8 , 0.18)	(0.39, 2.09, 0.11)	x_{14}	U(0.29, 0.93, 0.42)	(0.74, 1.61, 0.19)
x_5	U(0.14, 1.5 , 0.32)	(0.27, 1.33, .12)	x_{15}	U(0.1, 0.76, 1)	(0.33, 1.45, 0.66)
x_6	U(0.6, 1.6 , 0.94)	(0.4, 0.84, 0.36)	x_{16}	U(0.4, 1.65, 0.58)	(0.12, 1.08, 0.3)
x_7	U(0.1 , 0.61 , 0.4)	(0.19, 1.19, 0.61)	x_{17}	U(0.09, 0.88, 0.25)	(0.22, 1.5, 0.62)
x_8	U(0.32, 1.6, 0..52)	(0.15, 1.33, 0.21)	x_{18}	U(0.44, 1.07, 0.5)	(0.4, 1.61, 0.33)
x_9	U(0.12 , 0.9 , 0.4)	(0.36, 1.96, 0.34)	x_{19}	U(0.1, 1.7, 0.16)	(0.24, 1.61, 0.26)
x_{10}	U(0.4 , 1 , 0.21)	(0.17, 1.73, 0.42)	x_{20}	U(0.2, 0.99, 0.8)	(0.09, 0.7, 1)

time to find the optimal solution. In order to lessen the computational work, we employ neural networks (NNs). NNs are famous for approximating any nonlinear continuous functions over a closed bounded set [20].

In order to neural network modeling, we use one input layer, one hidden layer and two neurons as output layer. In this research, one training data set for objective function employed and for training theses data, back propagation algorithm investigated. And also, logistic sigmoid function used in hidden layer.

Also, for running genetic algorithm we define an integer pop-size = 30 as the number of chromosomes and initialize pop-size chromosomes randomly to produce feasible chromosomes explicitly. Expected values and chances were calculated by them. The probability of crossover and probability of mutation are $P_c = 0.5, P_m = 0.3$ respectively.

The hybrid intelligent algorithm was described as follows:

Step 1: Generate training input-output data for objective function like

$$U = Mininize\left\{ \lambda\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j\right] + (1-\lambda)\left\{-\sum_{i=1}^{n}\tilde{r}_i X_i\right\} \right\}$$

by the nonlinear integer fuzzy simulation.

Step 2: Train a neural network to approximate the objective function according to the generated training input-output data.

Step 3: Initialize *pop size* chromosomes whose feasibility may be checked by the trained neural network.

Step 4: Update the chromosomes by crossover and mutation operations.

Table 2: Comparisons of object values by hybrid intelligent system

Population size	P_m	Crossover function	f
20	0.2	0.5	0.4294131
20	0.2	0.8	0.483819
20	0.3	0.5	0.5550317
20	0.3	0.8	0.600388
30	0.2	0.5	0.633944
30	0.2	0.8	0.6401543
30	0.3	0.5	0.6443701
30	0.3	0.8	0.642452

Step 5: Calculate the objective values for all chromosomes by the trained neural network

Step 6: Compute the fitness of each chromosome according to the objective values.

Step 7: Select the chromosomes by spinning the roulette wheel according to the different fitness values.

Step 8: Repeat the fourth to seventh steps for a given number of cycles.

Step 9: Report the best chromosome as the optimal solution of portfolio selection problem.

A run of the hybrid intelligent algorithm (1000 cycles in fuzzy simulation, 500 training data in NN and 500 generations in GA) shows the optimal solution as Table 2, whose objective value $\overline{f} = 0.6443701$ is highest achieved goal value.

CONCLUSION

Portfolio selection is one of critical decision making process for any stockholders. This paper considered a fuzzy mixed portfolio selection with fuzzy return and linear and integer constraints to set output as integer values and hybrid algorithm approach for solving it. Fuzzy set theory is applied to model uncertain and flexible information. To solving mixed fuzzy model, a hybrid intelligent algorithm is provided to estimate objective function with combinational constraints that used fuzzy linear integer programming to selection stocks. An example was given to illustrate the proposed fuzzy integer portfolio selection using real data from Tehran Stock Exchange results were showed the high fitness of model.

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