World Applied Sciences Journal 11 (1): 100-105, 2010
ISSN 1818-4952

A Novel Method for Ranking of Fuzzy Numbers Using Center of Mass

[1]N. Mikaeilvand, [2]M. Barkhordary and [3]N.A. Kiani

[1]Department of Mathematics, Ardabil Branch, Islamic Azad University, Ardabil, Iran
[2]Department of Mathematics, Bandar Abbas Branch, Islamic Azad University, Bandar Abbas, Iran
[3]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

Abstract: In this paper, the ranking of fuzzy numbers by center of mass is proposed. This method is based on the center of mass at some r-levels of a fuzzy number. Our method can rank more than two fuzzy numbers simultaneously and some properties of method are analyzed. At last, we present numerical examples to illustrate our proposed method and compare with other ranking methods.

Key words: Ranking fuzzy numbers · Central of mass point · Center of mass

INTRODUCTION

For ranking of fuzzy numbers, one fuzzy number needs to be evaluated and compared with the others, but this may not be easy. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than another.

Fuzzy set ranking has been studied by many researchers. Some of these ranking methods have been compared and reviewed by Bortolan and Degain [1], more recently by Chen and Hwang [2] and it still receives much attention in recent years [3-6]. Many methods for ranking fuzzy numbers have been proposed, such as representing them with real numbers or using fuzzy relations. Wang and Kerre [6, 7] proposed some axioms as reasonable properties to determine the rationality of a fuzzy ranking method and systematically compared a wide array of existing fuzzy ranking methods. Almost each method, however, has pitfalls in some aspect, such as inconsistency with human intuition, indiscrimination and difficulty of interpretation. What seems to be clear is that there exists no uniquely best method for comparing fuzzy numbers and different methods may satisfy different desirable criteria.

In the existing fuzzy number ranking methods, many of them are based on the area measurement with the integral value about the membership function of fuzzy numbers[3, 5, 6, 8-16]. A commonly used technique is the centroid based ranking method. Yager [17] proposed centroid index ranking method with weighting function. Cheng [18] proposed a centroid index ranking method that calculates the distance of the centroid point of each fuzzy number and original point to improve the ranking method of [19]. They also proposed a coefficient of variation (CV index) to improve Lee and Lis method [16]. Recently, Tsu and Tsao [20] pointed out the inconsistent and counter intuition of these two indices and proposed ranking fuzzy numbers with the area between the centriod point and original point. Chen [6] also proposed a centroid point and standard deviations based ranking index to overcome the drawbacks of [16, 19, 17]. In this paper, we propose a new method for ranking of fuzzy numbers by means of the center of mass calculating the distance of original point and center of mass point. Thus, we use the ranking method *Cm(A)* which can rank more than two fuzzy numbers simultaneously. In Section 2, we introduce some preliminaries. In Section 3, we present a center of mass distance ranking method and investigate some properties of it. In Section 4, Some numerical examples are illustrated and are compared with some previous methods. The paper ends with conclusion in Section 5.

Background Information: First the notations and basic definitions for developing the ranking center of mass distance are delineated in the following.

Fuzzy Numbers: A real fuzzy number A is a fuzzy subset of the real line $\Re$ with membership function $\mu_A(x) = 0$, $x \in \Re$, which possesses the following properties:

Corresponding Author: N. Mikaeilvand, Department of Mathematics, Ardabil Branch, Islamic Azad University, Ardabil, Iran, E-mail: mikaeilvand@aol.com.

- $\mu_A(x)$ is a continuous function from $\Re$ to closed interval $[0, w]$, $0 \le w \le 1$;
- $\mu_A(x) = 0$, for all $x \in (-\infty, \alpha]$
- $\mu_A(x) =$ is strictly increasing on $[a, b]$;
- $\mu_A(x) = w$, for all $x \in (b, c)$;
- $\mu_A(x)$ is strictly decreasing on $[c, d]$;
- $\mu_A(x) = 0$, for all $x \in (d, \infty]$;

Where $a \le b \le c \le d$ are real numbers. Unless other wise specified, it is assumed that A is convex, normal and bounded, i.e. w = 1. We adopted this definition in this study and represented the membership function of a fuzzy number as (a,b,c,d). Particularly, the membership function of a triangular fuzzy number will have b equivalent to c. We assume that the fuzzy numbers in this study are defined on $\Re$.

$[A]_{r_i}$:the i^{th} r-cut of a fuzzy number A and $r_i = \frac{i}{n}$, $i \in \{0,...,n)$, $n \in N$, where n is the number of r-cuts with $r \ge 0$ and is defined as a crisp interval value

$$[A]_{r_i} = \{x \mid \mu_A(x) \ge r_i, \quad x \in \Re\}$$
$$[A]_{r_i} = [l_i, r_i]$$

Where

$$l_i = \min\{x \in [A]_{r_i}\} \quad r \in (0, 1]$$
$$r_i = \max\{x \in [A]_{r_i}\} \quad r \in (0, 1]$$

Ranking of Fuzzy Numbers Based on the Center of Mass: In this section, we propose the ranking of fuzzy numbers associated with a new method . Consider a fuzzy number A. Then center of mass distance method for ranking fuzzy numbers is defined as:

$$Cm(A) = \frac{2}{n(n+1)}\left(\sum_{i=1}^{n} i\delta_i \sqrt{x_i^2 + y_i^2}\right)$$

and

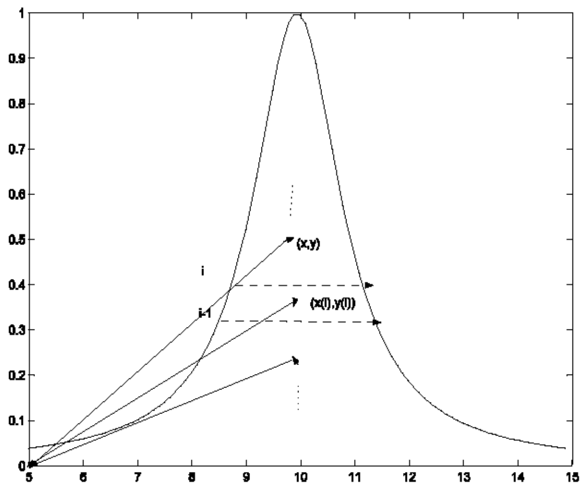
$$\delta_i = \begin{cases} +1 & 1.cmx_i \ge 0 \\ -1 & 1.cmx_i < 0 \end{cases} \qquad (1)$$

for $I = 0,.....,n$

Where (x_i, y_i) is center of mass at i - 1, i-level of fuzzy number A, the calculated as follow:

fuzzy number in i-1 and i^{th}, r-cut corresponds to a x on the horizontal axis and a y_i on the vertical axis. The center of mass (x_i, y_i) for a fuzzy number A in i-1, i^{th}, r-cut defined as

$$x_i(A) = \frac{1}{4i-2}[i(l_i + r_i) + (i-1)(l_{i-1} + r_{i-1})]$$
$$y_i(A) = \frac{1}{n(4i-2)}[4i^2 - 4i + 2 \qquad (2)$$



We can write simplify the above formula for triangular, trapezoidal.

- Let $A = (a,b,c)$ be triangular fuzzy numbers then $[A]_{r_i} = [l_i, r_i]$ where

$$l_i = a + (b-a)\frac{i}{n}$$

and

$$r_i = c + (b-c)\frac{i}{n}.$$

According to (2), we have that:

$$x_i(A) = \frac{1}{2}(a+c) + \left(\frac{1}{2n}\right)\left(\frac{2i^2}{2i-1} - 1\right)(2b - a - c)$$
$$y_i(A) = \frac{1}{n}\left(\frac{2i^2}{2i-1} - 1\right)$$

- If $A = (a,b,c,d)$ be trapezoidal fuzzy numbers then

$[A]_{r_i} = [l_i, r_i]$ where

$$l_i = a + (b-a)\frac{i}{n},$$

and

$$r_i = d + (c-d)\frac{i}{n}.$$

According to (2), we have that:

$$x_i(A) = \frac{1}{2}(a+d) + \left(\frac{1}{2n}\right)\left(\frac{2i^2}{2i-1} - 1\right)(b + c - a - d)$$
$$y_i(A) = \frac{1}{n}\left(\frac{2i^2}{2i-1} - 1\right)$$

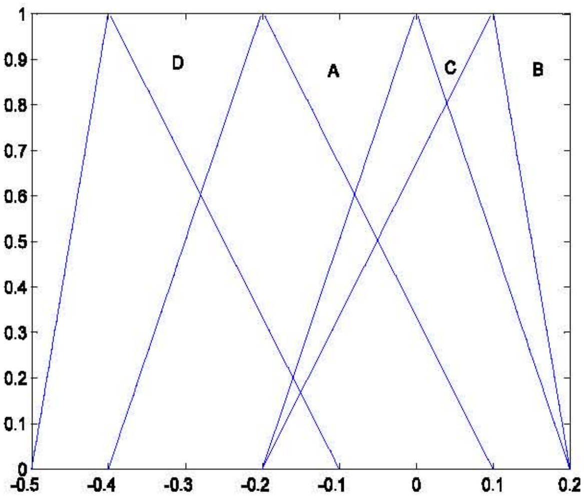


Fig. 1: Example 1

Table 1: Comparative results of Example 1

Fuzzy number	Sign Distance p=1	Sign Distance p=2	Center of mass distance
A	0.25	-0.321455024	-0.7144
B	0.23	0.182574185	0.6911
C	-0.20	0.163299316	-0.6843
D	-0.70	-0.56715665	-0.7834
Results	$D<C<B<A$	$D<A<C<B$	$D<A<C<B$

- If $A = (0,b,0)$ can be fined as

$$x(A) = b$$
$$y(A) = 1$$

$$Cm(A) = \delta(\sqrt{b^2+1})$$

We can use the "Matlab" package to obtain the value of Cm(A). Here, the Cm(A) is used to rank fuzzy numbers. Therefore, for any fuzzy numbers A_i and A_j,

If $Cm(A_i) > Cm(A_j)$ then $A_i > A_j$
If $Cm(A_i) < Cm(A_j)$ then $A_i > A_j$
If $Cm(A_i) = Cm(A_j)$ then $A_i > A_j$

Properties: We consider the following reasonable properties for the ordering approaches, see [21].

A_1 For an arbitrary finite subset Γ of E and $A \in \Gamma$, $A \pm A$.
A_2 For an arbitrary finite subset Γ of E and $(A,B) \in \Gamma^2$, $A \pm B$. and $B^\circ A$, we should have $A \approx B$.
A_3 For an arbitrary finite subset Γ of E and $(A,B,C) \in \Gamma^3$, $A \pm B$. and $B \pm C$. we should have $A \pm C$.

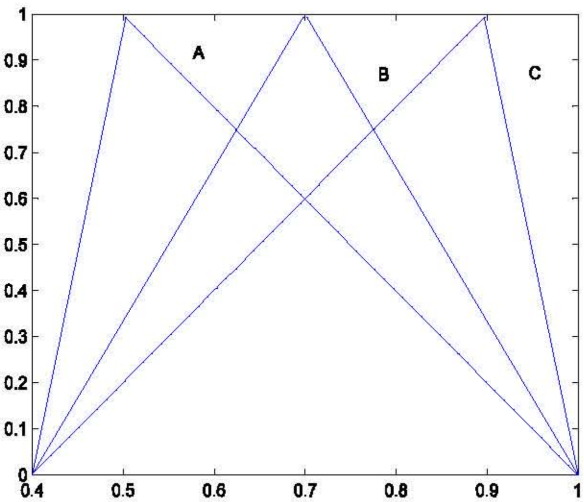


Fig. 2: Set 1.

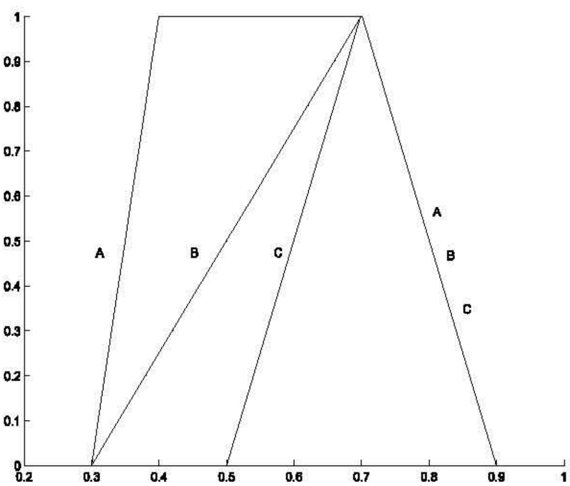


Fig. 3: Set 2.

A_4 For an arbitrary finite subset Γ of E and $(A,B) \in \Gamma^2$, inf *supp (A)> sup supp(B)*, we should have $A \succ B$.
A_4': For an arbitrary finite subset Γ of E and $(A,B) \in \Gamma^2$, inf *supp (A)> sup supp(B)*, we should have $A \succ B$.
A_5 Let Γ and Γ' be two arbitrary finite subsets of E in which A and B are in $\Gamma \cap \Gamma'$. We obtain the ranking order $A \succ B$ by (2) on Γ' if and only if $A \succ B$ by (2) on Γ.

Remark 3.1: center of mass distance method has the properties $A_1, A_2, A_3, \ldots. A_5$

Remark 3.2 : If $A \pm B$. then $-B \pm -A$.

Numerical Example

Example 1: Consider the four fuzzy numbers, A=(-0.4,-0.2,0.1), B=(-0.2,0.1,0.2), C=(-0.2,0,0.2) and D=(-0.5,-0.4,-0.1).

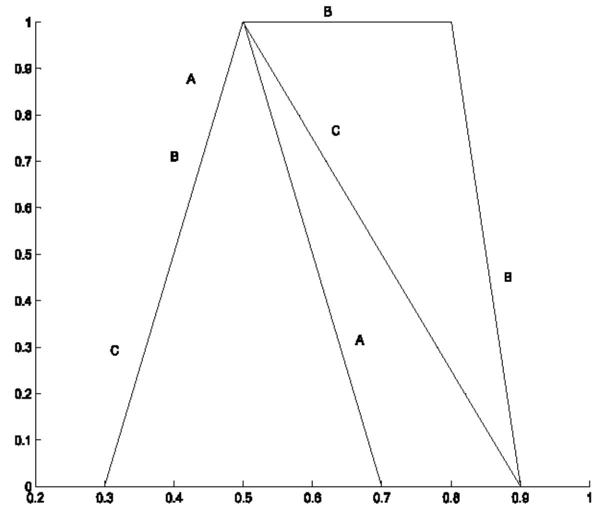


Fig. 4: Set 3.

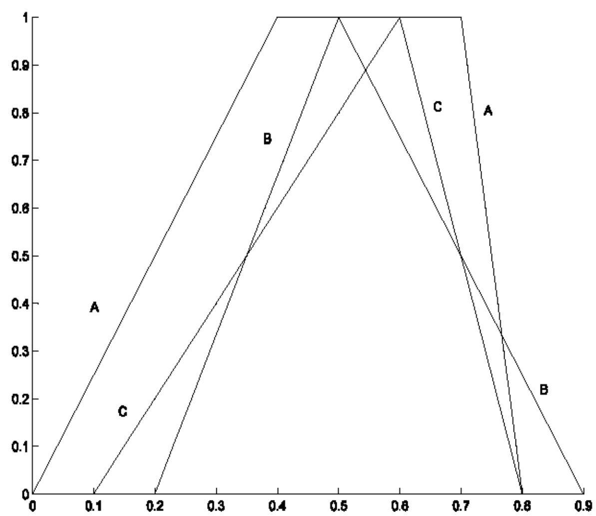


Fig. 5: Set 4.

Table 2: Comparative results of Example 2

Authors	Fuzzy number	Set 1	Set 2	Set 3	Set 4
Center of mass Distance	*A*	0.9045	0.9259	0.8592	0.8601
method	*B*	0.9926	0.9678	0.9707	0.8702
n=10	*C*	1.0913	0.9926	0.8815	0.9316
Results		$A<B<C$	$A<B<C$	$<B<C$	$A<B<C$
Choobineh	*A*	0.333	0.458	0.333	0.50
and Li	*B*	0.50	0.583	0.4167	0.5833
	C	0.667	0.667	0.5417	0.6111
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Yager	*A*	0.60	0.575	0.5	0.45
	B	0.70	0.65	0.55	0.525
	C	0.80	0.7	0.625	0.55
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Chen	*A*	0.3375	0.4315	0.375	0.52
	B	0.50	0.5625	0.425	0.57
	C	0.667	0.625	0.55	0.625
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Baldwin and Guild	*A*	0.30	0.27	0.27	0.40
	B	0.33	0.27	0.37	0.42
	C	0.44	0.37	0.45	0.42
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Chu and Tsao	*A*	0.299	0.2847	0.25	0.24402
	B	0.350	0.32478	0.31526	0.26243
	C	0.3993	0.350	0.27475	0.2619
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Yao and Wu	*A*	0.6	0.575	0.5	0.475
	B	0.7	0.65	0.625	0.525
	C	0.8	0.7	0.55	0.525
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Sign Distance method	*A*	1.2	1.15	1	0.95
p=1	*B*	1.4	1.3	1.25	1.05
	C	1.6	1.4	1.1	1.05

Table 2: Continued

Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Sign Distance method	A	0.8869	0.8756	0.7257	0.7853
p=2	B	1.0194	0.9522	0.9416	0.7958
	C	1.1605	1.0033	0.8165	0.8386
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Cheng Distance	A	0.79	0.7577	0.7071	0.7106
	B	0.8602	0.8149	0.8037	0.7256
	C	0.9268	0.8602	0.7458	0.7241
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Cheng CV uniform distribution	A	0.0272	0.0328	0.0133	0.0693
	B	0.0214	0.0246	0.0304	0.0385
	C	0.0225	0.0095	0.0275	0.0433
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$
Cheng CV proportional distribution	A	0.0183	0.026	0.008	0.0471
	B	0.0128	0.0146	0.0234	0.0236
	C	0.0137	0.0057	0.0173	0.0255
Results		$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$

Table 3 : Comparative results of Example 3

Fuzzy number	Sign Distance p=1	Sign Distance p=2	Chu and Tsao	Cheng Distance	CV index
A	6.12	8.52	3	6.021	0.028
B	12.45	8.82	3.126	6.349	0.0098
C	12.5	8.85	3.085	6.3519	0.0089
Results	$A<B<C$	$A<B<C$	$A<B<C$	$A<B<C$	$C<B<A$

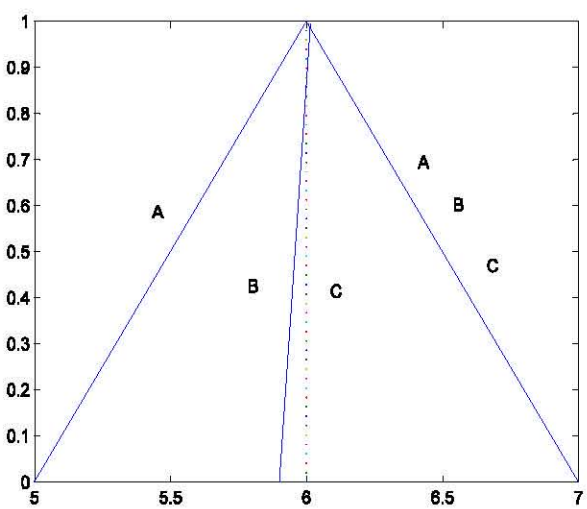


Fig. 6: Example 3.

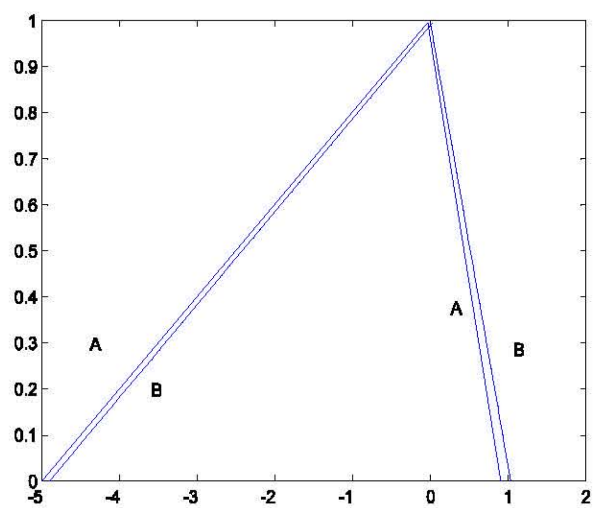


Fig. 7: Example 4.

Example 2: Consider the following sets, see Yao and Wu [22].

Set 1: A = (0.4,0.5,1), B=(0.4,0.7,1),C=(0.4,0.9,1),
Set 2: A = (0.3,0.4,0.7,0.9), B=(0.3,0.7,0.9),C=(0.5,0.7,0.9),
Set 3: A = (0.3,0.5,0.7), B=(0.3,0.5,0.8,0.9),C=(0.3,0.5,0.9),
Set 4: A = (0,0.4,0.7,0.8), B=(0.2,0.5,0.9),C=(0.1,0.6,0.8).

Example 3: Consider the three triangular fuzzy numbers, $A = (5,6,7)$, $B = (5.9,6,7)$ and $C = (6,6,7)$. See Fig. 6.

By using our method $Cm(A) = 6.0427$, $Cm(B) = 6.1841$ and $Cm(C) = 6.998$.

Table 2 : Comparative results of Example 4

Fuzzy number	Sign Distance p=3	Center of mass distance
A	3.155913976	-1.0430
B	3.09633916	-1.0151
Results	$B<A$	$B<A$

Thus the ranking order is *A<B<C*.

As you see in Table 2, the results of Chu-Tsao method and Cheng CV index are unreasonable. The results of sign distance method [23] and Cheng distance method, are the same as our new approach.

Example 4: Consider the four fuzzy numbers, A = (-5,0,0.9), B = (-4.9,0,1.03).

CONCLUSION

In this paper a new method for ranking is proposed. For this propose, center of mass is calculated for each level of fuzzy number. This method has the useful properties in practice, which are consistent by intuition, for instance if *A>B* then *-A<-B*. Finally it is compared with previous method. And by a numerical example it show that in same cases the proposed method is more consist by intuition.

ACKNOWLEDGEMENT

This paper was essential supported by research grant from Islamic Azad University, Ardabil branch.

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