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Influence of Neutral Surface Position on Deflection of Functionally Graded Beam under Uniformly Distributed Load

H. Yaghoobi and A. Fereidoon

Department of Mechanical Engineering, Semnan University, Semnan, Iran

Abstract: Bending analysis of a functionally graded (FG) simply supported beam subjected to a uniformly distributed load has been investigated. The material properties of the beam vary continuously in the thickness direction according to the power-law form. The neutral surface position for such FGM beams is determined. The present model is compared with the conventional mid-surface based formulation. In this study, the effect of power law index on the deflection of the beam is examined. Numerical results indicate that position of neutral surface is very important in functionally graded materials.

Key words: Functionally graded materials · Neutral surface position · Deflection · Simply supported beams

INTRODUCTION

Recently, a new class of composite materials known as functionally graded materials (FGMs) has attracted considerable attention. Typically, FGMs are made from a mixture of metals and ceramics and are further characterized by a smooth and continuous change of the mechanical properties from one surface to another. It has been reported that the weakness of the fiber reinforced laminated composite materials, such as debonding, huge residual stress, locally largely plastic deformations, etc., can be avoided or reduced in FGMs [1, 2].

Static and dynamic analyses of FGM structures have attracted increasing research effort in the last decade because of the wide application areas of FGMs. For instance, Sankar [3] gave an elasticity solution based on the Euler-Bernoulli beam theory for functionally graded beam subjected to static transverse loads by assuming that Young's modulus of the beam vary exponentially through the thickness. Chakraborty [4] proposed a new beam finite element based on the first-order shear deformation theory to study the thermoelastic behavior of functionally graded beam structures. In [4], static, free and wave propagation analyses are carried out to examine the behavioral difference of functionally graded material beam with pure metal or pure ceramic. Zhong and Yu [5] presented an analytical solution of a cantilever FG beam with arbitrary graded variations of material property distribution based on two dimensional elasticity theories. Kapuria [6] presented a finite element model for static and free vibration responses of layered FG beams using an efficient third order zigzag theory for estimating the effective modulus of elasticity and its experimental validation for two different FGM systems under various boundary conditions. Li [7] proposed a new unified approach to investigate the static and the free vibration behavior of Euler-Bernoulli and Timoshenko beams. Kadoli [8] studied the static behavior of a FG beam by using higher order shear deformation theory and finite element method. Benatta [9] proposed an analytical solution to the bending problem of a symmetric FG beam by including warping of the cross section and shear deformation effect. Sina [10] used a new beam theory different from the traditional first order shear deformation beam theory to analyze the free vibration of a FG beams.

In the above studies neutral surface is coincide with geometric mid-surface. Neutral surface of functionally graded beam may not coincide with its geometric mid-surface, because of the material property variation through the thickness.

In the present paper, first, the position of neutral surface for functionally graded beam is obtained then Influence of neutral surface position on deflection of functionally graded beam under uniformly distributed load is studied.

Corresponding Author: H. Yaghoobi, Department of Mechanical Engineering, Semnan University, Semnan, Iran.
E-mail: yaghoobi.hessam@gmail.com.

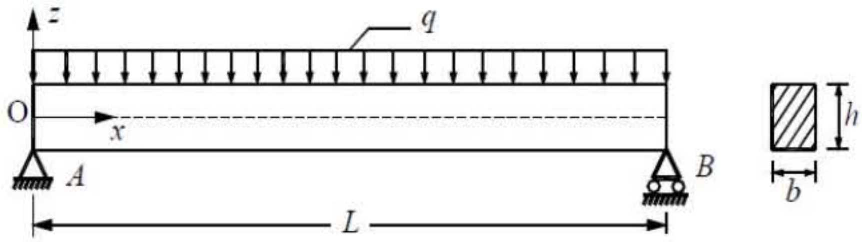

Fig. 1: A functionally graded simply supported beam subjected to a uniformly distributed load

Theoretical Formulation: A functionally graded simply supported beam of length L, width b, thickness h, with coordinate system $Oxyz$ having the origin O is shown in Fig. 1. The beam is subjected to a uniformly distributed load q.

In this study, it is assumed that the FG beam is made of ceramic and metal and the effective material properties of the FG beam, i.e., Young's modulus E vary continuously in the thickness direction (z axis direction) according to power-law form introduced by Praveen and Reddy [11]

$$E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k \tag{1}$$

Where k is the power law index that takes values greater than or equal to zero, m and c stand for metal and ceramic constituents, respectively.

Position of the Neutral Surface: Prior to the determination of a desirable solution, the location of the neutral surface must be given. Clearly, due to varying young's modulus of the beam, the neutral surface is no longer at the midplane, but shifted from the midplane unless for a beam with symmetrical young's modulus. To determine the position of the neutral surface, we construct a new coordinate system such that the new x-axis is placed at the neutral axis, which will be determined below. Then we have

$$x = x_1, z = z_1 + h_0 \tag{2}$$

Where h_0 is the distance of the neutral surface from the midplane of the beam. In this case, similar to the usual treatment in the Euler-Bernoulli beam theory, we can directly write the strain ε_{xx} and σ_{xx} as

$$\varepsilon_{xx} = -\frac{z_1}{\rho} = -z_1\frac{d^2w}{dx^2} \tag{3}$$

$$\sigma_{xx} = E(z_1)\,\varepsilon_{xx} \tag{4}$$

Where w is the deflection of the functionally graded beam and ρ is the curvature radius of the neutral surface. Here the small deformation assumption has been employed. The position of the neutral surface can be determined by choosing h_0 such that the total axial force at cross- section vanishes

$$\sum F_x = 0 \to \int_{-\frac{h}{2}-h_0}^{\frac{h}{2}-h_0} \sigma_{xx} dA = 0 \tag{5}$$

Substituting (3) together with (4) into (5) result in

$$\int_{-\frac{h}{2}-h_0}^{\frac{h}{2}-h_0} b \times E(z_1) \times \frac{z_1}{\rho} dz_1 = 0 \tag{6}$$

By changing interval of integral we have

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} b \times E(z) \times \frac{z-h_0}{\rho} dz = 0 \tag{7}$$

Then

$$\frac{b}{\rho}\left(\int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) \times z dz - h_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) \times dz\right) = 0 \tag{8}$$

The position of neutral surface can be determined from below equation

$$h_0 = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) z dz}{\int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) dz} \tag{9}$$

Substituting Eq. (1) into Eq. (9) and integrating gives

$$\frac{h_0}{h} = \frac{\left(E_c - E_m\right)\dfrac{k}{(k+2)(2k+2)}}{E_m + \dfrac{E_c - E_m}{k+1}} \tag{10}$$

Also, value of k that maximize h_0/h, is calculated from below equation

$$k = \sqrt{2\frac{E_c}{E_m}} \tag{11}$$

Deflection of Functionally Graded Beam: Now let us consider a FGM simply supported beam that is shown in Fig. 1. Bending moment in equilibrium can be expressed as an integral in terms of internal stress:

$$M = \int_{-\frac{h}{2}-h_0}^{\frac{h}{2}-h_0} \sigma_{xx} z dA \tag{12}$$

Plugging Eqs. (3) and (4) into Eq. (12) leads to a differential equation

$$\frac{d^2 w}{dx^2} = \frac{M(x)}{D} \tag{13}$$

Where D denotes bending rigidity of the FGM beam, defined by

$$D = \int_{-\frac{h}{2}}^{\frac{h}{2}} bE(z)\left(z - h_0\right)^2 dz \tag{14}$$

For the case of beam subjected to a uniformly distributed load q we have

$$M(x) = \frac{qLx}{2} - \frac{qx^2}{2} \tag{15}$$

Boundry condition for simply supported beam is

$$w = 0 \; at \; x = 0,l \tag{16}$$

After inserting (15) into (13) we integrate both sides (15) with respect to x twice and by applying the boundry conditions (16) we obtain

$$w = \frac{q}{24D}\left(L^3 x - 2Lx^3 + x^4\right) \tag{17}$$

Maximum deflection is occurred in middle of beam

$$w_{\max} = \frac{5qL^4}{384D} \tag{18}$$

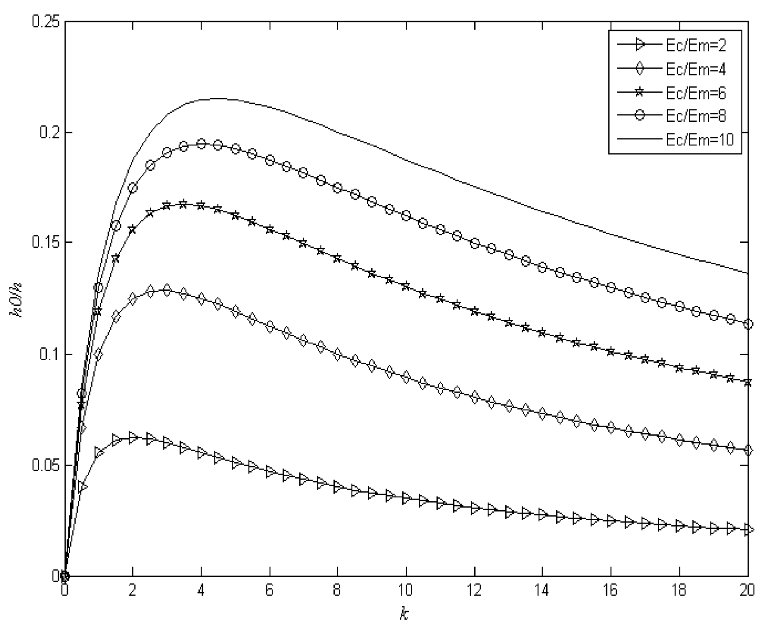

Fig. 2: Effect of the gradient index on the position of the neutral surface of the beam

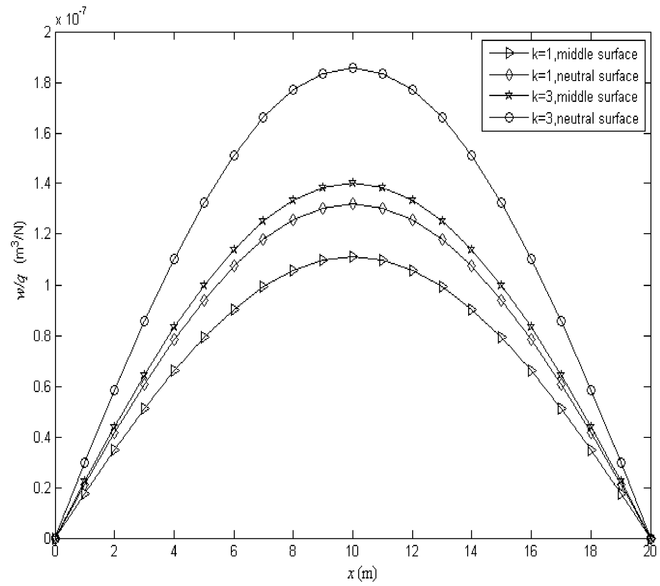

Fig. 3: Influence of neutral surface on deflection of FG beam along the length of beam for k=1, 3

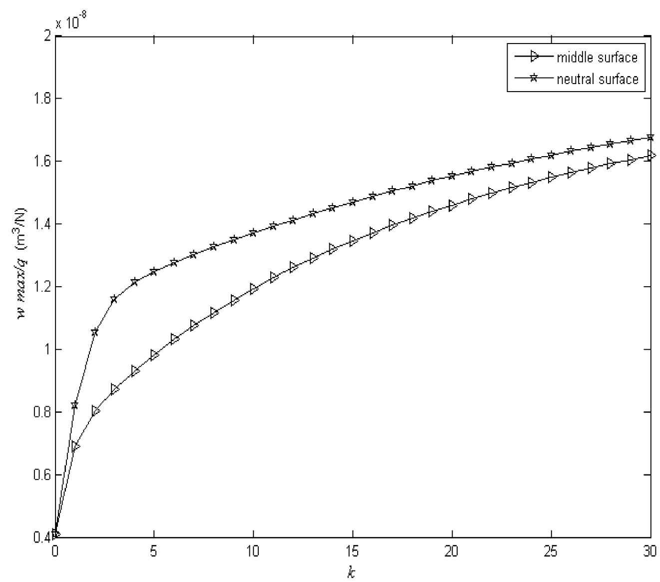

Fig. 4: Maximum deflection respect to k

Numerical Results: A ceramic-metal functionally graded beam is considered. The combination of materials consists of aluminum and alumina. The Young's modulus for aluminum and alumina are $E_m = 70GPa$, $E_c = 380GPa$. The boundary conditions are assumed to be simply supported. The shift (h_0) of the neutral surface from the geometric mid-surface is calculated using Eq. (10) and plotted in Fig. 2. It can be noted from the figure that the nondimentional shift (h_0/h) increases with the increase in k and reaches a maximum value for k which is determined from (11) after which the curve drops down and reaches zero asymptotically. It can be seen that the nondimentional shift is zero for the isotropic plates, i.e., for the pure ceramic beam ($k = 0$) as well as for the pure metallic beam ($k = \infty$).

Fig. 3 shows the deflection of functionally graded beam under uniformly distributed load for $k = 1,3$, $L = 20m$ and two state of neutral surface and middle surface respect to x. We can see that the values of deflection for neutral surface are more than middle surface. Fig. 4 shows difference of maximum deflection for neutral surface and middle surface respect to k for $L = 10m$.

When the power-law exponent is increased, the maximum deflection of the FG beam is increased. Maximum difference for two cases occurs in $k = 3.295$ and then it is decreased. For all of above cases $b = h = 1m$.

CONCLUSION

In this article, first, the position of neutral surface for functionally graded beam is obtained then influence of neutral surface position on deflection of functionally graded beam under uniformly distributed load is studied. It is concluded that:

- The neutral surface for functionally graded beams shifts towards ceramic rich surface. The distance between neutral surface and mid-surface increases with the increase in gradient index k, reaches a maximum value, after which drops down and reaches zero asymptotically.
- When the difference between young's modulus of ceramic and metal is decreased, the nondimentional shift (h_0/h) is decreased.
- The values of deflection for neutral surface are more than middle surface.
- When the power-law exponent is increased, the maximum deflection of the FG beam is increased.

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