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Improvement of One Factor at a Time Through Design of Experiments

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Abstract: Experiments can discover many unexpected things and highlighted issues for further detailed study. The emerging of advanced products and processes are changing rapidly, customers are more demanding and product life-cycle and time to market are shrinking. In this environment engineers and scientists need a strategic approach to overcome this demand. Design of experiments (DOE) is the answer to these challenges. It allows a researcher to understand what happen to the output (response) when the settings of the input variables in a system are purposely changed. Unfortunately there are many scientists and engineers still practice the study one-factor-at-a-time (OFAT). DOE offers a number of advantages over the traditional OFAT approach to experimentation. One of the important advantages of DOE is that it has the ability to discover the presence of interaction between the factors of the process, while OFAT does not. The objective of this paper is to demonstrate how DOE approach works. This paper describes a case study on rubber glove manufacturing process. It illustrates interaction between factors that cannot be found when varying only one factor at a time. Model that describes the relationships between the inputs and output variables were then developed and used to indicate areas where operations may be improved.

Key words: Analysis of variance · Fractional factorial · Replication · Blocking · Optimization · randomization · Interaction · Regression · Orthogonal · Resolution

INTRODUCTION

Knowledge can serve as a driver of change for any organization, businesses, researchers or academicians. In scientific and engineering disciplines, knowledge about a product, process or system is often derived from experimentation. Experiments can unveil many unexpected things such as key opportunities and alert important issues for further detailed study. In today's highly competitive environment, businesses cannot afford to experiments by trial and error. This is because markets and technologies are changing rapidly, cost pressures are increasing, customers are more demanding and product life cycles and time-to-market are shrinking. However, the key problems facing researchers are to ensure research outputs are delivered to the market place on time and ahead of competitor with minimum resources. A strategy of experimentation which could address the above challenges is obviously needed in order to stay ahead of the game.

Design of Experiments or DOE is an alternative answer to the above challenge. Despite its widespread use and economic impact, many scientists, engineers and professionals are not aware of design experiments approach. They perform one-factor-at-a-time experiment (OFAT) to examine or develop a product/process. However, OFAT can prove to be inefficient and unreliable leading to false optimal conditions. Moreover they often consist largely of trial and error, relying on luck, intuition, guesswork and experience for their success [1]. It is applicable to almost every industry. DOE is used to identify the factors that cause changes in the response and predict them in a simple mathematical form. These designs also allow the researcher/investigator to

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study a large number of variables in a small number of experimental runs. They have proven to be extremely useful tool in research and industrial development applications. The use of DOE is most beneficial in multidisciplinary application, where traditional engineering analysis, simulation and verification are difficult to achieve [2]. Experimental designs are often utilized during the development phase and the early stages of manufacturing, rather than as a routine on-line or in-process control procedure. Also, these techniques offer a potentially useful methodology for examining cause and effect relationships [3].

DOE uses statistical experimental methods to develop the best factor and level settings to optimize a process or a design. The statistical analysis of the data can be performed quickly through the use of specialized software analysis packages such as Minitab, Design expert, JUMP etc.

Why it is possible to study several factors simultaneously and yet obtained useful information. This will be discussed in more detailed in the next section. This simultaneous look at the process variables can be a big time-saver according to Sola optical plant manager in California. The company claimed that the throughput of its lens coating line after DOE implementation revealed optimal set-up and gained enough capacity. In this way the company avoided a $250K equipment purchase despite increase in lens demand.

The objective of this paper is to demonstrate how DOE approach works. This paper describes a case study on rubber glove manufacturing process. It illustrates interaction between factors that cannot be found when varying only one factor at a time.

Experimentations

One-Factor-at-A-Time Approach to Experimentation: In the traditional one factor at a time (OFAT) experiments, no advanced statistical knowledge is needed in its execution and data analysis. The OFAT approach is still popular in many organizations when carry out experiments to determine the setting of main factors [4]. Historically, scientists and engineers perform OFAT experiments by changing one factor at a time and keeping others fixed. This factor is varied until its best setting is found. It is then fixed at this level. Next, the other factor is then changed until its best setting is found and held constant at this setting. The whole process is repeated with another factor. One of the pitfalls of this approach is that the estimation of the factor effects is less precise because the experimental conclusions are drawn after collecting the data of each trial run by comparing the observed outcome with previous result. Typically only 2 of the observations in OFAT experiments are used to estimate the effect of each factor. This form of experimentation can be regarded as trial and error which requires luck, experience and intuition for its success [5].

Another pitfalls in this approach is that one will miss the best settings if the factors interact that is if the effect of a factor depends on the setting of other factor(s). This suggests that an interaction exists. It indicates that there is a relationship between the independent factors. An example of interaction between the factors. Suppose Z represents the event of stirring a cup of coffee and Y represents the event of adding sugar to a cup of coffee.

The effect of these factors on sweetness of the coffee depends on the levels of both factors. Neither factor has an effect on its own but together they make the coffee sweet. Factor Z and Y interact. This interaction can be estimated.

Another example of interaction occurs when fertilizer and water are added together. The combined effects are greater than either factor on their own. Using OFAT approach interaction between factors cannot be estimated because there is no information and this can misleading the optimal conditions of the process.

What is Design of Experiments (DOE) Approach: A designed experiment is a modern approach in planning an experiment based on sound statistical practices. DOE constitutes a wide range of techniques such as factorial design, fractional factorial design, response surface method, EVOP etc. It is a structured method of changing multiple factors simultaneously to investigate their effect on one or more outputs in which combinations of factors (run) are allotted to one or more experimental units. This typed of experiments is defined as a factorial experiment. In a full (complete) experiment where every possible combination is run at least once, information about individual and joint effects of the factors on the mean response could be obtained. These are called “main effect” and “interaction effect”, respectively. However, as the number of factors in the factorial experiment increases, the number of runs for full replicate of the experiment rapidly exhausts the resources of most experiments. For instance, to test 8 factors each at 2 levels, a full factorial design would require $2^8 = 256$ runs.

It is not possible to run all combinations. Fortunately, [6] proposed the use of fractional factorial experiments in such situations. These designs contain a fraction of the runs in the complete factorial experiment, allowing the estimation of all main effects and often lower order interactions under the assumption of zero higher order interactions. That is, a fractional experiment is a subset of combinations from a full factorial experiment. Information on higher order is discarded to accommodate extra factors or to reduce the number of testing runs, for example a screening experiment. However, this can lead to difficulties which could distort our view of the main effects. Then, in the late 1940's, Galois field theory was found to be useful in the construction of fractional factorial experiments which gave orthogonal estimates of the factorial effect. [7] presented the general theory of symmetrical factorial experiments, leading [8] to further improve upon and to propose the use of orthogonal arrays in factorial designs. Experimental designs were initially applied in agricultural experiments. It was then introduced to the manufacturing industries, initially the chemical industry, after the statistical designed experiments had been further developed by statisticians such as [9]. See also the publications of [10-12].

In fractional factorial experiments, "Plans" which permit estimates of all main effects when all interaction effects are zero are called Resolution III plans. The expression "Resolution R plan" was originated by [13]. In the case where very few known lower-order interactions are non-zero, using search designs are quite useful. [14] proposed the theory of search designs that allows estimates of lower-order interaction effects and the search of non-zero higher-order interaction effects. This method permits inference about those non-zero higher-order interaction effects as well as on lower-order interaction effects.

With so many confounding patterns, it is helpful to have a way to classify the "degree of confounding". The term Resolution (R) describes the confounding with a number that signifies the number of factors that are tied together (confounded). A higher resolution number indicates less confounding.

According to [15], resolution IV designs are used more often because they seem to provide a good balance of useful information versus the number of trials required. For a small trial experiment, resolution III designs give a lot of information but can be misleading if there are too many confounding of factors. Resolution V designs are probably too costly for many situations.

Fundamentals of Experimental Designs: There are three basic principles in experimental design, namely, replication, randomization and blocking. Replication is simply repeating the basic experiment again. [16] reported that replication has two essential properties; it allows to estimate the experimental error and secondly, it permits to obtain a more precise estimate, if the sample mean is used to estimate the effect of a factor.

Randomization is very important principle. It is a procedure for running the experiments in random order. This is to avoid subjective decisions or bias and to minimize the effects of unexpected or uncontrollable changes.

Blocking is the process of grouping the trials of an experiment into subgroups or "blocks". Trials in the same block are performed at the same time or day. The idea is to improve the comparison of treatments by randomly allocating treatments within each block or subgroup. The need to block an experiment can occur under a variety of situations.

Benefits of Doe over Ofat Experimentation: DOE has many advantages over one-factor-at-a-time (OFAT) experimentation. It requires far few tests than OFAT for valid results. This means less resources for the amount of information obtained and crucial factor for an industry. Approximation of the effect of each factor is more precise through the use of more observations and thus reduced variability in the experiment. It detects the interactions among the factors considered for the experiment which cannot be found when changing one factor at a time. It is these interactions that most often prove to be the prime breakthrough improvements.

Another important aspect of this approach is the possibility of allowing sequential experimentation. It allows researchers to study the effect of a factor when the conditions of other factors vary. Thus, leads a better understanding of how the existing process inputs influence the performance of the process. In this way, factors that are critical can be identified. Once, the best settings for the critical factors are identified the performance of the processes can be better optimize than the OFAT.

In addition, it can be used as a tool for troubleshooting a manufacturing process such as determining the cause of high rejection rates. This approach is simpler, more efficient and will need fewer experimental runs to examine the impact of two or more factors on a response of interest. The estimation flexibility leads to substantial savings in run size.

A Case Study: To illustrate how interactions are captured in DOE, part of the experimentation data that had been obtained by [17] will be used. [17] used DOE to plan an experiment in a rubber glove production plant in Malaysia. The objective was to determine the effects of 8 variables on several important dimensions of rubber gloves. Fractional factorial was employed. The experiment was replicate twice, resulting in a total of 32 runs. All of the runs were conducted in a random order.

The tensile strength of the rubber glove was one of the critical dimension identified while three of the variables investigated were latex temperature (B) in the dip tank in degrees Centigrade, oven temperature before latex dip (G) and curing temperature profile (A). All the three factors were varied over two levels; a low and high level. The low and high levels for latex temperature were 25-26 and 29-30 degrees Centigrade.

The statistical software package Minitab was used to perform the regression modeling. The result of the regression fit to the data is presented in Table 1 from which deductions could be made. The analysis of variance (ANOVA) of the linear regression fit is presented in Table 2. The data was further analyzed using a graphical method known as main effect plot and interaction plot as shown in Table 3 and 4 respectively. The regression method produced a model that relates average tensile strength to latex temperature, curing temperature profile, oven temperature and their interaction.

Table 1 and 2 were analyzed to determine if the different factor levels affect tensile strength. The analysis of variance showed that the regression equation is very significant at $p<0.000$. The mean square regression is many times larger than the mean square error. The R^2 and R^2 (adjusted) values are 69.6% and 65.1% respectively. This means that the model could explain about 65.1% of the variability in the response about the mean tensile strength of 27.8MPa.

A complete response table for this data appears in Table 3. Factors having strong effects on mean tensile strength are shown in Table 1. We were tempted to interpret the main effects separately which in this case could be quite misleading. This is because of the presence of interaction effect between factor B and G. The estimated effects (based on contrast effects) of this interaction are further summarized in Table 4.

A graphical representation of the estimated effects of the interaction is shown in Figure 2. Figure 1 revealed that when oven temperature after coagulant dip (G) and curing temperature profile (A) are set at their low levels, the highest average tensile strength was achieved.

Table 1: Multiple regression result for mean tensile strength

Predictor	Coefficient	Standard Deviation	t-ratio	p-value
Constant	27.8462	0.1519	183.28	0.000
A	-0.4609	0.1519	-3.03	0.005
G	-0.8401	0.1519	-5.53	0.000
BD	0.3557	0.1519	2.34	0.027
BG	0.6203	0.1519	4.08	0.000

Table 2: Analysis of variance for mean tensile strength regression model

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	F-value	p-value
Regression	4	45.742	11.436	15.48	0.000
Error	27	19.944	0.739		
Total	31	65.686			

Table 3: Main effects of factors A and G on mean strength

	Factors	
Level	A (MPa)	G (MPa)
Average Response at High Level (2)	27.385	27.006
Average Response at Low Level (1)	28.307	28.686
Main Effect	0.915	1.680

Table 4: Interaction effect of BG on mean tensile strength

G_1 (Low)	G_2 (High)	
29.346MPa	26.425MPa	B_1 (Low)
28.027MPa	27.587MPa	B_2 (High)

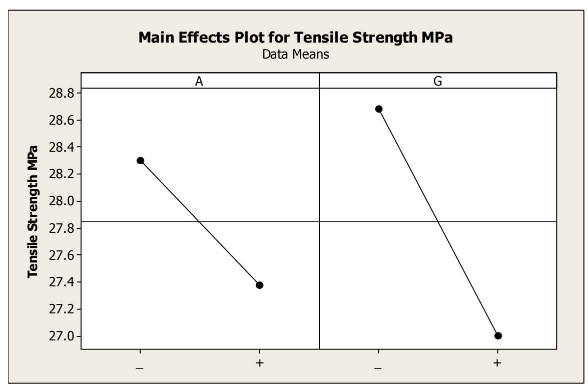

Fig. 1: Main effects for tensile strength

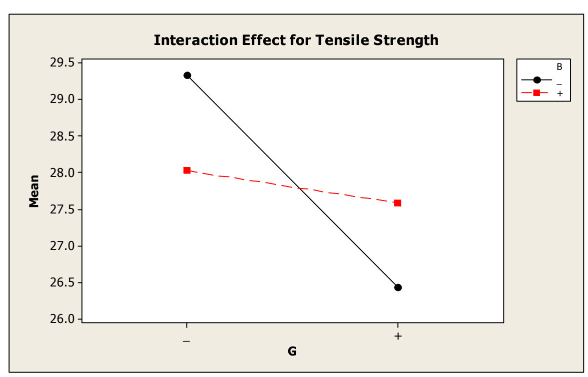

Fig. 2: Interaction effect for tensile strength

Table 5: Estimation of 95% Confidence Intervals for mean tensile strength

Factors	95% Confidence Intervals	Factors	95% Confidence Interval
CD	0.0178±0.689	DF	0.3428±0.689
C	0.0408±0.689	E	0.3927±0.689
B	0.0780±0.689	H	0.4884±0.689
BF	0.0948±0.689	BD	0.7114±0.689
AE	0.1781±0.689	A	0.9218±0.689
D	0.2656±0.689	BG	1.2406±0.689
F	0.2656±0.689	G	1.6847±0.689

However, a significant interaction was found between the main controllable factors, latex temperature (B) and oven temperature after coagulant dip (G). This interaction is important. Although factor B is not significant by itself, its interaction with factor G which is highly significant requires B to be considered. Figure 2 indicates that setting factors G and B at low is the optimal choice in order to maximize the mean strength. Though BD interaction was statistically significant at ($p \leq 0.027$) was not considered significant. This is because both B and D themselves were not statistically significant. According to [18], if the main factors are not significant, their interaction is not significant. The rest of the factors B, C, D, E, F and H have very small F-values and appear to be insignificant. They should be set at their most economical levels which are at low levels, since these are the least expensive levels. The best level settings or the preferred optimal settings are A_1 B_1 C_1 D_1 E_1 F_1 G.

We have also calculated the estimated standard error by replacing $ó^2$ by its estimate S^2 given by the error mean square in the ANOVA Table 1 and substituting these in equation below.

$$\overline{x}_{A1} - \overline{x}_{A2} \pm t_{n-1\ a/2}\sqrt{\frac{\sigma^2}{7}}$$

These intervals are approximately 95% Confidence Intervals as depicted in Table 5. This analysis confirms that there is some evidence that G, BG, A and BD interaction are important as they are the only factor estimates for which the intervals do not include zero. These findings tally with Table 1.

The preferred model for mean tensile strength is a simple mathematical model that depicts the relationship between the tensile strength of the examination gloves and the key factors and interactions which influence it. This would assist in predicting the tensile strength for various combinations of factor levels. The predicted model is:

$$\text{Mean Strength} = 27.8 - 0.461A - 0.840G + 0.356BD + 0.62BG$$

The predicted response was calculated by substituting the coded optimal setting of the factors. This would yield a value of 30.08MPa. From the results it suggests that the oven temperature before latex dip and the curing temperature profile have a very strong impact on the mean tensile strength of the examination gloves.

CONCLUSIONS

These investigations showed that the tensile strength response was significantly affected by the oven temperature before latex dip (G), curing temperature profile (A) and BG interaction. It appears that factor G has the largest influence, followed by factor A.

As we can see from the above discussion, a more informed decision can be made regarding the preferred settings of the controllable factors about the mean response. DOE is a valuable experimental strategy for designing and conducting experimentation. The information gained from such experiments can be used to improve the performance of the process and product. The technical knowledge acquired from the experiment has increased our understanding of the process behavior and our ability to monitor the process. This information is more reliable than that obtained from OVAT. DOE should therefore be part of every scientists and engineers' toolbox.

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