

## Optimizing a Multi-Objectives Time-Cost-Quality Trade-Off Problem by a New Hybrid Genetic Algorithm

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**Abstract:** Project managers are often facing with different and conflicting objectives in optimizing resources in projects. Time, cost and quality are among the crucial aspects of each project. In recent years, demands of project stakeholders are considered in order to maximize the project quality while minimizing the project cost and time simultaneously. This leads researchers to developing models that incorporate the quality factor to previously existing time cost trade-off models. We develop a model for discrete time-cost-quality trade-off problem that uses the planner-specified weights for handling a multi-objective optimization problem. These weights represent the priority of project objectives presented by managers. In this model, for each activity, an execution mode can be selected from a number of possible ones. This paper proposes a new metaheuristic-based genetic algorithm, called NHGA, for optimizing a multi-objectives time-cost-quality trade-off problem. By considering a case example, the performance of the proposed NHGA for solving the presented model and its flexibility for making decision by project managers is illustrated. The results of the proposed algorithm and classic genetic algorithms are analysed by the analysis of variance (ANOVA) method.

**Key words:** Time-cost-quality trade-off problem • Bi-objective model • Hybrid genetic algorithms • ANOVA

### INTRODUCTION

One important aspect of project management is to know about the information related to the optimum balance between the project's objectives. Time, cost and quality are important objectives of a project. Heretofore, extensive researches to develop cost-time trade-off problems have been conducted. Nowadays, in engineering contracts, the quality of a project is also added to the project time and cost. In this case, if it is required to finish the project before a specified due time, the duration of performing some activities must be decreased. To achieve this goal, it is necessary to either increase some resources or change the execution methods of the activities which results in the increased cost and changed quality. The aim of a time-cost-quality trade-off problem (TCQTP) is to select a set of activities for

crashing and an appropriate execution method for each activity such that the cost and time of the project is minimized while the project quality is maximized.

In most time cost trade-off models, the relationship between the decreased duration and the increased cost is assumed to be linear. Furthermore, the aim is to finish the project before a due time and by minimizing the total cost. To solve such a linear model, numerous methods have been put forward [1-7]. Although nonlinear models have also been proposed for the time cost trade-off problem [8, 9]. For real world applications, the discrete time-cost trade-off problem (DTCTP) is applied. Unlike linear models, there are a few studies regarding the DTCTP. This is why discrete time-cost relationship is more relevant to the real-world projects since resources, execution methods and technology types in the real-world projects are represented by discrete values [10].

The solution methods of the DTCTP are classified as exact and heuristic methods. The weaknesses of exact methods have been documented in several matters [11] however; the major weakness is its inability to optimize multi-objectives. In fact, the DTCTP is known as an NP-hard problem [12]. Thus, no exact solution method can be found to have the required efficiency for solving the DTCTP. However, there are many approaches based on heuristics for solving DTCTP [11-20].

The total project quality is affected by project crashing. Thus, it is necessary that we include the quality factor in the time-cost trade-off problem. Including the quality to the model leads us toward the time-cost-quality trade-off problem. For a discrete case, this problem is represented by a discrete time-cost-quality trade-off problem (DTCQTP). In this problem, each project activity can be executed by one of some execution modes. The execution mode of any activity is related to the resources, execution methods and technology of the activity. In this paper, measuring the quality of each activity and the total quality of the project are adapted the procedures used in [21].

The aim of a time-cost-quality trade-off problem is to accomplish the project by considering the minimal cost and time and the maximal quality simultaneously. Not many studies can be found in the literature for this optimization problem [21-28]. By relaxing some non-realistic assumptions, more realistic methods should be considered. In this paper, we consider the problem in the discrete case (i.e., DTCQTP) and relax the assumption of linear relationship between time and cost as well as between time and quality. What makes this study different in several respects from the previous ones are in modeling the multi-objective combinational optimization and introducing a new approach for solving the given problem. In This paper, project managers have been asked to prioritize the objectives of the project then apply a new hybrid genetic algorithm (NHGA) for solving the presented model. The high speed of the algorithm and quick convergence of solutions make this approach suitable for large projects with large numbers of activities.

This paper is organized as follows. In Section 2, we present the problem definition and the problem formulation. In Section 3, a solution procedure is introduced. We develop an algorithm, namely NHGA. To illustrate the proposed approach, a number of examples are presented in Section 4. The related results of this algorithm are analyzed by the analysis of variance (ANOVA) method in Section 5. Finally, the remarking conclusion is given in Section 6.

## PROBLEM FORMULATION

A project is represented by a direct acyclic graph  $G=(V,E)$  consisting of  $m$  nodes and  $n$  arcs, in which  $V=\{1,2,\dots,m\}$  is the set of nodes and  $E=\{(i,j),\dots,(l,m)\}$  is the set of direct arcs. Arcs and nodes represent activities and events, respectively. Each project activity, say  $(i,j) \in E$ , can be executed by a set of modes,  $M_{ij}$ . Each  $k \in M_{ij}$  needs a execution time of  $t_{ijk}$ , cost of  $c_{ijk}$  and quality of  $q_{ijk}$ . Let  $k$  and  $r$  be two modes for activity  $(i,j)$  and  $k < r$ ; then, it is assumed that  $t_{ijk} > t_{ijr}$ ,  $c_{ijk} < c_{ijr}$  and  $q_{ijk} \neq q_{ijr}$ . Although in the literature, it is assumed that any activity time decreasing leads to activity quality decreasing, it is noteworthy that, in real world projects, this is not always the case. For instance, a new technology can be employed to reduce the required time, while this reduction can be accompanied by an increase in the cost and quality.

The aim of this paper is to obtain the optimal combination  $(t_{ijk}, c_{ijk}$  and  $q_{ijk})$  of each activity for crashing the project network, such that along the cost and time of the project is minimized while the project quality is maximized. Notations used for the problem formulation are as follow:

$M_{ij}$	Set of available execution modes for activity $ij$ ; where $(ij) \in E$ ;
$C_{ijk}$	Direct cost of activity $ij$ if performed by execution mode $k$ ;
$t_{ijk}$	Duration of activity $ij$ if performed by execution mode $k$ ;
$q_{ijk}$	Quality of activity $ij$ if performed by execution mode $k$ ;
$\omega_{ij}$	Quality weight of activity $ij$ in the project, $\sum_{ij \in E} \omega_{ij} = 1$
$\omega_{ijl}$	Weight of quality level $L$ in activity $ij$ , $\bullet$ ;
$\sum_{l=1}^L \omega_{ijl} = 1$	
$x_i$	Earliest time of event $i$ , ( $i = \{1, 2, \dots, m\}$ )
$C$	Total project cost;
$T$	Total project duration;
$Q$	Total project quality;

### The Problem of the Dtcqtp Is Formulated By:

$$\text{Min } C = (\sum_{ij \in E} \sum_{k \in M_{ij}} c_{ijk} y_{ijk}) \quad (1)$$

$$\text{Min } T = x_n - x_1 \quad (2)$$

$$\text{Min } Q = (\sum_{ij \in E} \sum_{l \in L} \sum_{k \in M_{ij}} \omega_{ijl} \omega_{ijl} q_{ijk} y_{ijk}) \quad (3)$$

s.t.

$$x_j - x_i \geq \sum_{k \in M_{ij}} t_{ijk} y_{ijk} \quad : \quad ij \in E, i, j \in V \quad (4)$$

$$\sum_{k \in M_{ij}} y_{ijk} = 1 \quad : \quad ij \in E \quad (5)$$

$$x_i \geq 0 \quad : \quad i \in V \quad (6)$$

$$y_{ijk} \in (0,1) \quad (7)$$

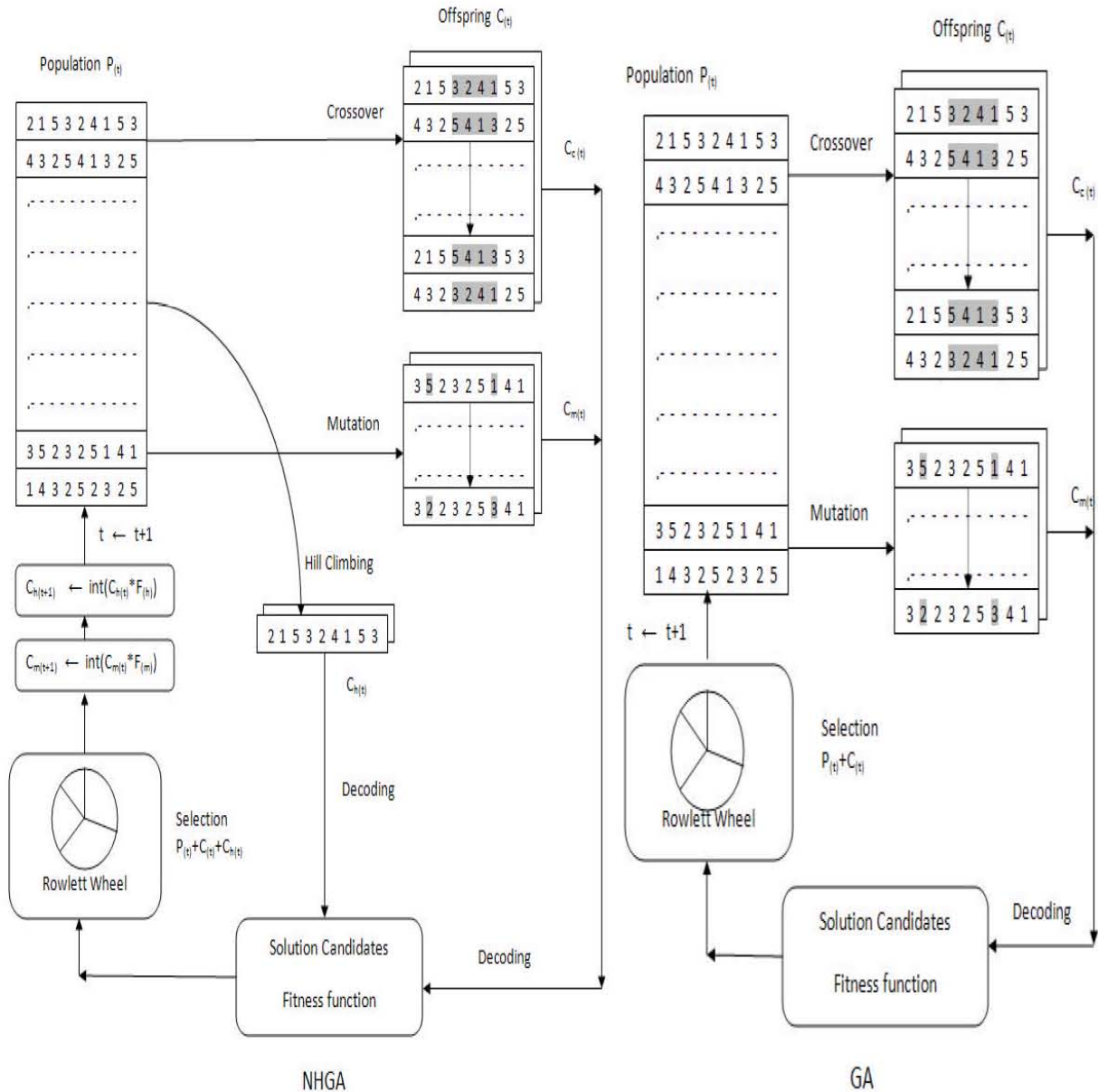


Fig. 1: NHGA and GA flowchart

Eqs. (1) and (2) minimize the total project cost and duration. Eq. (3) maximizes the total project quality. Eq. (4) preserves the precedence relations between project activities. In Eq. (5), one and only one execution mode is assigned to each activity.

**Solution Procedure:** In the DTCQTP, there are a number of execution modes to select for each activity. If the number of project activities is  $n$  and there are  $k$  execution modes for each activity to choose from, then there are  $k^n$  solutions, which

results in a very large search space. Therefore, it is necessary for to develop an efficient evolutionary algorithm.

**New Hybrid Genetic Algorithm:** A new hybrid genetic algorithm (NHGA) is proposed to obtain the optimal (or near-optimal) solution. The high speed of this algorithm and quick convergence of the solutions makes it suitable for solving the given problem. The proposed NHGA is developed by some modifications in the classical GA as shown in Figure 1.

### These Modifications Are Given Below:

- Addition of the hill-climbing method.
- Reduction in a hill-climbing selection rate by introducing a new function.
- Reduction in the mutation rate in the next generations by using a function.

**Nhga Implementation:**  $G(V,E)$  is demonstrated by a node-arc incidence matrix,  $A_{m \times n}$ , in which  $m$  and  $n$  denote the number of nodes and arcs,  $A = [a_{ie}]$ .

$$a_{ie} = \begin{cases} 1 & \text{if node } i \text{ starts arc } e \\ -1 & \text{if node } i \text{ ends arc } e \\ 0 & \text{otherwise} \end{cases} \\ t = 1, \dots, m \quad e = 1, \dots, n$$

A chromosome is a set of integer values (genes) that represents the set of modes for each activity. In each chromosome, only one mode is selected for each activity, which leads to the combination of  $(t, c, q)$  for executing the activity. When reading the numbers for all genes of a chromosome is completed, then an execution mode is selected for all project activities and a chromosome is produced with feasible genes. The main steps of the NHGA are as follows:

**Step 1:** The data are read and then  $N$  chromosomes with feasible genes are randomly produced as primary solutions. The project data consisting of:

- Project network matrix ( $A_{m \times n} = [a_{ie}]$ )
- Available execution modes for each activity  $e$  and their expected activity cost, duration and quality ( $M_e$  and  $(c_{ek}, t_{ek}, q_{ek})$ ).
- Weight of activity  $e$  compared to other activities in the project ( $\omega_e$ ).

### The Required Nhga Parameters Include:

- String size ( $n$ ).
- Number of generation ( $G$ ).
- Population size ( $N$ ).
- Hill-climbing rate.
- Weight of exponential function ( $W_{exp}$ ).
- Weight of linear function ( $W_{lin}$ ).
- Two point crossover rate.
- Uniform crossover rate.
- Mutation rate.

**Step 2:** The following notation is used to describe this step:

- $c_{se}$  Direct cost of activity  $e$  for chromosome  $S$ ;
- $t_{se}$  Duration of activity  $e$  for chromosome  $S$ ;
- $q_{se}$  Quality of activity  $e$  for chromosome  $S$ ;
- $a_{ie}$  Entry of incidence matrix, as defined before;
- $b_i$  Available supply in node  $i$ ;

$$l_e = \begin{cases} 1 & \text{if the active } e \text{ is in the path;} \\ 0 & \text{otherwise.} \end{cases}$$

For each chromosome, the total direct cost of project  $C_{(s)}$ , the total project duration  $T_{(s)}$  and the quality of project  $Q_{(s)}$  are calculated as follows:

Total cost of project: the sum of direct costs of all project activities.

$$C_{(s)} = \sum_{e=1}^n c_{eg} \quad s = 1, 2, \dots, N \quad (8)$$

**Total Time of Project:** The sum of activities duration in the critical path.

$$T_{(s)} = \text{Max} \sum_{e=1}^n l_e t_{eg} \quad s = 1, 2, \dots, N \quad (9)$$

s.t.

$$\sum_{e=1}^n a_{ie} l_e = b_i$$

$$l_e \in (0, 1) \quad \forall e \in E \quad (10)$$

Total quality of project: weighted sum of activities quality

$$Q_{(s)} = \sum_{e=1}^n \omega_e q_{eg} \quad s = 1, 2, \dots, N \quad (11)$$

**STEP 3:** Determining the fitness function ( $F_{(s)}$ ) and the probability of selection ( $P_{(s)}$ ) for each parent chromosome “ $S$ ” by using the following equations:

$$F_g = w_c \times \frac{c_s - c_{\min} + Y}{c_{\max} - c_{\min} + Y} + w_t \times \frac{T_s - T_{\min} + Y}{T_{\max} - T_{\min} + Y} + w_q \times \frac{Q_{\max} - Q_s + Y}{Q_{\max} - Q_{\min} + Y} \quad (12)$$

$$P_s = \frac{\sum_{s=1}^N F_{(s)}}{\sum_{s=1}^N F_{(s)}} \quad (13)$$

Where,  $w_c$ ,  $w_t$  and  $w_q$  are the planner-specified weight.

They indicate the relative importance of project cost, duration and quality respectively. The values of weight coefficients  $w_c$  and  $w_t$  and  $w_q$  are subjectively selected in the range [0,1] by project managers and they should satisfy the equation,  $w_t + w_c + w_q = 1.0$ .

$C_{max}$ ,  $C_{min}$ ,  $T_{max}$ ,  $T_{min}$  and  $Q_{max}$ ,  $Q_{min}$  are the maximal and minimal values of cost, duration and quality in the current population.  $\gamma$  is a very small positive number in order to prevent dividing by zero in the fitness function and also does not permit fitness function to be become zero because the model uses the inverse of fitness function for the reproduction scale in the proposed NHGA. As chromosomes with lower  $F_{(s)}$  are more desirable,  $P_{(s)}$  should be defined so that the probability of selecting chromosome "S" with lower  $F_{(s)}$  is more than other chromosomes. So, "Eq. (13)" is introduced for  $P_{(s)}$ .

**Step 4:** In this step, we produce offspring from parents for entering the next generation. Before applying crossover and mutation operators, first a small number ( $c_{(h)}$ ) of the best parents are directly transferred to the next generation. The number of ( $c_{(h)}$ ), however, decreases from one generation to the next generation. The decline rate function ( $F_{(h)}$ ) is a combination of a linear function and an exponential function with pre-specified weights.

$$F_h = \frac{w_{exp} \times \left(1 - \left(e^{(n_g - c)}\right)\right) + w_{lin} \times \left(1 - \frac{n_g}{c}\right)}{w_{exp} + w_{lin}} \quad (14)$$

Where,  $n_g$  is the generation number index. The direct transfer of the best parents to the next generation with the decline rate function (i.e., Eq. (14)) is an innovation for improving GA for this problem, which enhanced the performance of the algorithm. After hill climbing, to produce the rest of the offspring ( $c_{(t)}$ ), crossover ( $c_{c(t)}$ ) and mutation ( $c_{m(t)}$ ) operators should be applied. The operators are designed such that, after they are applied, the genes of the chromosome are still feasible. In this problem, for the number of activities less than 50, combination of a two-point crossover and a uniform crossover with pre-specified weights are used and for the number of activities greater than 50, only a uniform crossover is used. The uniform crossover was introduced by [29]. Besides, depending on the number of activities, one point to multi-point mutation is used. For instance, in a project with nine activities, two random chromosomes with feasible genes can be as follows:

Parent 1 = [2,1,5,3,2,4,1,5,3]  
Parent 2 = [4,3,2,5,4,1,3,2,5]

In this example, since the number of activities is small, two-point and uniform crossover and two-point mutation are used. By applying the above-mentioned operators, the offspring produced from these parents are as follows:

Two-cut-point crossover with random points ( $e_1=4$ ,  $e_2=7$ ).

Offspring 1 = [2,1,5,5,4,1,3,5,3]  
Offspring 2 = [4,3,2,3,2,4,1,2,5]

Uniform crossover with a random mask chromosome [1,1,0,1,0,0,1,0,0].

Offspring 1 = [4,3,5,5,2,4,3,5,3]  
Offspring 2 = [2,1,2,3,4,1,1,2,5]

Two point mutation with random points ( $e_1=6$ ,  $e_2=9$ ).

Offspring 1 = [2,1,5,3,2,3,1,5,1]

It should be mentioned that the mutation rate ( $F_{(m)}$ ) decreases and uses Eq. (15), so that in the final generation, the mutation rate will be zero.

$$F_{(m)} = 1 - \frac{n_g}{g} \quad (15)$$

**Step 5:** Repeat Steps 2 to 4 until the chromosomes do not change from one generation to the next generation.

The steps of our proposed NHGA are summarized in Figure 1.

**Illustrative Example:** As an example, a project consisting of nine activities is presented in this section as depicted in Figure 2. The network matrix is as presented in Table 1. Different execution modes of each activity associated with its time, cost and quality are presented in Table 2. The effect weight of each activity in the total quality ( $\omega_e$ ) is also considered in Table 2.

Table 1: Network matrix

	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$	$e_8$	$e_9$
1	1	1	0	0	0	0	0	0	0
2	-1	0	0	1	1	0	0	0	0
3	0	-1	1	0	0	0	0	0	0
4	0	0	-1	-1	0	1	1	0	0
5	0	0	0	0	-1	-1	0	1	0
6	0	0	0	0	0	0	-1	-1	1
7	0	0	0	0	0	0	0	0	-1

Table 2: Activities executions modes

Mode	Activities	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$	$e_8$	$e_9$
1	T	7	8	8	10	14	8	11	11	11
	C	160	140	110	100	160	130	150	140	150
	Q	90	85	90	88	92	85	87	91	90
2	T	6	7	7	9	13	7	10	10	10
	C	180	150	120	130	170	140	180	150	170
	Q	85	82	85	90	90	82	90	88	88
3	T	5	6	6	8	12	6	9	9	9
	C	190	170	140	140	180	150	190	160	180
	Q	80	80	84	85	86	80	85	85	85
4	T	4	5	5	7	11	5	8	8	8
	C	200	180	150	150	200	170	200	170	200
	Q	70	75	80	75	70	85	90	75	90
5	T	3	4	4	6	10	4		7	
	C	230	200	170	165	220	190		265	
	Q	85	80	90	80	80	90		85	
6	T					9				
	C					240				
	Q					90				
	$\omega_g$	0.1	0.1	0.14	0.11	0.12	0.15	0.08	0.12	0.08

Table 3: Final outputs of the NHGA

$W_t$	$W_c$	$W_q$	$T$	$C$	$Q$	Solution Chromosome								
0.5	0.2	0.3	31	1675	88.4	5	1	5	2	2	5	2	5	4
0.5	0.3	0.2	31	1645	88.2	5	1	5	2	2	5	1	5	4
0.4	0.3	0.3	33	1540	88.2	5	1	5	2	2	5	1	3	4
0.3	0.3	0.4	39	1420	89.5	1	1	1	2	1	5	2	2	4
0.3	0.4	0.3	40	1350	89.4	1	1	1	1	1	5	1	1	4
0.2	0.3	0.5	43	1360	89.9	1	1	1	2	1	5	2	1	1
0.2	0.4	0.4	43	1300	89.4	1	1	1	1	1	5	1	1	1
0.2	0.5	0.3	46	1250	88.2	1	1	1	1	1	2	1	1	1
0.1	0.6	0.3	47	1240	88.6	1	1	1	1	1	1	1	1	1

The model is programmed in the Microsoft Excel 2007 software using the Visual Basic Application (VBA). The project data given in Tables 1 and 2 are entered in the application software. In this example, there are 1500000 solutions. The presented model is solved in order to obtain the optimal solution. The planner-specified weights are selected by the project manager ( $w_t=0.4$ ,  $w_c=0.3$ ,  $w_q=0.3$ ). Since the number of activities is small in the given example, a combination of a two-point crossover and a uniform crossover with pre-specified weights and two-point mutation have been used. The NHGA parameters are set as follows.

$G=100$ ,  $N=70$ , two-point crossover rate=0.6, uniform crossover rate=0.2, mutation rate=0.2, hill-climbing rate=0.15,  $W_{exp}=0.4$ ,  $W_{lre}=0.6$ .

The program is ran on a Pentium 4 PC with CPU 2.8 Ghz, which chromosome [5,1,5,2,2,5,1,3,4] and its corresponding time, cost and quality ( $T=33$ ,  $C=1540$ ,  $Q=88.16$ ) are obtained as the output.

The project manager may then obtain other optimum solutions by changing the value for the planner-specified weights, which are presented in the results in Table 3.

**Experimental Evaluation:** This section evaluates the performance of our proposed NHGA and the classical GA. These algorithms are coded and implemented in Excel 2007 by the VBA and are ran on a Pentium 4 PC with CPU 2.8 GHz and 512 MB of RAM memory. We use the weight relative deviation (WRD) as a common performance measure to compare these algorithms that is computed by:

$$WRD = \left( w_c \times \frac{c_{alg} - c_{min}}{c_{min}} + w_t \times \frac{T_{alg} - T_{min}}{T_{min}} + w_q \times \frac{Q_{alg} - Q_{min}}{Q_{min}} \right)$$

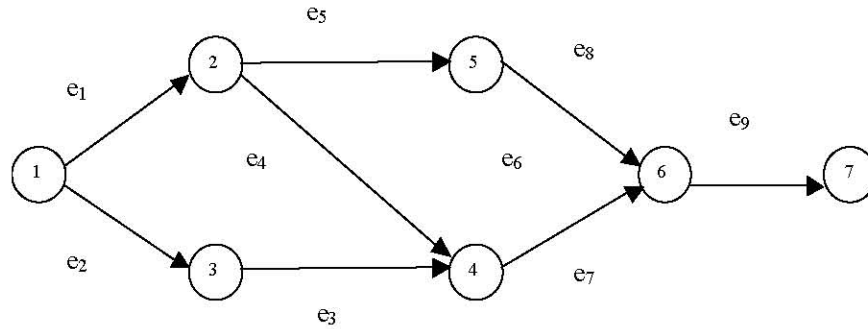


Fig. 2: Project network

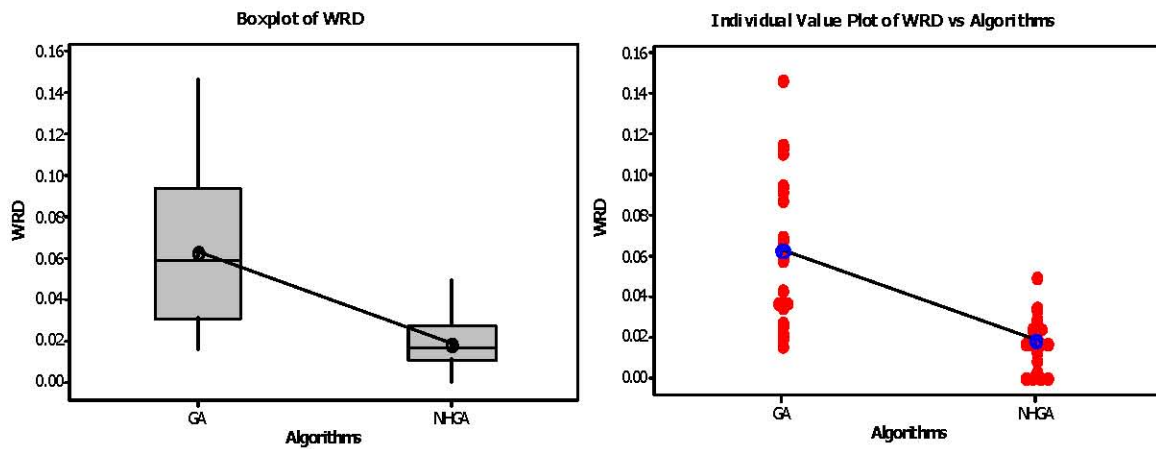


Fig. 3: Means plot and LSD intervals for the NHGA and GA

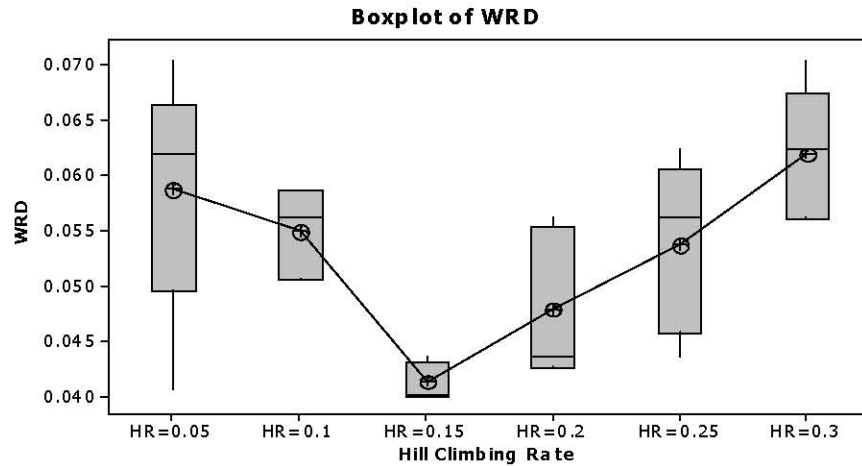


Fig. 4: Means plot and LSD intervals for the hill-climbing rate

Where,  $C_{alg}$  and  $T_{alg}$  and  $Q_{alg}$  are the total project cost, duration and quality for a given algorithm, respectively.  $C_{min}$  and  $T_{min}$  and  $Q_{max}$  are the best solutions obtained by each algorithm for a given instance.

The NHGA and GA are implemented with same parameters for twenty five times. Their

results are analysed via the analysis of variance (ANOVA) method. The means plot and least significant different (LSD) interval for the NHGA and GA are shown as Figure 3. It demonstrates that the NHGA gives better outputs than GA statistically.

The hill-climbing rate (HR) is an important parameter of the NHGA. The considered levels of this parameter are as 0.5, 0.1, 0.15, 0.2, 0.25 and 0.3. The NHGA is implemented five times for each level of the HR. The results are analysed by the ANOVA method. The means plot and LSD interval for the levels of the HR is shown in Figure 4. This figure demonstrates that HR= 0.15 results better outputs than the other HR levels statistically.

## CONCLUSION

Project managers perform project crashing with the aim of reducing the total cost and time along with maximizing the total quality. The model presented in this paper addresses real world projects. To be as realistic as possible, the problem was considered in the discrete time-cost-quality trade-off problem (DTCQTP), in which each project activity can be executed by one of several modes. Associated with each execution mode of any activity, there are specific resources, execution methods and technology. For each mode of an activity, there is a triple combination of time, cost and quality ( $t, c, q$ ). A problem solving algorithm finding an optimal solution considers the time, cost and quality of the project in term of the planner-specified weights. These weights represent the priority of project objectives that should be selected by the managers. Other optimal solutions have been obtained by changing these weights. Having these optimal solutions on hand and analyzing the environmental conditions, project managers can make effectively decisions. To solve the given problem, a new hybrid genetic algorithm (NHGA) was also developed. The project managers have been asked to prioritize the objectives of the project and then we have applied the proposed NHGA for solving the presented model. The high speed of the proposed algorithm and its quick convergence makes it desirable for large projects with a large number of activities. Furthermore, we have used the weight relative deviation (WRD) measure to compare the performance of the NHGA and GA by the ANOVA method. We have also demonstrated that the hill-climbing rate of 0.15 have resulted in statistically better output than the other HR levels. By considering uncertainty in duration, cost and quality factors, this model can be extended to the cases which can be more realistic.

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